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PREDICTING METHOD OF FATIGUE STRENGTH OF CONCRETE BEAMS REINFORCED WITH PLURAL WELDED BARS

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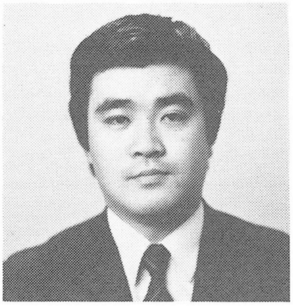

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SYNOPSIS

It is recognized that fatigue strength of concrete beams reinforced with welded bars decreases compared with that with normal deformed bars. It is also predicted that the extent of the decrease in fatigue strength of beams is changed according to the number of contained welded bars. The authors considered that the extent of the decrease in fatigue strength of beams would depend on not only the average fatigue strength of welded bars but also the variation. In order to clarify this assumption, fatigue tests of fifty concrete beams were carried out. In the tests the number of welded bars was changed from one to three. As a result of processing data by a statistical method, it was assured that the fatigue strength of concrete beams could be estimated fairly well if the average and the variation in the fatigue strength of welded bars would be obtained.

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PREDICTING METHOD OF FATIGUE STRENGTH OF CONCRETE BEAMS REINFORCED WITH PLURAL WELDED BARS

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ABSTRACT

Fatigue tests of fifty concrete beams reinforced with welded bars were carried out. The main purpose of these experiments was to estimate the extent of the decrease of the fatigue strength of concrete beams reinforced with plural welded bars as main reinforcement in comparison with the fatigue strength of concrete beams reinforced with only one welded bar. According to experimental and analytical study, it became clear that the distribution of the difference between a logarithm of measured fatigue strength of a welded bar and calculated one was the normal distribution. By considering this fact, the fatigue strength of concrete beams reinforced with three welded bars could be estimated with reasonable accuracy. This statistical approach might be furthermore utilized to predict the fatigue strength of concrete members reinforced with a large number of welded bars.

1. INTRODUCTION

As the standard specification of concrete structures of JSCE will be renewed and published in April 1986, the Concrete Committee in JSCE has been revising the design method of concrete structures taking account of the limit state design method. In this situation, the fatigue limit state is chosen as one of the limit states which should be examined in design procedures. In order to examine the fatigue limit state, it is necessary to clarify the fatigue characteristics of reinforced concrete members subjected to repeated loading. In this paper, the predicting method of the fatigue strength of reinforced concrete members subjected to repeated flexural loading is presented, especially the whole fatigue strength of concrete beams reinforced with plural welded bars.

The fatigue characteristics of reinforced concrete members may be normally dependent on the fatigue characteristics of steel rather than concrete. Therefore, it is important to clarify the fatigue characteristics of steel. Actually many research activities have been carried out in this field. In these researches, however, the attention was almost directed to estimate the fatigue strength of steel only as structural material. In the case of reinforced concrete beams subjected to repeated flexure, the

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whole fatigue strength of concrete beams reinforced with plural bars was not distinguished from the fatigue strength of steel itself. The reason why these two fatigue characteristics are regarded as identical may be due to the fact that the fatigue break of a deformed bar may normally occur at a foot of a lug and moreover a deformed bar may contain a large number of lugs. If the fatigue strength of deformed bars will be controlled by the fatigue strength of the weakest lug, as there are many lugs in one deformed bar itself, the fatigue strength of a deformed bar may be nearly close to the fatigue strength of the absolutely weakest lug. Consequently even if the number of bars will increase to some extent, the fatigue strength of the weakest lug in these bars may not decrease significantly.

The weak point of deformed bars for fatigue is a lug but on the other hand the weak point of welded bars for fatigue may be a welded part. As weak points of welded bars are very few in one welded bar, the effect of the increase of the number of welded bars is relatively large. Therefore, it is necessary to consider the decrease of the whole fatigue strength of reinforced concrete beams according to the increase of the number of welded bars.

As the fatigue break of welded bars may be controlled by the fatigue strength of a welded part and the fatigue strength of a welded part may be varied to some extent, it is reasonable to assume that the whole fatigue strength of concrete members reinforced with plural welded bars will not increase proportionally according to the number of welded bars and decrease to some extent in accordance with the increase of the number of welded bars.

