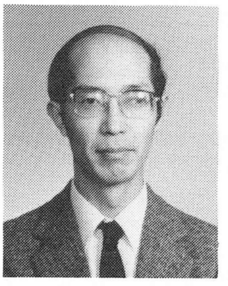
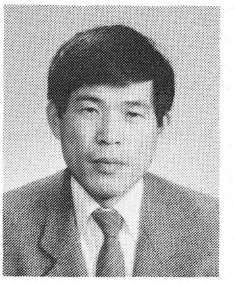
CONCRETE LIBRARY OF JSCE NO. 5, AUGUST 1985

STUDY ON FATIGUE OF CONCRETE UNDER VARIABLE REPETITIVE COMPRESSIVE LOADING

(Rearrangement in English of papers in Journal of the Society of Materials Science Japan, Vol. 31, No. 350, Nov. 1982 and Vol. 32, No. 353, Feb. 1983)



Shinzo NISHIBAYASHI   Kenji SAKATA   Kiyoshi YAMURA   Shoichi INOUE

SYNOPSIS

Experimental research on the behaviour of concrete subjected to variable repetitive loads is described. Constant-amplitude fatigue tests, two- and three-stage fatigue tests and variable load fatigue tests are performed. Fatigue lives are evaluated by Miner sum (M) and the distributions of M and $\bar{M}$ (mean value of M) are discussed. The applicability of Miner's hypothesis to fatigue of concrete is also examined.

S. Nishibayashi is a professor of civil engineering at Tottori University, Tottori, Japan. He recieved his Doctor of Engineering Degree from Kyoto University in 1968. His research interests cover fatigue properties of concrete and concrete members, durability of concrete, the properties of superplasicized concrete and alkali-aggregate reaction. He was awarded a JSCE prize(Yoshida Prize) in 1969 for a study of the properties of artificial lightweight aggregate concrete. Also, he and his coauthors received an award for this paper from JSMS in 1984. Dr. Nishibayashi is a member of ACI, JSCE, JSMS, and JCI.

K. Sakata is an associate professor of civil engineering at Okayama University, Okayama, Japan. He received his Doctor of Engineering Degree from Kyoto University in 1976. His reseach interests include problems concerning prediction of shrinkage and creep of concrete and fatigue properties of concrete. He is a member of JSCE, JCI, and JSMS.

K. Yamura is an associate professor of civil engineering at Tottori University. He recieved his Doctor of Engineering Degree from Kyoto University in 1980. His research interests include dynamic behaviours of reinforced concrete members, durability of concrete, and fatigue properties of concrete. He is a member of JSCE, JSMS, JCI, and ACI.

S. Inoue is a lecture of civil engineering at Tottori University. He recieved his Master of Engineering Degree from Gifu University in 1975. His research interests include the properties of fatigue of concrete and concrete members. He is a member of JSCE, JSMS, and JCI.

1. Introduction

Loads acting on concrete structures are completely random in nature with respect to frequency, magnitude and order of loading. In order to design concrete members and structures rationally and to ensure that their functions will be amply demonstrated throughout their service lives, it is necessary to investigate the fatigue characteristics of concrete under variable repetitive loading. It is, however, difficult to perform a laboratory fatigue test which reproduces an actual environmental situation of random loading. Therefore, a method of predicting fatigue life of concrete on the basis of constant-amplitude fatigue test must be established. Miner's hypothesis is one such procedure, and is often adopted in fatigue design of metal structures[1].

Very few investigations on fatigue of concrete under random loading have been carried out, and therefore, it is not clear whether or not Miner's hypothesis is appropriate for estimating fatigue life of concrete.

The object of this study is to establish a method of estimating fatigue life of concrete under variable repetitive compressive loading. In this study, two- and three-stage fatigue tests (Series A) and multi-stage fatigue tests (Series B, hereinafter called variable load fatigue test) are carried out. In the variable load fatigue test, a test specimen is loaded with a number of successive stress blocks. The relationship between stress level and frequency in a stress block can be expressed by a number of probability density functions.

The fatigue lives obtained by these fatigue tests are evaluated by using the "Miner sum: M" which will be defined later, and the probability distribution of the fatigue lives are discussed. Furthermore, the possibility of predicting fatigue life of concrete under random loading based on the result of the constant-amplitude fatigue test, that is, the applicability of Miner's hypothesis to fatigue of concrete is discussed.

2. Experiments

2.1 Materials and Specimens

The cement used in these tests was ordinary portland cement. The coarse aggregate was crushed stone and the fine aggregates were river sand and marine sand. Table 1 shows the mechanical properties of the aggregates. The mix proportions and properties of concretes are presented in Table 2. The specimens were cylinders, 10 cm in diameter and 20 cm in height for Series A, and 7.5 cm in diameter and 15 cm in height for Series B. They were cured in water maintained at 20 ± 1°C for 28 days after casting. Thereafter, the specimens were stored in the laboratory under normal conditions until the age of testing (more than 100 days).

