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DESIGN OF SLENDER REINFORCED CONCRETE COLUMNS

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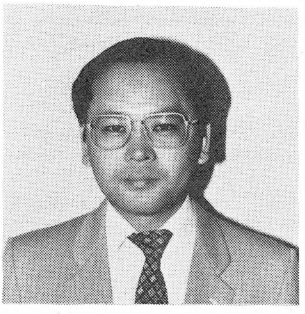

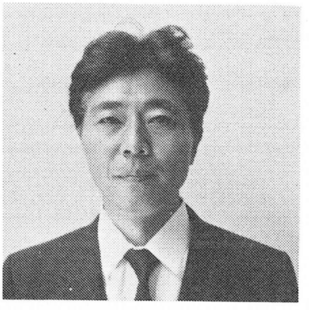

Koji SAKAI　　Yoshio KAKUTA　　Sumio NOMACHI

SYNOPSIS

The purpose of this study is to present an approximate design method on slender reinforced concrete columns. Numerical tests are carried out on slender reinforced concrete columns subjected to short time and sustained loads using non-linear analysis, in which both geometrical and material non-linearities are considered. Based on the regression equation introduced from the results of numerical tests, additional eccentricity for design is proposed. In order to examine the validity of the proposed design formula, comparisons are made with 626 test data cases. The comparisons show that the proposed design formula is adequately effective for practical use. Finally, the design procedures are presented.

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1. INTRODUCTION

The purpose of design in structures is to check whether each member of structures have an adequate strength to the loading condition or not. Therefore, it is necessary to analyze the structures as a system. Analysis methods for reinforced concrete structures may be classified into two:(1)linear analysis based on the theory of elasticity:(2)non-linear analysis in which the geometrical non-linearities and/or the material non-linearities are taken into account. There can be little doubt that the best way to examine the load-carrying capacity of structures is by the analysis in which both non-linearities are considered. However, such an analysis is not practical since it requires a iterative procedure.

In designing the members in reinforced concrete structures,which include columns, according to the results of linear analysis, a problem occures. When the bending moments in each section of the structure are not influenced by the displacements, the correspondence of the bending moments due to linear analysis and those in the ultimate state may be possible. In the particular case of columns, however, a significant additional moment may occur due to deflections and axial forces. In this case the results of linear analysis do not reflect the actual behavior. The effect of additional moment, called secondary effect, should be considered in design. There are three ways to take the effect into consideration in design:(1)the influences of displacements on the behavior are rationally considered in the analysis:(2)the second-order elastic analysis is carried out using a constant rigidity:(3)the analysis is based on linear elastic analysis and secondary effect is additionally considered. The 3rd method is the most simple and easiest to design.

In this study numerical tests are carried out to examine the behavior of reinforced concrete slender columns using non-linear analysis, in which both geometrical and material non-linearities are considered. From the results of numerical tests, an additional eccentricity formula is introduced by a regression analysis and a design formula is then proposed. To examine the validity of the proposed formula, extensive comparisons are made with the results of tests available from references. Finally, the design procedures are presented.

2. REVIEW OF PREVIOUS RESEARCH AND DESIGN CODES

2.1 Previous Research

A slender ideal column built in vertically at the base, free at the upper end and subjected to an axial force, was solved originally by L. Euler in 1744. The introduction of steel as a structural material has made the complicated structures, in which slender bars or thin plates are used, possible. The use of such structural elements has given rise to the necessity of examining stability and many theoretical and experimental investigations have been done. Concrete has also been used as a structural material and concrete columns as a structural member. In the early studies on reinforced concrete columns, the problems were the application of the Engesser-Karman theory to reinforced concrete. In 1963, however, W.F. Chan, et al. investigated the influence of varied cracking of the section on column deflection and critical length and a numerical method was presented to calculate the deflected shape of columns under a given load. In 1964 E.O. Pfrang, et al. developed an analytical model of a structural system consisting of a column and its adjoining members. Up to the present, many analytical studies have been done on columns subjected to sustained load or biaxial bending, or on frame structures. At the same time, experimental studies have also been carried out. The results of these experimental studies will be described later.

The accumulation of analytical and experimental studies has contributed to the development of design methods for slender reinforced concrete columns. The design methods adopted in various design codes will be reviewed in the following section.

2.2 Design Codes

Slender reinforced concrete columns should be fundamentally designed on the basis of a rational second-order analysis of the structure. However, as this kind of calculation requires complicated procedures, the method is not convenient for use. Therefore, in addition to the fundamental philosophy an approximate procedure should be also presented in design code. The basic forces and moments to be used in the approximate procedure are obtained by conventional frame analysis. Approximate design procedures would reduce expenses and the time necessary for design.