In this research, fatigue tests of concrete beams reinforced with plural welded bars were carried out. Specimens were divided into three types. The first was the type in which a welded bar was employed as single. The second was the type in which welded bars were employed as plural. The last was the type in which a welded bar was not employed at all. Based on the experimental data, the fatigue characteristics of a welded bar itself was estimated. Finally the whole fatigue strength of concrete members reinforced with plural welded bars was predicted.

2. METHODS OF EXPERIMENTS

Three types of beams were tested. Two types of them contained welded bars as main reinforcement. Type 2 contained one deformed bar and one welded bar. Type 3 contained three welded bars only. In order to know the fatigue strength of a deformed bar itself, Type 1 beams which contained two deformed bars only were also tested.

The section of beams was a rectangle with 30cm depth and 20cm width for Type 1 and Type 2 or a rectangle with 30cm depth and 30cm

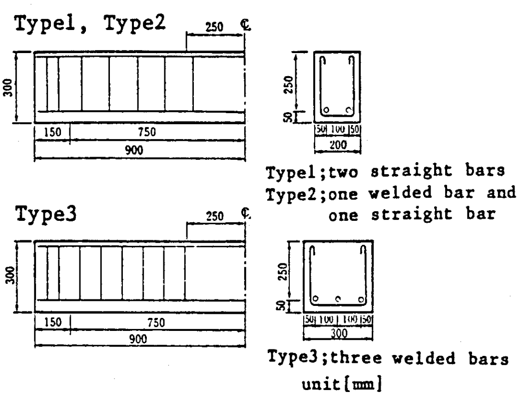

Fig.1 Detail of Specimens

Table 1 Mechanical Properties of A Bar

Size	Spec.	Area (cm²)	f_{sy} (kg/mm²)	f_{su} (kg/mm²)	E_s (kg/cm²)
D19	SD35	2.865	41.3	58.7	1.82×10^6

width for Type 3. The effective depth was 25cm for all types. The length of beams was 180cm for all types (Fig.1).

Deformed bars employed in this research were D19 (SD35), the yield strength was 41.3kg/mm^2 and the tensile strength was 58.7kg/mm^2 (Table 1). The lugs were inclined to the longitudinal direction of a bar and the inclination angle was 60 degrees. An arc was not created at the surface of a foot of a lug.

Welded bars were jointed by the automatic welding method. The welded part of a welded bar was placed at the center of the beam. As the purpose of these experiments was to make beams failed due to the fatigue break of bars, for beams not to be failed due to shear, vertical stirrups which were made by D10 deformed bars were arranged in a shear span at intervals of 12cm for Type 1 and Type 2 and of 9cm for Type 3. All of welded bars and vertical stirrups were offered by Mitsui Engineering & Ship Building Ltd.

Concrete was ready-mixed concrete of which the maximum size of aggregate was 20mm and the slump was 8cm. In order to remove the effect of the variation of the concrete strength during tests, loading tests were begun past 2 months after the casting of concrete. The compressive strength of concrete; fc' ranged between 327 and 360kg/cm^2 during fatigue tests (Table 2).

The loading method was a symmetrical two points loading and the span was 150cm and the shear span was 50cm (Fig.1). After the determination of the minimum and the maximum stress of bars, denoted $\sigma_{s.min}$ and $\sigma_{s.max}$, respectively, the applied fatigue loads were calculated neglecting tensile resistance of concrete and