Table 1 Mechanical properties of aggregate

Series	Coarse aggregate				Fine aggregate		
	Max.size of agg. (mm)	F.M.	Specific density	Absorption (%)	F.M.	Specific density	Absorption (%)
A	20	7.13	2.67	0.91	2.73	2.59	1.88
B	15	6.23	2.69	0.78	2.59	2.59	2.03

Table 2 Mix proportions and compressive strengths of concrete

(a) Mix proportions

Series	Slump (cm)	Air (%)	W/C (%)	S/a (%)	Unit weight(kg/m^3)				Admixture
					W	C	S	G	
A	5	5	66	43	165	250	786	1074	Pozzolith No.8
B	5	5	61	46	170	280	823	1004	Pozzolith No.70

(b) Compressive strength of concrete

Series	Batch No.	28 days Comp. strength (Mpa)	Age at test Comp. strength (Mpa)	Numbers of specimen	Coeff. of vari.(%)
A	1	19.1	22.6	10	4.4
	2	20.9	25.0	8	5.8
	3	20.4	24.5	10	4.9
	4	21.5	24.7	10	5.5
	5	20.2	23.8	10	3.9
B	6	28.1	40.5	8	3.2
	7	29.3	40.1	6	3.3
	8	31.9	42.2	6	2.3

2.2 Test procedure

The tests consisted of a constant-amplitude fatigue test, two- and three-stage fatigue tests and six kinds of variable load fatigue tests. These tests were carried out in the test setup with a 20-ton pulsator (Series A) and a 25-ton servo-hydraulic control fatigue testing machine (Series B). The relation between load and time is shown by a sine waveform. The tests were performed with a constant frequancy of 5 Hz. The magnitude of the repetitive stress was determined by the amplitude of stress ratio S to the static strength for each batch (Series A : 22.5 - 25.2 MPa, Series B : 40.1 - 42.6 MPa). Figs. 1 and 2 show the magnitudes and frequencies of the repetitive loads.

The distributions of the upper stresses in Fig.2 are the models of distributions of cumulative frequency of random loading to which actual concrete structures are subjected. The triangular distribution (T) is that of frequency of wheel load more than 11 tons on an actual highway bridge [2]. The normal distribution (N) is determined by the frequency distribution of a wheel in an automobile lane passing through a cross section perpendicular to the axis of the bridge[2]. The expornential distributions (E and F) are the models established by the relation between wave height and its frequency in an offshore environment.

As an example, the loading procedure in the case of the triangular distribution is explained as follows. The upper stress ratios are 72, 74.5, 77, 79.5 and 82% and the order of loading is decided by a five-figure random number. The frequency of each stress ratio in a stress block is shown in Fig. 2, and the sum of repetitions of loading in the first stress block is 10000.

The lower stress ratio was fixed at 10 percent of the static strength of concrete in all types of fatigue tests.

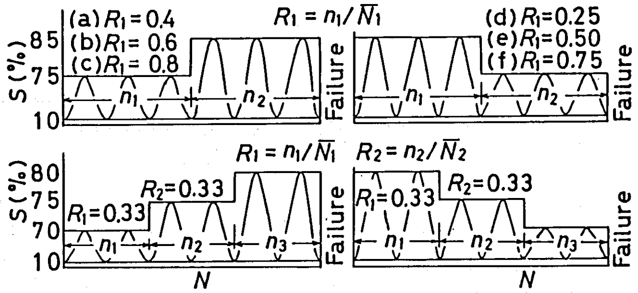


Fig. 1 Details of test programs
(Two- and three-stage fatigue test)

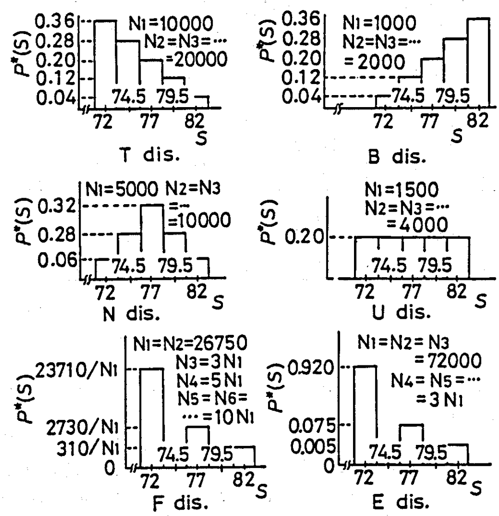


Fig. 2 Details of test programs
(Variable load fatigue test)

3. Results and Discussions

3.1 Constant-Amplitude Test

It is well known that the distribution of fatigue life of concrete may be considered as a logarithmic normal distribution[3].

Fig.3 shows the relationships between fatigue lives for individual stress ratios and survival probabilities calculated by Eq. (1) on logarithmic normal probability paper.

$$p = 1 - r/(1+l)\ , \qquad t = \phi^{-1}(p) \qquad \text{------------------- (1)}$$

where l is total number of specimens under the same test condition, r is the order statistic or the ordinal number of fatigue lives arranged in order from young to old, and t is distance from symmetric axis of normal distribution curve.