Approximate design procedures of slender reinforced concrete column in various codes could be roughly classified into the following three categories:(1)the reduction factor method which is the first concept of slender columns. This method implies the maintenance of the same eccentricity in both the slender column and analogous short column, i.e. in general the reduction in sectional strength due to slenderness effects is represented as a function of slenderness ratio. The method has been adopted in the JSCE Concrete Code and introduced in the ACI Building Code Commentary as an alternate design method. However, taking the actual behavior of slender columns into consideration, this method is not rational;(2)the moment magnifier method which has been adopted in the ACI Building Code. In this method columns are designed using the values which multiply the moment based on conventional analysis by a magnifier. The magnifier was introduced from an approximation of maximum moment in an elastic beam-column bent in single curvature;(3)the "additional eccentricity method" in which additional eccentricities or moments are provided for. This method seems to be the most rational since it can most closely reflect the actual behavior of slender columns. The BSI CP110, the DIN1045 and the CEB-FIP Model Code give the design method. The variables considered in the additional eccentricity formula of the CEB-FIP Model Code are the slenderness ratio and effective height. The DIN1045 uses the slenderness ratio and initial eccentricity, whilst the BSI CP110 uses only the slenderness ratio.

It seems that the estimation of additional eccentricity by these design formulas may be considerably coarse since the number of variables considered are limited. Therefore, there is a need to introduce rational design formula in which a great number of influence factors are considered.

3. ANALYSIS

3.1 Analysis for Short Time Loads

3.1.1 Assumption

Although slender reinforced concrete columns have various types of cross section shapes, only columns with rectangular cross sections are considered in this study(see Fig.1). The reinforcing steel is assumed to be an idealized elasto-plastic material, as shown in Fig.2. The instantaneous stress-strain

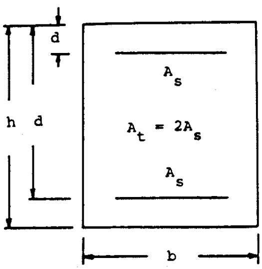

Fig. 1 Cross section

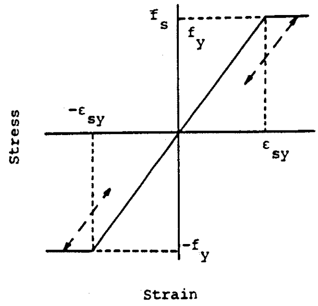

Fig. 2 Stress-strain curve for steel

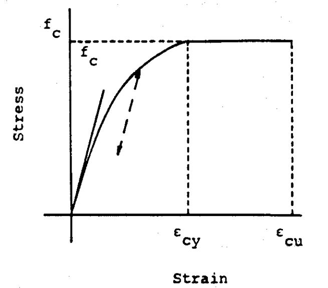

Fig. 3 Stress-strain curve for concrete

relationship for concrete is assumed to be a combination of parabola and straight line(see Fig.3). The influence of shrinkage and tensile resistance of concrete is neglected. Modulas of elasticity of reinforcing steel, E_s, is taken to be 2.06×10^6 MPa. Plastic strain and ultimate strain of concrete, ε_{cy} and ε_{cu}, is assumed to be 0.002 and 0.0035 regardless of the strength of concrete.

3.1.2 Method of Analysis

The method of analysis used in this study is an iterative procedure consisting of two parts:(1) a second-order elastic analysis using finite element method to consider the geometric non-linearity and (2)a cross-sectional analysis to consider the material non-linearity. The method is fundamentally similar to the procedure developed by K. Aas-Jakobsen(19).

3.1.3 Load-Carrying Behavior of Slender Columns under Short Time Loads

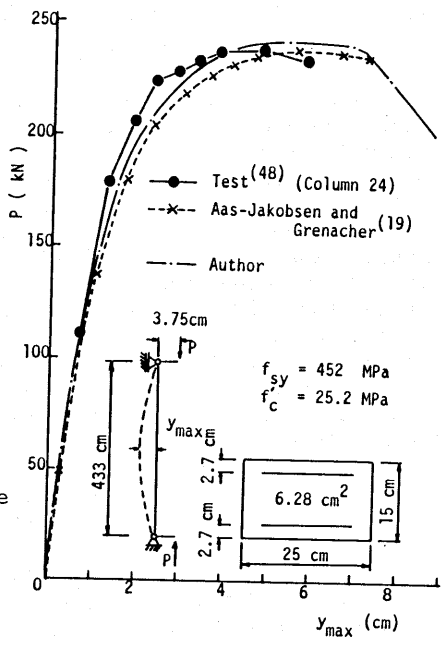

Fig. 4 Load-displacement curve under short time load

Figure.4 shows the comparison between the test result by Ramu, et al.(48), the calculated result by Jakobsen, et al.(19), and the calculated result by the authors. The result by the authors is greater than the test result by approximately 1.6% in the maximum load. The difference between the calculated result by the authors and Jakobsen, et al. seems to be due to the fact that Jakobsen, et al. used only a curvature controlled procedure and the increment of curvature is large, while the authors used both a load and curvature controlled procedure.