Table 2 Results of Fatigue Tests

No.	Name	f_c' (kg/cm²)	$\sigma_{s.min}$ (kg/mm²)	$\sigma_{s.max}$ (kg/mm²)	N (×1 000)	Mode*	n**
1	I-1	347	5.0	20.1	2 080	N	0
			5.0	25.1	1 860	N	0
			5.0	30.1	556	S	0
2	I-2	343	5.0	30.1	953	B	0
3	I-3	360	5.0	25.1	2 120	N	0
			5.8	28.9	364	S	0
4	I-4	356	5.8	28.9	810	B	0
5	I-5	351	5.5	27.6	940	B	0
6	I-6	352	5.3	26.3	2 210	B	0
7	I-7	327	14.7	36.8	232	B	0
8	I-8	332	14.7	36.8	646	B	0
9	I-9	360	9.4	31.5	3 550	B	0
10	I-10	360	9.9	33.0	1 323	S	0
11	II-1	347	5.0	20.1	2 104	B	1
12	II-2	347	5.0	25.1	264	B	1
13	II-3	343	4.4	22.1	1 081	B	1
14	II-4	342	5.0	25.1	591	B	1
15	II-5	360	4.4	22.1	611	B	1
16	II-6	356	5.0	20.1	1 806	B	1
17	II-7	356	5.0	25.1	268	B	1
18	II-8	346	10.0	25.1	897	B	1
19	II-9	351	10.0	25.1	790	B	1
20	II-10	352	8.7	21.7	762	B	1
21	II-11	352	8.7	21.7	1 372	B	1
22	II-12	348	11.8	29.4	565	B	1
23	II-13	327	7.4	18.4	2 200	N	1
			8.0	20.1	1 160	B	1
24	II-14	327	7.4	18.4	2 160	N	1
			8.0	20.1	1 520	B	1
25	II-15	332	7.3	24.4	555	B	1
26	II-16	332	5.6	18.6	2 190	N	1
			6.4	21.5	1 763	B	1
27	II-17	357	7.4	29.7	163	B	1
28	II-18	357	6.7	26.8	163	B	1
29	II-19	360	6.0	20.1	1 595	B	1
30	II-20	360	8.7	21.7	1 520	B	1
31	III-1	347	5.0	20.0	594	B	3
32	III-2	347	5.0	22.1	267	B	3
33	III-3	343	4.4	22.1	349	B	3
34	III-4	343	3.3	16.3	1 144	B	3
35	III-5	360	3.1	15.3	965	B	3
36	III-6	356	3.3	16.3	1 087	B	3
37	III-7	356	5.0	20.0	306	B	3
38	III-8	346	3.1	15.3	1 924	B	3
39	III-9	351	8.7	21.8	805	B	3
40	III-10	351	8.7	21.8	658	B	3
41	III-11	352	7.4	18.4	896	B	3
42	III-12	348	7.4	18.4	1 254	B	3
43	III-13	327	6.7	16.7	2 130	N	3
			7.0	17.5	2 130	B	3
44	III-14	332	6.0	18.4	1 503	B	3
45	III-15	332	5.2	17.2	1 850	B	3
46	III-16	332	10.8	27.1	119	B	3
47	III-17	357	4.7	15.7	2 230	N	3
			8.2	22.3	249	B	3
48	III-18	357	5.6	18.6	1 533	B	3
49	III-19	360	8.2	22.3	658	B	3
50	III-20	360	8.0	20.0	1 290	B	3

* B:Break, N:Not Break, S:Shear Failure

**n:number of welded bars

assuming that the ratio of Young's modulus of steel to concrete was 7.

To make sure of the occurrence of the intended strain of bars, strain of bars was measured by electric strain gauges attached to the bottom surface of bars in Type 2 and Type 3. Avoiding a welded part, strain gauges were attached at the section 20cm apart from the center of a beam. The small artificial slit was arranged on the bottom surface of a beam where strain gauges were placed so that flexural cracks might occur at this section.

The range of stress; σ_r (= $\sigma_{s.max}$ - $\sigma_{s.min}$) was calculated from measured strains. The calculated range of stress was compared with the prescribed value. As a result of comparison, the average ratio of the calculated range of stress to the prescribed value in the whole type was 1.00 and 0.94 and the coefficient of variation was 0.049 and 0.062 for Type 2 and Type 3, respectively. As the coefficient of variation in any types was sufficiently small, the prescribed stress value of bars was directly used to process the experimental data in this research.

3. RESULTS OF EXPERIMENTS

3.1 EXPERIMENTAL RESULTS OF TYPE I

In order to estimate roughly the fatigue strength of deformed bars employed in the experiments, fatigue tests of a deformed bar itself were carried out previous to welded bars. Ten concrete beams reinforced with two deformed bars only were tested as Type 1. The applied range of stress; σ_r was from 15.1 to 25.1kg/mm^2 and the stress ratio of the minimum to the maximum ; r (= $\sigma_{s.min}/\sigma_{s.max}$) was from 0.17 to 0.40. These values were determined at random. Experimental results are represented in Table 2. Three beams in Type 1 failed in shear due to the propagation of a diagonal crack up to the upper surface of a beam before the break of deformed bars. As other two beams in Type 1 had not failed by the arranged repeated loading up to 2 million cycles, the range of stress was increased after 2 million cycles. The rest except for beams which failed in shear had failed due to the break of deformed bars by the arranged repeated loading. Experimental data of fatigue failure due to the break of deformed bars are plotted in Fig.2. In Fig.2, the relationship of log σ_{ro} and log N is shown, where σ_{ro} is the range of stress in the perfectly one directional loading. Following the modified Goodman's theory, σ_{ro} can be calculated by eq.(1), where f_{su} is the tensile strength of a deformed bar.