Table 3 Results of two- and three-stage fatigue tests.

r	Two-stage stress															Three-stage stress			
	p	S_1=85% → S_2 =75%						p	S_1 =75% → S_2 =85%							S_1=80% → S_3=70%		S_1=70% → S_3=80%	
		R_1 =0.25, n_1 =160		R_1 =0.50, n_1 =320		R_1 =0.75, n_1 =480			R_1 =0.40, n_1 =64340		R_1 =0.60, n_1 =96500		R_1 =0.80, n_1 =128670			R_1 =R_2=0.33, n_1 =3040, n_2 =53610		R_1 =R_2=0.33, n_1 =170080, n_2 =53610	
		n_2 (x10)	M	n_2 (x10)	M	n_2 (x10)	M		n_2 (x10)	M	n_2 (x10)	M	n_2 (x10)	M		n_3 (x10)	M	n_3 (x10)	M
1	0.957	*12	0.19	*12	0.19	*7	0.11	0.952	*3250	0.20	*3627	0.23	*3693	0.23		*140	0.15	*9555	0.19
2	0.913	*16	0.25	*17	0.27	*14	0.22	0.905	*3950	0.25	*4160	0.26	*4070	0.25		*243	0.27	*13219	0.26
3	0.870	19	0.25	*20	0.31	*17	0.27	0.857	*4770	0.30	*5810	0.36	*5530	0.34		*250	0.27	*16476	0.32
4	0.826	124	0.26	*22	0.34	*24	0.38	0.810	*5988	0.37	*6436	0.40	*5623	0.35		*295	0.32	**827	0.38
5	0.783	169	0.26	*26	0.41	*30	0.47	0.762	28	0.84	*8315	0.52	*6200	0.39		**56	0.34	**2077	0.46
6	0.739	284	0.27	*28	0.44	*35	0.55	0.714	33	0.92	*8660	0.54	*8750	0.54		**111	0.34	**2602	0.50
7	0.696	526	0.28	143	0.51	*42	0.66	0.667	42	1.06	21	0.93	*10285	0.64		**236	0.35	**3454	0.55
8	0.652	605	0.29	373	0.52	*45	0.70	0.619	43	1.07	64	1.60	*11400	0.71		**564	0.37	**4542	0.62
9	0.609	751	0.30	570	0.54	*48	0.75	0.571	52	1.21	79	1.83	39	1.41		**1158	0.41	102	0.78
10	0.565	1106	0.32	1388	0.59	15	0.75	0.524	76	1.59	83	1.90	50	1.58		**1901	0.45	633	1.36
11	0.522	1354	0.33	1606	0.60	177	0.76	0.476	82	1.68	84	1.91	67	1.85		**2570	0.49	872	1.62
12	0.478	1955	0.37	3290	0.70	362	0.77	0.429	87	1.76	115	2.40	70	1.89		**2571	0.49	1494	2.31
13	0.435	3349	0.46	3440	0.71	763	0.80	0.381	111	2.13	139	2.77	75	1.97		**2886	0.51	1682	2.51
14	0.391	3479	0.47	4030	0.75	1013	0.81	0.333	123	2.32	150	2.94	78	2.02		**3664	0.56	1703	2.54
15	0.348	4764	0.55	4148	0.76	1347	0.83	0.286	151	2.76	162	3.13	113	2.57		2065	0.71	1937	2.79
16	0.304	4873	0.55	4443	0.78	1789	0.86	0.238	264	4.53	225	4.12	119	2.66		3841	0.74	2397	3.30
17	0.261	10486	0.90	16613	1.53	2997	0.94	0.190	327	5.51	255	4.58	155	3.22		13866	0.94	5782	7.01
18	0.217	21849	1.61	18077	1.62	7197	1.20	0.143	334	5.63	264	4.73	278	5.14		17054	1.00	22255	25.10
19	0.174	37026	2.55	19635	1.72	9337	1.33	0.095	401	6.67	268	4.79	347	6.22		21315	1.08	23992	27.00
20	0.130	46131	3.02	21435	1.83	11347	1.46	0.048	661	10.73	518	8.69	889	14.69		67796	2.00	26877	30.17
21	0.087	63949	4.23	26627	2.16	11697	1.48												
22	0.043	100009	6.48	95075	6.42	16682	1.79												

* Failed at first stress level
** Failed at second stress level

$\bar{N}(S)$: $\bar{N}$(85%)=641, $\bar{N}$(75%)=160840, $\bar{N}$(80%)=9110, $\bar{N}$(70%)=510250

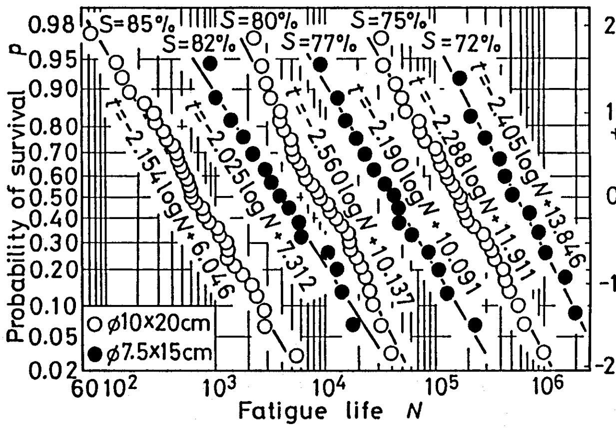
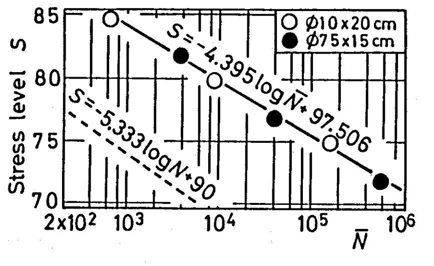