The load-carrying behavior of slender reinforced concrete columns is very complicated since it is influenced by the shape of cross section, the reinforcement states, the slenderness ratio, the degree of end constraints, the loading conditions, etc. However, the behavior can be divided into two types. A schematic diagram of the relationships between the interaction curve of section strength and and the typical load-carrying behavior of columns is shown in Fig.5. TYPE A is called material failure, in which concrete collapses at point C. TYPE B is called stability failure. TYPE B implies the case in which the column becomes unstable before the material failure occurs. In this case, the maximum load-carrying capacity does not generally correspond with the ultimate one. The design in this case should be fundamentally done by the method which corresponds to such behavior. However, since the rigidities of columns usually vary in the longitudinal direction and the load levels, it is not real to consider the design as a instability problem. Furthermore, it should be noted that reinforced concrete columns are generally designed to the ultimate limit state of cross section. In the case of TYPE B, therefore, the instability load is considered as ultimate load and the ultimate moment is taken as the point E on the interaction curve corresponding to the ultimate load.

3.2 Analysis for Sustained Loads

3.2.1 Assumption

In addition to the fundamental assumptions described in SECTION 3.1.1, for instantaneous unloading or reloading a linear relation between stress and strain is assumed both for steel and concrete and the slope is taken equal to the modulus of elasticity, as shown in Figs.2 and 3. Total strain for concrete subjected to sustained stress is represented by the summation of instantaneous and creep strains, ε_{IN} and ε_{CR}. The creep strain used in this study is that proposed by Manuel, et al.(8), based on the test results by Rüsch(28):

$$\varepsilon_{CR}=F1(n)(f_c/f_c')^3+F2(n)(f_c/f_c')^2+F3(n)(f_c/f_c') \quad (1)$$

$$F1(n)=0.0009(0.64\ \log_{10}t+0.4)$$

$$F2(n)=0.0008(0.64\ \log_{10}t+0.4)$$

$$F3(n)=0.0007(0.64\ \log_{10}t+0.4)$$

where t is the duration of sustained loading in days and f_c' is concrete strength. Equation (1) has been derived under the condition in which the age of concrete at the first application of load is 28 days and

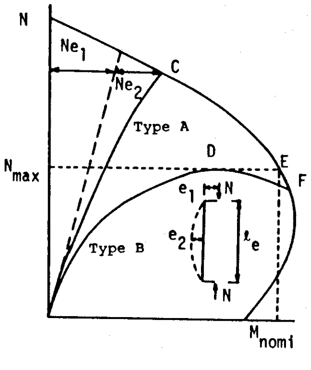

Fig. 5 Behavior patterns for columns

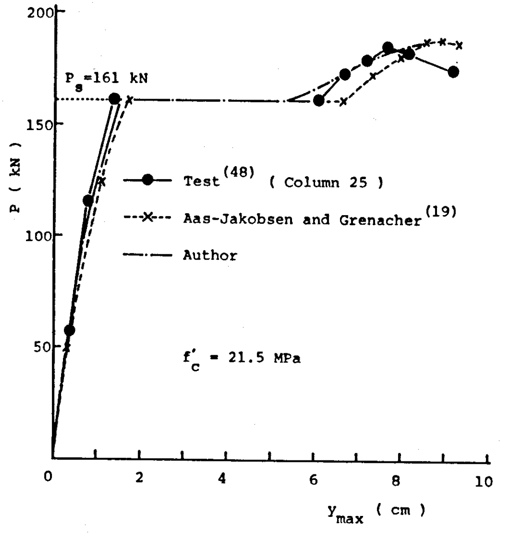

Fig. 6 Load-displacement curve including sustained load

the term of concrete curing is 7 days at 20°C and 100% relative humidity and remainder at 20°C and 50% relative humidity. The rate of creep method, in which the creep is not dependent on the previous stress history, is used to apply the creep strain to a variable stress problem.