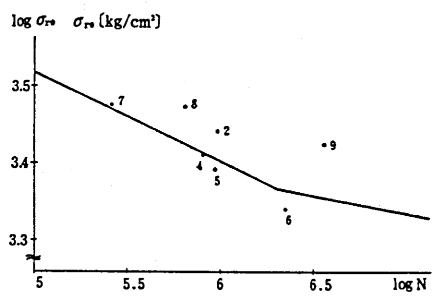

Fig.2 Results of Fatigue Tests of Typel and S-N Curve[1]

$$\sigma_{ro} = \frac{\sigma_{s.max} - \sigma_{s.min}}{1 - \sigma_{s.min} / f_{su}} \quad (1)$$

The S-N curve in Fig.2 is the proposed equation for the fatigue strength of deformed bars by the authors[1]. This equation represents the mean value of the fatigue strength. According to Fig.2, experimental data of Type 1 appear somewhat larger than the proposed equation. Except for I-9, however, the experimental data accord well with the proposed equation.

Strictly speaking, the experimental data of Type 1 should show the weaker fatigue strength in two deformed bars employed in a beam. Actually the whole fatigue strength of Type 1 beams did not differ largely from the fatigue strength of a deformed bar predicted by the proposed equation. This can be explained as follows. As a single deformed bar contains a large number of lugs, the fatigue strength of the weakest lug in a bar may be nearly close to the fatigue strength of the absolutely weakest lug. Consequently even if the number of deformed bars becomes two, the whole fatigue strength is not much influenced by the increase of the number of bars.

As the fatigue strength of deformed bars employed in the experiments can be estimated fairly well by the proposed equation, it may be concluded that deformed bars used are not specially strong or weak and they have the average fatigue strength.

3.2 EXPERIMENTAL RESULTS OF TYPE II

Twenty beams were tested as Type 2. As the fatigue strength of welded bars was relatively weaker than that of deformed bars, the range of stress was arranged smaller compared with deformed bars. σ_r was arranged from 11.0 to 22.3 kg/mm^2. The stress ratio of the minimum to the maximum was from 0.20 to 0.40. These values were determined at random. Experimental results are shown in Table 2. All of Type 2 beams failed due to the break of a welded bar. As three beams did not fail due to the arranged repeated loading up to 2 million cycles, the range of stress was increased after 2 million cycles of repeated loading.

Type 2 beams contained two bars. One was a welded bar and the other was a deformed bar. As the fatigue strength of welded bars is somewhat smaller than that of deformed bars, a welded bar will break first due to repeated loading and then the range of stress of a remained deformed bar will increase twice the initial value. Therefore, after the break of a welded bar, beams may fail instantly. The loading cycles up to the whole failure of Type 2 beams should be almost equal to the loading cycles up to the break of a welded bar.

The relationship of log σ_{ro} and log N is plotted in Fig.3. The S-N curve in Fig.3 was calculated as follows. The curve for less than or equal to 2 million cycles was obtained as the regression line for the experimental data. However, it was very difficult to determine the curve for more than 2 million cycles, because the experimental data were

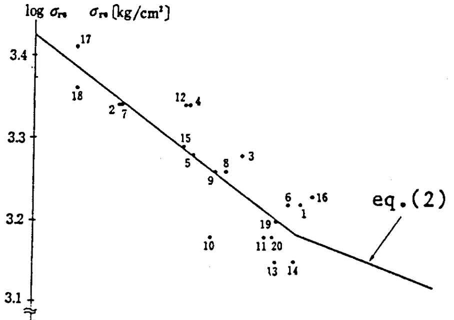

Fig.3 Results of Fatigue Tests of Type2 and S-N Curve(eq.(2))

very few. It was observed that the tendency of data for more than 2 million cycles was somewhat different from that for less than or equal to 2 million cycles. Namely the slope of the data for more than 2 million cycles on Fig.2 was relatively flatter than the slope of the data for less than or equal to 2 million cycles. In the case of deformed bars, the authors already proposed 1/2 of the slope for less than or equal to 2 million cycles as the slope for more than 2 million cycles. Therefore, in the case of welded bars, the slope for more than 2 million cycles was assumed to be half of the slope for less than or equal to 2 million cycles. Finally the fatigue strength equation for Type 2 is derived as follows.

$$\log \sigma_{ro} = 4.35 - 0.185 \log N \qquad N \leq 2 \times 10^6$$
$$\log \sigma_{ro} = 3.77 - 0.093 \log N \qquad N > 2 \times 10^6 \qquad (2)$$

The unit of σ_{ro} is kg/cm^2.