Fig. 3 *P-N* diagram (φ 10x20 cm, σ_c=25 MPa)



Fif. 4 *S-N̄* diagram

It is clear from Fig. 3 that the distribution of fatigue lives at each stress ratio is treated as a logarithmic normal distribution. The regression line is given as follows by the method of least squares.

$$t = A \log N + B \qquad \text{--} \; (2)$$

From Eq.(2), the mean of log *N*, *m* (log *N*), and the standard deviation of log *N*, *V* (log *N*), are given as follows:

$$m\,(\log N) = -B/A \;, \quad V\,(\log N) = 1/|A| \qquad \text{--------------------} \; (3)$$

The relationship between stress ratio and the mean of fatigue lives is given as follows:

$$S = A_1 \log \bar{N} + B_1 \qquad \text{----------------------------} \; (4)$$

There are differences in mix proportions, strengths and sizes between the specimens indicated by black and white dots in Fig. 4. In the figure, the broken line shows the *S-N* line recommended in the "Tentative Recommendations for the Limit State Design of Concrete Structures" published by the Japan Society of Civil Engineers (JSCE equation). The fatigue strength at 2 x 10^6 repetitions calculated by the experimental and JSCE equation in Fig. 4 are respectively 69.5 and 56.3 percent of static strength.

3.2 Two- and Three-Stage Fatigue Tests

The tests results are summarized in Table 3.

The cumulative fatigue lives ($N = \sum_{i=1}^{j} n_i$; j=2 or 3) in two- and three-stage fatigue tests are estimated according to the Miner sum (*M*) defined by Eq.(5).

$$M = \sum_{i=1}^{j} (n_i/\bar{N}_i) \;, \qquad (\; \bar{N}_i=\bar{N}(S_i);j=2 \text{ or } 3) \;\text{----------------------------}(5)$$

where, N_i is number of stress cycles giving failure in a constant-amplitude fatigue test at stress ratio $S=S_i$, which is given by substituting t=0 for the equation in Fig. 3.

If the distribution of fatigue lives at the same stress ratio can be regarded as a logarithmic normal distribution in the constant-amplitude fatigue test, the following equations are given[4].

$$\log M = \log N - m(\log N)$$

$$m(\log M) = 0 \qquad \text{------------------------------(6)}$$

$$V(\log M) = V(\log N) = 1/|A_1|$$

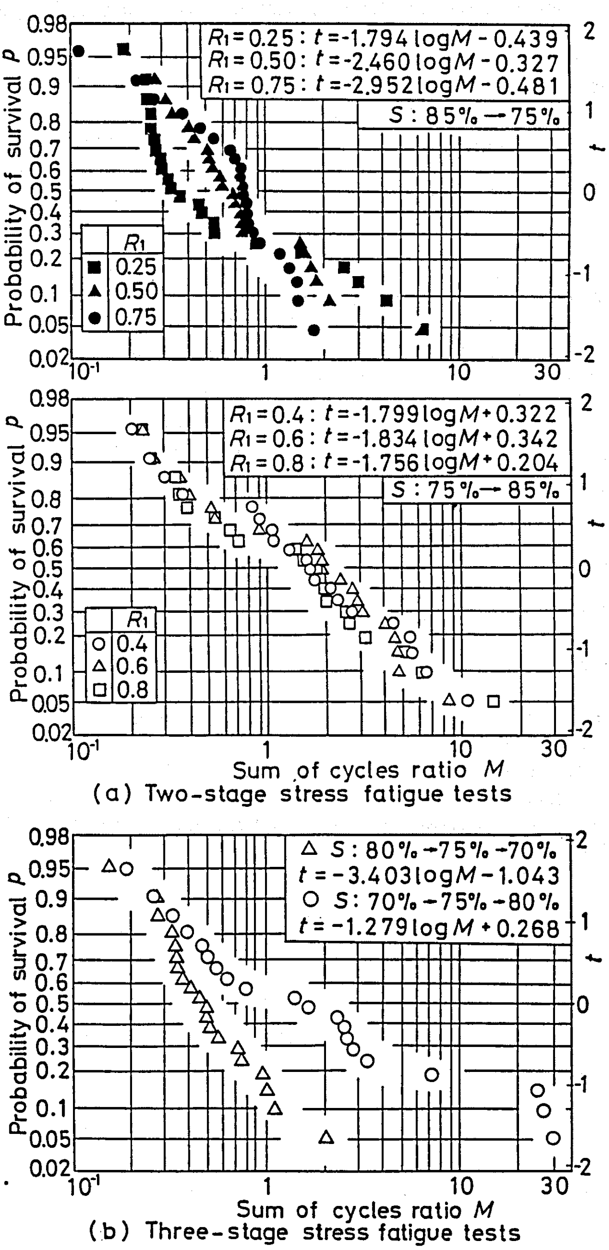


Fig. 5

Relationships between probability of survival and sum of cycles ratio.

That is to say, M is in the form of logarithmic normal distribution when N is distributed likewise. The mean of log M is equal to zero. The standard deviation of log M is equal to that of log N.

The regression line of M on logarithmic normal probability paper is given as follows:

$$t = \phi^{-1}(p) = A \log M + 0 \qquad \text{------------------------------------(7)}$$

In Fig.5, the relationships between probabilities of survival and Miner sums in two- and three-stage fatigue tests are plotted on logarithmic normal probability paper. It is clear from Fig. 5 that there are differences in the shapes of curves according to the magnitudes of the initial stress level S_1 and the following stress levels S_2 and S_3. When the initial stress level is higher than the following ones, the slope of the curve becomes steeper and the mean of M which is calculated by Eq.(8), is smaller than 1.0 in the range in which the specimen is subjected to initial load. On the other hand, the mean of M is larger than 1.0 when the initial stress level is lower than the levels that follow.

In Table 4, it is checked by the Kolmogrov-Smirnov test[5] whether the distribution of M can be regarded as a logarithmic normal distribution. It is clear from Table 4 that the distribution of M for each test condition in the two- and three-stage fatigue tests can be regarded as logarithmic normal distribution at a 5-percent significance level.

Fig. 6 shows the relationships between the repetition ratio of initial load and the mean of M ($\bar{M}$) and the standard deviation of log M ($V(\log M)$), which are calculated by the following equations:

$$t = A_2 \log M + B_2$$

$$\bar{M} = 10^{-B_2/A_2}, \qquad m(\log M) = -B_2/A_2 \qquad \text{---------------------------(8)}$$

$$V(\log M) = 1/|A_2|$$

When the initial stress level is lower than stress levels that follow, the values of $\bar{M}$ and $V(\log M)$ are larger than those in the constant-amplitude test. On the other hand, the values of $\bar{M}$ and $V(\log M)$ become smaller than those in the constant-amplitude fatigue test when the initial stress level is higher than the stress levels following. The values of $\bar{M}$ are not affected by the repetition ratio of the initial stress level.

Table 4 Results of Kolmogrov-Smirnov test.

$S_1 \to S_2 \to S_3$ (%)	R_1	D_{max}	$D_l^{\alpha=5\%}$	D_{max}/D_l^{α}	l
85 → 75	0.25 0.50 0.75	0.198 0.201 0.155	0.282	0.70 0.71 0.55	22
75 → 85	0.4 0.6 0.8	0.100 0.136 0.096	0.290	0.34 0.47 0.33	20
80→75→70 70→75→80	0.33	0.127 0.147	0.290	0.44 0.51	20

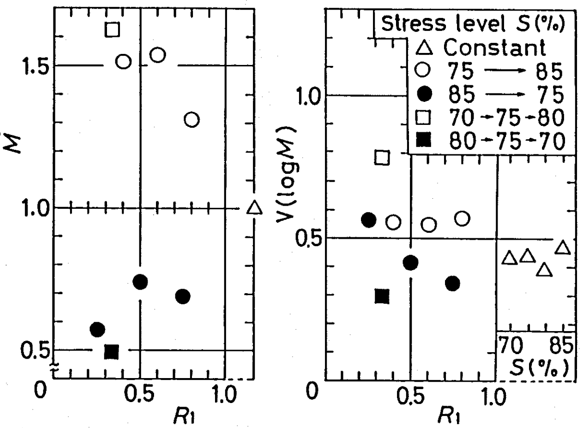


Fig. 6

Relationships between M or $V(\log M)$ and R_1

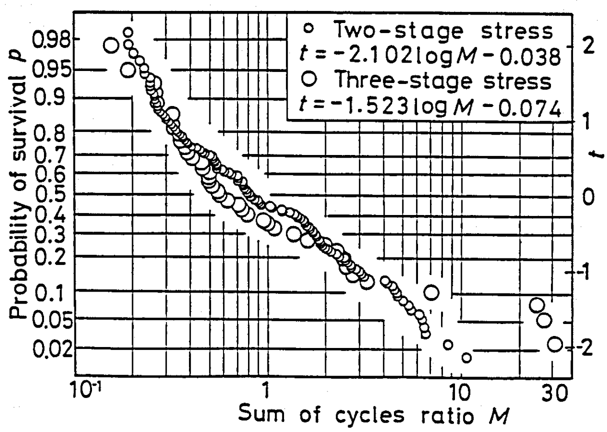


Fig. 7

Relationships between probability of survival and sum of cycles ratio (for all specimens used in two- and three-stage fatigue test)

Fig. 7 shows the relationships between the values of probability of survival and Miner sums for all specimens used in the two- and three-stage fatigue tests (two-stage ; 126 specimens, three-stage ; 40 specimens). It is clear that almost linear relations are obtained and the distribution of M can be regarded as a logarithmic normal distribution in the two-stage fatigue tests, but the plotted results of the three-stage fatigue tests have rather poor agreement with the logarithmic normal distribution. The regression equations given in Fig.7 are calculated by the method of least squares. The mean values of M in the two- and three-stage fatigue tests caluculated by the abovementioned regression equations are 0.96 and 0.89, respectively.

If the order of loading is not considered, the mean value of M is approximately equal to 1 at p=50 percent. That is to say, Miner's hypothesis is applicable to fatigue of concrete.