According to Fig.3, it is admitted that the experimental data of Type 2 distribute along the S-N curve almost uniformly.

3.3 EXPERIMENTAL RESULTS OF TYPE III

Twenty beams were tested as Type 3. σ_r was from 11.3 to 20.0 kg/mm^2. σ_r was arranged somewhat smaller than Type 2. r was from 0.20 to 0.40. These values were also determined at random. Experimental results are shown in Table 2. All of Type 3 beams failed due to the break of welded bars. As two beams did not fail due to the arranged repeated loading up to 2 million cycles, the range of stress was increased after 2 million cycles of repeated loading.

Type 3 beams contained three welded bars. If the fatigue strength of welded bars did not scatter at all, the fatigue strength of Type 3 beams could be estimated quite easily by the experimental data of Type 2. In fact the fatigue strength of Type 3 could not be predicted by the experimental data of Type 2.

The failure mechanism of Type 3 is considered as follows. As welded bars have the scatter in the fatigue strength, the weakest one in three welded bars will break first. Then the range of stress of the remained two welded bars will increase to 1.5 times the initial value. After the break of the weakest one, the remained two welded bars may break one after the other and finally the beam will fail. The loading cycles up to the whole failure of Type 3 beams will be nearly equal to the loading cycles up to the break of the weakest welded bar.

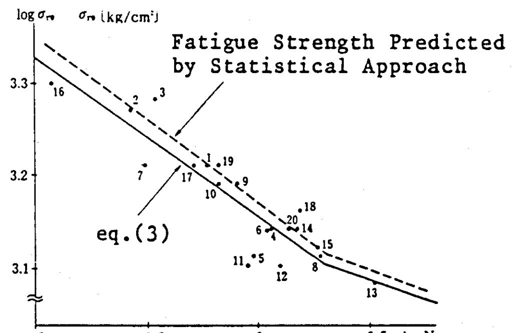

Fig.4 Results of Fatigue Tests of Type3 and S-N Curve(eq.(3))

As the whole fatigue strength of Type 3 beam may be dependent on the weakest welded bar, the fatigue strength of Type 3 beam can not be predicted easily by the experimental data of Type 2 which should represent the average fatigue strength of one welded bar.

The relationship of log σ_{ro} and log N of Type 3 beams is plotted in Fig.4. The curve in Fig.4 is the regression line derived from the experimental data of Type 3 by the same method as Type 2. The slope of the regression line is also changed at 2 million cycles. Finally the fatigue strength equation for Type 3 is obtained as follows.

$$\log \sigma_{ro} = 4.20 - 0.174 \log N \qquad N \leq 2 \times 10^6$$
$$\log \sigma_{ro} = 3.65 - 0.087 \log N \qquad N > 2 \times 10^6 \qquad (3)$$

The unit of σ_{ro} is kg/cm^2.

From Fig.4, it is observed that the number of the experimental data which are below the regression line are less than the number of data which are beyond the line and a few data, for example, III-5, III-11 which are fairly weaker than the rest make the regression line decrease. It is also admitted that the scatter of data to the line is relatively small except for these few data.

4. THE FATIGUE STRENGTH OF THE WEAKEST ONE IN THREE WELDED BARS

4.1 QUANTITATIVE ESTIMATION OF THE FATIGUE STRENGTH DISTRIBUTION OF A WELDED BAR

It is considered that the experimental data of Type 2 may represent the average fatigue strength of a welded bar. If the fatigue strength of Type 3 can be predicted by the information of Type 2, it will be very effective in design for fatigue. As it seemed that the fatigue strength of Type 3 beams was dependent on the fatigue strength of the weakest one in three welded bars, an effort was made to predict the fatigue strength of Type 3 beams by the statistical approach.

Before the prediction of the fatigue strength of Type 3, it is necessary to clarify the distribution of Type 2 data. As the loading conditions of Type 2 were not uniform but random, the fatigue strength or the loading cycles up to failure can not be directly used in order to examine the fatigue strength distribution of Type 2.