3.3 Variable Load Fatigue Tests

The results of the variable load fatigue tests are summarized in Table 5. In the table, *O.L.*, u and S_k are respectively order of loading, number of stress block and stress ratio at failure. M and M_1 are Miner sums caluculated by the experimental S-$\bar{N}$ equation in Fig.4 and JSCE equation, respectively.

In Fig. 8, the results of the variable load fatigue tests are plotted on logarithmic normal probability paper. The Miner sums are in the form of

Table 5 Results of variable load fatigue tests. (1)

r	P	t	T distribution						B distribution					
			O.L.	At failure			M	M_1	O.L.	At failure			M	M_1
			S_j	u	S_k	n			S_j	u	S_k	n		
1	0.952	1.665	54231	2	82	10160	0.33	43	34215	2	82	2480	0.25	29
2	0.905	1.311	42153	2	74.5	12690	0.47	64	23451	2	82	2900	0.40	45
3	0.857	1.067	21534	2	77	27040	0.64	85	53124	3	82	3390	0.52	58
4	0.810	0.876	51234	2	79.5	27730	0.66	89	45312	3	82	3880	0.54	61
5	0.762	0.712	31254	2	79.5	29580	0.81	109	15432	4	82	5220	0.71	80
6	0.714	0.566	25314	3	82	35750	0.92	125	25413	4	82	5490	0.74	84
7	0.667	0.431	43215	3	79.5	32220	1.02	137	51342	5	82	7010	0.94	106
8	0.619	0.303	51423	3	79.5	39050	1.17	152	24135	5	79.5	7450	0.95	110
9	0.571	0.180	54321	3	79.5	32180	1.19	153	54321	5	77	8400	1.18	135
10	0.524	0.060	12453	3	77	47250	1.34	178	42153	6	82	10120	1.32	150
11	0.476	-0.060	42531	4	79.5	50300	1.42	192	51243	6	82	9720	1.42	159
12	0.429	-0.180	25341	4	82	55930	1.53	206	43512	7	82	12080	1.56	178
13	0.381	-0.303	21453	4	79.5	64140	1.63	213	35241	8	79.5	14640	1.93	224
14	0.333	-0.431	14235	4	77	65390	1.64	225	25314	9	82	15730	2.16	243
15	0.286	-0.566	34152	4	82	64120	1.84	248	13254	10	79.5	18490	2.51	282
16	0.238	-0.712	15234	5	82	77890	2.17	288	41523	13	82	24040	3.25	367
17	0.190	-0.876	43512	7	82	117070	3.55	476	12345	14	77	25560	3.36	380
18	0.143	-1.067	12345	9	82	169610	4.64	629	52431	16	82	29590	4.06	458
19	0.095	-1.311	25413	10	79.5	178280	5.18	693	32145	22	77	41310	5.50	623
20	0.048	-1.665	45132	14	82	252990	7.36	987	13542	30	77	57220	7.65	864

r	P	t	N distribution						U distribution					
			O.L.	At failure			M	M_1	O.L.	At failure			M	M_1
			S_j	u	S_k	n			S_j	u	S_k	n		
1	0.952	1.665	13542	1	74.5	4900	0.24	32	35421	2	82	2710	0.26	30
2	0.905	1.311	25143	2	82	8140	0.36	46	51432	2	82	2240	0.34	37
3	0.857	1.067	54321	2	82	5530	0.40	49	14523	2	82	3740	0.38	43
4	0.810	0.876	34512	2	82	11150	0.58	79	24351	3	79.5	6970	0.50	60
5	0.762	0.712	12543	2	77	12660	0.68	88	43251	3	82	8200	0.62	74
6	0.714	0.566	52413	2	77	14030	0.71	93	32415	4	77	9690	0.77	91
7	0.667	0.431	24351	2	72	14440	0.73	96	21435	4	72	10310	0.77	91
8	0.619	0.303	41253	3	79.5	16490	0.85	112	45312	4	77	11150	1.07	124
9	0.571	0.180	14352	3	77	18630	0.96	128	15342	5	82	14450	1.14	133
10	0.524	0.060	12345	3	79.5	24060	1.02	138	52431	5	82	14090	1.27	146
11	0.476	-0.060	31452	3	82	21920	1.12	148	41352	5	82	16270	1.29	151
12	0.429	-0.180	34215	3	82	24990	1.21	160	31254	5	79.5	16790	1.36	158
13	0.381	-0.303	25134	4	82	28310	1.29	180	25134	5	79.5	17060	1.38	159
14	0.333	-0.431	14253	5	79.5	36380	1.77	234	41325	6	72	18690	1.48	174
15	0.286	-0.566	51324	6	82	45500	2.34	305	12345	6	82	21130	1.63	192
16	0.238	-0.712	23145	7	79.5	64010	2.96	395	23514	6	79.5	20830	1.69	196
17	0.190	-0.876	45132	7	82	58230	3.04	397	15324	10	79.5	37480	3.04	355
18	0.143	-1.067	53214	9	82	75520	3.82	498	54321	13	82	46270	3.92	455
19	0.095	-1.311	42315	11	79.5	95670	4.69	618	53142	19	79.5	72610	5.95	694
20	0.048	-1.665	43521	13	82	121540	6.15	798	32514	24	82	91790	7.48	872

Table 5 Results of variable load fatigue tests. (2)

r	P	t	F distribution O.L. S_j	F At failure u	F At failure S_k	F At failure n	F M	F M_1	E distribution O.L. S_j	E At failure u	E At failure S_k	E At failure n	E M	E M_1
1	0.952	1.665	77 72 82	2	72	48760	0.28	48	82 77 72	1	72	21810	0.25	38
2	0.905	1.311	72 77 82	3	72	77650	0.41	70	77 72 82	2	77	73390	0.36	64
3	0.857	1.067	77 72 82	3	72	85830	0.54	87	77 82 72	2	77	76340	0.42	75
4	0.810	0.876	82 77 72	3	77	56320	0.61	91	77 72 82	2	72	104260	0.49	90
5	0.762	0.712	77 82 72	3	72	71930	0.70	104	77 72 82	2	72	115180	0.50	95
6	0.714	0.566	77 82 72	3	72	78870	0.71	107	82 77 72	2	72	124860	0.62	110
7	0.667	0.431	77 82 72	3	72	88800	0.72	111	72 77 82	3	72	174840	0.70	125
8	0.619	0.303	72 77 82	4	72	109780	0.76	120	82 77 72	3	82	145720	0.72	131
9	0.571	0.180	77 72 82	4	77	108450	0.78	124	77 82 72	3	82	149580	0.82	143
10	0.524	0.060	77 72 82	4	72	215800	1.20	209	82 77 72	3	72	152120	0.88	150
11	0.476	-0.060	82 77 72	4	72	144020	1.54	226	72 77 82	4	72	217290	0.98	177
12	0.429	-0.180	82 77 72	4	72	168150	1.58	236	77 72 82	4	72	327520	1.48	276
13	0.381	-0.303	82 77 72	5	82	241720	1.98	297	72 77 82	4	77	425470	1.53	284
14	0.333	-0.431	77 82 72	5	77	259550	2.10	336	77 72 82	4	72	398960	1.59	300
15	0.286	-0.566	72 77 82	5	77	486750	2.26	399	77 82 72	4	72	266480	1.70	306
16	0.238	-0.712	72 77 82	5	77	494460	2.42	428	77 82 72	4	72	415200	1.94	346
17	0.190	-0.876	72 77 82	6	72	714850	3.90	651	82 77 72	5	72	595160	2.86	509
18	0.143	-1.067	82 77 72	6	72	754440	5.42	851	72 77 82	6	72	765460	3.13	580
19	0.095	-1.311	77 82 72	7	77	777290	5.49	866	72 77 82	9	72	1344830	5.97	1081
20	0.048	-1.665	77 72 82	8	77	1055400	7.60	1201	77 82 72	10	72	1892080	7.78	1391

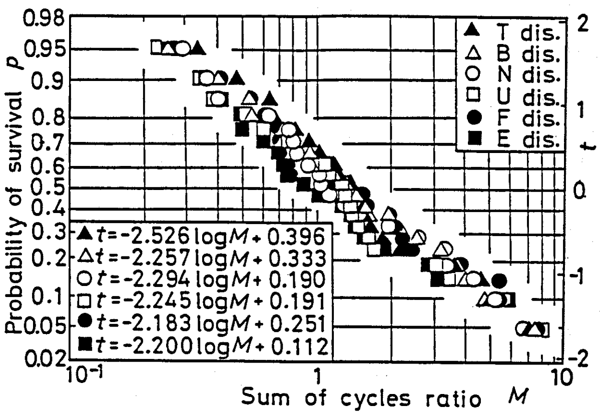


Fig. 8

P-M diagram (Variable load fatigue test)

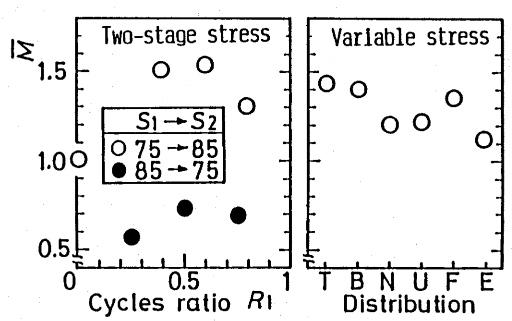


Fig. 9

Relationships between $\bar{M}$ and test condition.

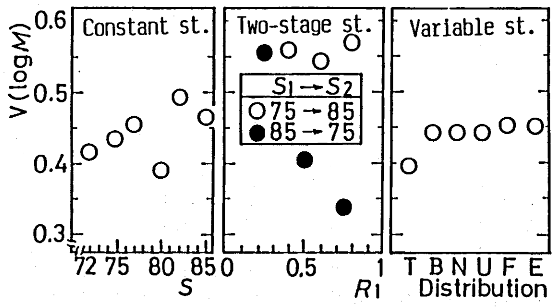


Fig. 10

Relationship between standard deviation and test condition.

logarithmic normal distribution for each of the cases of variable loads. The linear equations shown in Fig. 8 are determined by the method of least squares.