In this research, the attention was paid to the difference; x between a logarithm of σ_{ro} which was already prescribed in tests and a logarithm of $\sigma_{ro.cal}$ which was calculated by eq.(2) using the loading cycles up to failure.

$$x = \log \sigma_{ro} - \log \sigma_{ro.cal} \qquad (4)$$

The unit of σ_{ro} and $\sigma_{ro.cal}$ is kg/cm^2.

As x is the difference of logarithms, 10^x represents the ratio of σ_{ro} to $\sigma_{ro.cal}$. For example, if x is negative, 10^x becomes less than 1 and this implies that experimental data is weaker than predicted value.

By the examination of the distribution of x, the scatter of experimental data from the fatigue strength equation eq.(2) was checked.

Obtained x values for each experimental data are shown in Table 3. As it was expected that the distribution of x would follow the normal distribution, the frequency distribution table of x was drawn and the results were plotted on the paper of the normal probability (Fig.5).

According to Fig.5, it is admitted that the distribution of x is approximately a straight line on the paper of the normal probability. Therefore, the distribution of x can be assumed as the normal distribution. The average and the variance of the population of x can be calculated by the statistical method[2].

As it is considered that twenty data of x obtained from Type 2 tests are the sample of the population of x, the average μ and the variance σ^2 can be calculated as follows.

Table 3 x of Type 2

Specimen	x	Specimen	x
II- 1	0.081	II-11	−0.071
II- 2	−0.012	II-12	0.131
II- 3	0.111	II-13	−0.130
II- 4	0.137	II-14	−0.093
II- 5	0.005	II-15	0.008
II- 6	0.058	II 16	0.120
II- 7	−0.009	II-17	0.049
II- 8	0.026	II-18	−0.069
II- 9	0.003	II-19	−0.015
II-10	−0.180	II-20	−0.052

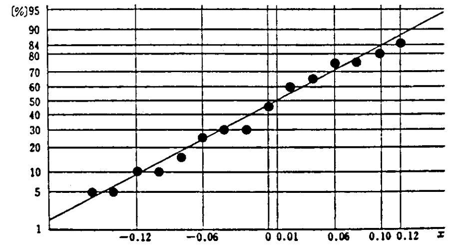

Fig.5 x on The Paper of The Normal Probability

$$\mu = \frac{\Sigma x_i}{20} \quad , \quad \sigma^2 = \frac{\Sigma (x_i - \mu)^2}{20 - 1}$$

After the calculation, μ became equal to 0.00 and σ^2 became equal to 0.088^2. The distribution of x could be assumed as the normal distribution of which the average was 0.00 and the variance was 0.088^2. According to this assumption, the probability density function of x is represented as eq.(5).

$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} \exp \left[- \frac{(x - \mu)^2}{2\,\sigma^2} \right]$$

$$= \frac{1}{0.221} \exp \left[- \frac{x^2}{0.0155} \right] \qquad (5)$$

4.2 THE FATIGUE STRENGTH OF THE WEAKEST ONE IN THREE WELDED BARS

When any one welded bar breaks in three welded bars, the range of stress of the remained two welded bars will increase 1.5 times the initial stress range. As log N up to the whole failure of a beam can not increase largely after the break of the weakest welded bar under this severe stress condition, the fatigue characteristics of Type 3 beam can be assumed as a serial system[3].

If the distribution of x_{min} which is the minimum value of x in three welded bars can be known, the average fatigue strength of the whole beam in which three welded bars are employed will be predicted easily.

As the probability density function of x is represented by eq.(5), the probability density function of x_{min} can be represented by eq.(6).

$$g(x_{min}) = 3\ f(x)\ [\ R(x)\ ]^2 \qquad (6)$$

where, $R(x) = 1 - F(x)$

$$F(x) = \int_{-\infty}^{x} f(t)\ dt$$

Eq.(6) represents the case in which the difference of the fatigue strength of any one welded bar from the average fatigue strength of a welded bar is x and the fatigue strength of the remained two welded bars are larger than x. By substituting actual numerical values into eq.(6), the distribution of $g(x_{min})$ can be calculated numerically. Fig.6 shows the shape of $g(x_{min})$. In Fig.6 the shape of f(x) is drawn at the same time.

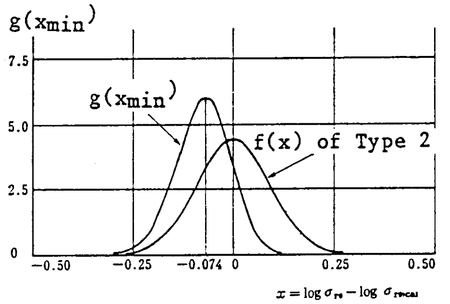

Fig.6 The Probability Density Function of x_{min} in Three Welded Bars, $g(x_{min})$

As being expected instinctively, the average of the distribution of $g(x_{min})$ is smaller than the average of the distribution of f(x) and the variance of $g(x_{min})$ is also smaller than the variance of f(x), that is, the shape of $g(x_{min})$ concentrates rather than f(x). The distribution of $g(x_{min})$ is not the normal distribution strictly. However, from Fig.6, it is admitted that the right and left of the distribution of $g(x_{min})$ is nearly symmetrical and the distribution is very close to the normal distribution.

As the average of $g(x_{min})$ was calculated as -0.074, the perfectly one directional fatigue strength; σ_{ro} of the weakest one in three welded bars should be smaller than the calculated average fatigue strength of one welded bar; $\sigma_{ro,cal}$ which was calculated by eq.(2). The extent of the decrease in the fatigue strength is calculated as follows.

$$10^{-0.074} = 0.843$$

Namely the extent of the decrease is predicted as 15.7%.

5. COMPARISON BETWEEN CALCULATED AND EXPERIMENTAL VALUES OF THE FATIGUE STRENGTH OF TYPE III BEAM

When the weakest one in three welded bars may break, if tensile force which the weakest one has maintained will be transferred to the remained two welded bars and they can resist the increase of the range of stress, the whole beam will not fail instantly. The loading cycles up to the whole failure of Type 3 beam is calculated by following procedure. At first it is assumed that welded bars may break by turns from a weaker one. The extent of the decrease of the fatigue strength of the weakest one from the average fatigue strength can be calculated by eq.(6). After the break of the weakest one, total tensile force will be redivided to the remained welded bars. In this redivision, the effect of the preceding stress hysteresis is estimated by the Miner's law. By performing the calculation in turn, the loading cycles up to the whole failure of Type 3 beam can be obtained.

The broken line in Fig.4 shows the fatigue strength of Type 3 beam predicted by this method. By this statistical approach, it is estimated that the fatigue strength of Type 3 beam decreases compared with the fatigue strength of Type 2 beam. The extent of the decrease is 15.3% and 14.9% when log N is 5 and 6, respectively. As the extent of the decrease of the fatigue strength of the weakest welded bar from the average fatigue strength is 15.7% as shown in Section 4.2, it is realized that after the break of the weakest welded bar the whole beam can resist the increase of the loading cycles to some extent.

The extent of the decrease of the fatigue strength of Type 3 compared with Type 2 was estimated as about 15%. On the other hand, the extent of the decrease observed by the experiments can be calculated by the difference between eq.(2) and eq.(3). By the experiments, the extent of the decrease is 19.6% and 17.6% when log N is 5 and 6, respectively. The extent of the decrease obtained by the experiments is somewhat larger than the calculated value. This tendency is also observed in Fig.4.

As already explained in Section 3.3, the regression line in Fig.4 was decreased by a few especially weaker data, that is, III-5 or III-11,etc. The reason why these data were weaker than the rest is uncertain. However, as represented in Fig.4, for the rest data except for these weaker ones the predicted fatigue strength according to this statistical approach can estimate the experimental results with reasonable accuracy.

6. TRIAL CALCULATION OF THE FATIGUE STRENGTH OF CONCRETE MEMBERS REINFORCED WITH A LARGE NUMBER OF WELDED BARS

In real construction, it is usual for a large number of deformed bars to be welded and jointed at just the same section of a member. In this Chapter, the whole fatigue strength of concrete members reinforced with a large number of welded bars will be discussed.

When n number of welded bars are employed at just the same section, the probability density function of x_{min} is generally represented by eq.(7).

$$g(x_{min}) = n\, f(x)\, [\, R(x)\,]^{n-1} \qquad (7)$$

If the average of $g(x_{min})$ represented by eq.(7) can be calculated numerically, the average value of the fatigue strength of the weakest one in n number of welded bars will be obtained. For trial, the variation of the fatigue strength of the weakest welded bar was examined. In this trial calculations, the fatigue characteristics of welded bars employed in this research was used. In calculation, after the break of welded bars, the effect of the preceding stress hysteresis was also estimated by the Miner's law. Calculations were continued in turn up to the break of the last welded bar.