Fig. 9 shows mean values of Miner sums ($\bar{M}$) calculated by Eq.(8). The values of $\bar{M}$ in the variable load fatigue tests are in the range from 1.12 to 1.43. The values of $\bar{M}$ in the two-stage fatigue test scatter affected by the order of loading. Those in the variable load fatigue tests, however, do not scatter widely, and the arithmetic mean $\bar{M}$ is 1.29. Therefore, fatigue life can be estimated by Miner's hypothesis to be on the conservative side.

Fig.10 shows the standard deviations of log M in the variable load fatigue tests. Those values are in the range of 0.40 to 0.46, and approximately equal to the results of the constant-amplitude test (0.39 - 0.46).

3.4 Fatigue Design

The safety from fatigue failure of concrete structures is evaluated by Miner sum and S-N equation. In the design of an offshore concrete structure, with regard to fatigue strength, M is estimated at 0.2[6].

In the "Tentative Recommendations for the Limit State Design of Concrete Structures" published by JSCE, the fatigue strength is reduced, which is shown in Fig.4.

Table 6 Results of regression analysis.

Test	Distribution		M=0.2 P_f	t= $A\log M + B$ $-A$	B	$\bar{M}$	$V(\log M)$	M=1 P_f
Variable load fatigue test	T		0.02	2.515	5.738	191	0.40	10^{-9}
	B		0.03	2.246	4.986	159	0.44	10^{-7}
	N		0.03	2.287	5.032	159	0.44	10^{-7}
	U		0.04	2.235	4.806	141	0.45	10^{-7}
	F		0.03	2.172	5.072	217	0.46	10^{-7}
	E		0.05	2.160	4.974	201	0.46	10^{-7}
	S	R_1	P_f	$-A$	B	$\bar{M}$	$V(\log M)$	P_f
Two-stage fatigue test	S_1=75%	0.4	0.06	2.727	6.227	192	0.37	10^{-10}
	↓	0.6	0.05	2.904	6.794	219	0.34	10^{-11}
	S_2=85%	0.8	0.08	2.577	6.015	216	0.39	10^{-10}
	S_1=85%	0.25	0.21	1.335	2.486	73	0.75	10^{-3}
	↓	0.50	0.08	1.648	3.163	83	0.61	10^{-4}
	S_2=75%	0.75	0.06	2.206	3.951	61	0.45	10^{-5}

Table 6 gives the values of probability of failure (P_f) calculated by substituting 0.2 for M in the experimental S-$\bar{N}$ equation in Fig.4. Table 6 also gives $\bar{M}$, $V(\log M)$ and P_f (approximate value at M=1) of those test results calculated by JSCE equation. The values of $\bar{M}$ calculated by JSCE equation are about a hundred times larger than those by the experimental equation. That is to say, the mean of Miner sums is significantly different according to the S-N equation used in the analysis.

4. Conclusions

Experimental work was carried out in this study with regard to the fatigue properties of concrete subjected to constant-amplitude repetitive loads, two- and three-stage repetitive loads and the variable repetitive loads. The following conclusions were obtained within the limits of this study:

1) The fatigue life for each stress ratio in the constant-amplitude fatigue test is in the form of a logarithmic normal distribution.

2) In the constant-amplitude fatigue test and two- and three-stage fatigue tests, the distribution of Miner sums for the individual test conditions are regarded as logarithmic normal distributions.

3) The means and scatters of Miner sums in the two- and three-stage fatigue tests are affected by the order of loading

4) The distributions of Miner sums in the variable load fatigue tests can be regarded as logarithmic normal distributions.

5) The means of Miner sums in the variable load fatigue tests are between the upper and lower limits of $\bar{M}$ given by the two-stage fatigue tests.

6) Miner's hypothesis is applicable to the estimation of fatigue life of concrete subjected to random loading.

7) The mean values of Miner sums are significantly different according to the S-N equation used in the analysis.

Acknowledgement

The authors gratefully acknowledge the guidance and encouragement received from of Dr. Kiyoshi Okada, Professor, Kyoto University this study. They also wish to thank Dr.Takayuki Kojima, Professor, Ritsumeikan University, Dr. Kazuo Kobayashi Associate Professor, Kyoto University, and Mr. Toyoaki Miyagawa, Research Associate, Kyoto University for their advices and help in carrying out the experimental works.

References

[1] Takeo Yokobori : Strength of Materials, Gihodo Co., Ltd., pp.213 - 218, 1955 (in Japanese).
[2] Tetsuo Kunihiro : Working Loads and Fatigue Dedign for Highway Bridge, The Proceedings of the Symposium on Structural Mechanics, Vol.22, 1976 (in Japanese).
[3] Kenji Sakata, Hideo Kiyama and Shinzo Nishibayashi : A Study on the Fatigue Life of Concrete by the Statistic Treatment, Proc.of JSCE, No.198, pp.107 - 114, 1972 (in Japanese).
[4] J.V. Leeumen and A.S.M.Siemes : Miner's Rule with Respect to Plain Concrete, Proc.of BOSS'79, pp.591 - 610, 1979.
[5] A.H.S. Ang and W.H. Tang : Probability Concepts in Engineering Planning and Design (translated by M. Ito and H. Kameda), Maruzen Co., pp. 274 -281, 1977 (in Japanese).
[6] K.Waagaard : Fatigue Strength Evaluation of Offshore Concrete Structures, IABSE Proc., P-56/82, pp. 97- 113, 1982.