Fig.7 shows the relationship of the loading cycles up to the whole failure of a reinforced concrete member and the number of welded bars. In this figure, the initial value of perfectly one directional range of stress, that is, $\sigma_{ro} = \sigma_{ro.i}$ is a parameter. According to Fig.7, it is admitted that for arbitrary $\sigma_{ro.i}$ the fatigue strength would decrease rapidly accompanied with the increase of the number of welded bars from one to three. The reason can be explained as follows. When the number of welded bars are few, the effect of the variance of the fatigue strength of welded bars is dominant and even if the number of welded bars may increase slightly, the extent of the decrease of the fatigue strength of the weakest welded bar will become larger and finally this effect will have a large influence on the decrease of the whole fatigue strength.

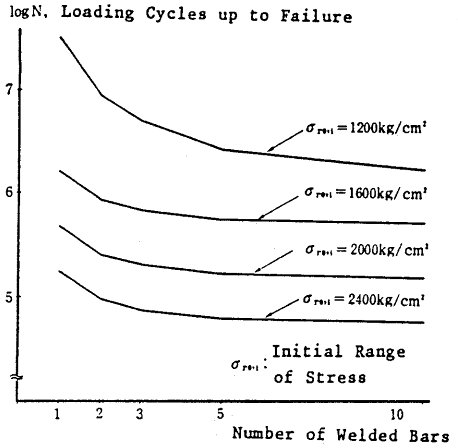

Fig.7 Relationship between Fatigue Strength and Number of Welded Bars

When the number of bars furthermore increase, for example, up to 10, if $\sigma_{ro.i}$ is larger to some extent, the fatigue strength is nearly equal to the case of three bars. With the increase of welded bars, the fatigue strength itself of the weakest one should decrease certainly. However, the range of stress of the remained bars will not increase significantly by the redivision after the break of the weakest one. Consequently the break of the weakest one does not connect directly with the failure of the whole reinforced concrete member.

According to trial calculation, in the case of welded bars employed in this research, if the number of welded bars becomes more than or equal to five, the decrease of the fatigue strength due to the increase of the number of welded bars can be almost prevented.

The number of welded bars which prevents the decrease of the fatigue strength of the whole member can not be represented generally. It naturally depends on the fatigue characteristics of employed welded bars. If the fatigue characteristics of employed welded bars is clearly known, by performing this statistical approach, the fatigue strength of the whole member can be easily predicted.

7. CONCLUSION

(1) According to the fatigue tests of Type 3 beams in which three welded bars were employed, the fatigue strength of Type 3 beam was clearly weaker than the fatigue strength of Type 2 beam in which only one welded bar was employed. The fatigue strength of Type 2 and Type 3 was represented by each regression lines derived from the experimental data, that is, eq.(2)

and eq.(3), respectively. When the perfectly one directional range of stress; σ_{ro} was $20kg/mm^2$ and $15kg/mm^2$, the loading cycles up to the whole failure decreased from log N = 5.7 for Type 2 to 5.2 for Type 3 and from log N = 6.4 for Type 2 to 5.9 for Type 3, respectively.

(2) It was found that the distribution of the difference between a logarithm of the fatigue strength of any one welded bar and a logarithm of the average fatigue strength calculated by the regression line followed nearly the normal distribution.

(3) By calculating the average of the minimum value of this difference in three bars, the fatigue strength of the weakest one in three welded bars could be predicted. After the break of the weakest one, by considering the redivision of stresses and the effect of the preceding stress hysteresis, the loading cycles up to the whole failure of Type 3 beam could be calculated. By comparing this calculated fatigue strength with the experimental data of Type 3, it was admitted that the fatigue strength of Type 3 beams could be predicted with reasonable accuracy.

(4) By trial calculation for the case in which a large number of welded bars were employed, it was found that when the number of welded bars increased largely, the decrease of the fatigue strength due to the increase of the number of welded bars could be limited fairly well.

REFERENCES

[1] Niwa,J., Maeda,S. and Okamura,H.,"Proposed Design Equation for Fatigue Strength of Deformed Bars," Proc. of JSCE, No.354/v-2, Feb. 1985

[2]Matsumoto,Y.,"Analytical Method in Civil Engineering (1)," Giho-do Publishing Co. Ltd., 1975

[3]Kitagawa,K.,"Primer to Probabilistic Engineering," Corona Publishing Co. Ltd., 1981