3.2.2 Method of Analysis

The loading considered in this study is short time loading to a load level, followed by a period of sustained load at this load level, followed by quick loading to failure. The method of analysis is fundamentally the same as that described in SECTION 3.1.2. Furthermore, in the case of sustained load the sustained loading period must be divided into some increment of time and the analysis for each increment of time is required. Manuel, et al. showed that in the case where sustained load is applied for 25 years, creep behaviors could be predicted by considering three steps of time(8):7 days, 9 months, and 25 years. The analysis used in this study is described in the following. By using elastic analysis for a increment of time, moments and strain distributions are determined and the corresponding stresses are adjusted so that each fiber strain of cross section is equal to the strain calculated using the assumption that the total strain of concrete is $\varepsilon_{IN}+\varepsilon_{CR}$. The internal cross section load and moment are calculated using the stress distributions. The internal load and moment are then compared to the results from elastic analysis and if compatibility does not exist, the assumed rigidities are changed and the calculation is repeated until the compatibility is attained within a permissible variation. This procedure is done in each division of time. In the calculation with respect to quick loading following the sustained load period, the creep strains retain the values which existed at the end of the sustained load periods.

3.2.3 Load-Carrying Behavior of Slender Columns Subjected to Sustained Loads

Figure.6 shows the comparison between the test result by Ramu, et al.(48), the calculated result by Jakobsen, et al.(19), and the calculated result by the authors on the load-carrying behavior of hinged slender reinforced concrete columns which are first loaded at 28 days and then the load sustained for 141

Table. 1 Comparison of test results(15) with calculated results

Specimen	Short-Term Loading: Maximum Load: Experimental P_{exp} (kN)	Short-Term Loading: Maximum Load: Calculated P_{cal} (kN)	Sustained Loading: Sustained Load P_s (kN)	$\frac{P_s}{P_{cal}}$	Sustained Loading: Maximum Load: Experimental $P_{s,max}$ (kN)	Sustained Loading: Maximum Load: Calculated $P_{s,cal}$ (kN)	$\frac{P_{s,exp}}{P_{s,cal}}$	$\frac{P_{s,cal}}{P_{cal}}$ (a)	$\frac{P_{s,exp}}{P_{exp}}$ (b)	$\frac{(b)}{(a)}$
A	33.14 33.36	33.59	20.15	0.6	31.98	31.92	1.00	0.95	0.96	1.01
B	33.14 33.36	33.59	13.44	0.4	32.29	32.56	0.99	0.97	0.97	1.00
K	46.57 45.59	45.63	27.38	0.6	40.88	39.25	1.04	0.86	0.89	1.03
L	46.57 45.59	45.63	18.25	0.4	43.82	42.77	1.02	0.94	0.95	1.01
O	82.30 92.39	80.45	48.27	0.6	89.19	69.94	1.28	0.87	1.02	1.17
R	33.45 31.14	31.70	19.02	0.6	24.07	26.38	0.91	0.83	0.75	0.90

days, followed by a quick load to failure. The time division used in the authors' calculation was determined by the number of days corresponding to one-third of the creep strain in 141 days. The relationship between the test result and the calculated results are satisfactory except for a little difference of the deflections at the end of sustained loading and at the maximum load. It appears that a relatively large difference of the deflections at the end of sustained loading between the calculated results by the authors and Jakobsen, et al., is due to the difference in the estimation of creep strains.

To examine furthermore the validity of the authors' method, six cases have been appropriately selected from the test results by Goyal, et al. on hinged slender reinforced concrete columns which are first loaded at 28 days and loaded for 6 months with a sustained load followed by a quick load to failure(15). Table.1 shows the comparison between the test results and the corresponding calculated results. From the comparison, it can be concluded that the authors' method is satisfactory.

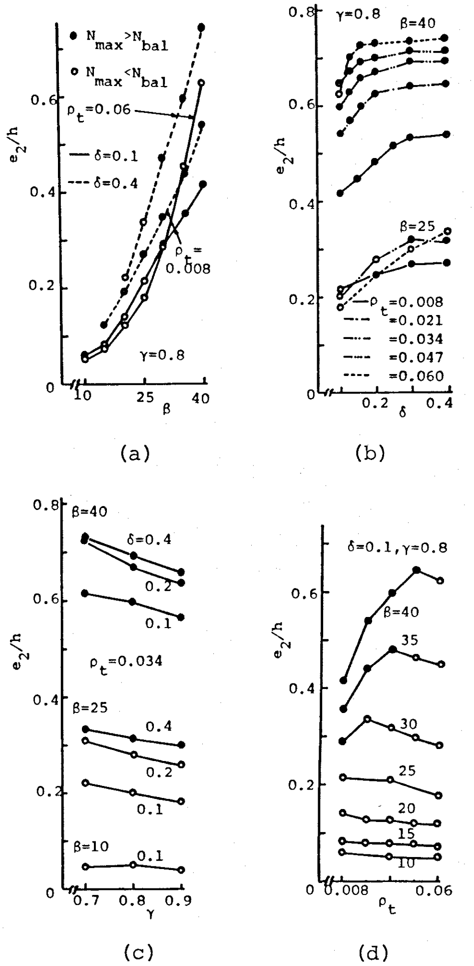

Fig. 7 Effect of β,δ,γ,and ρ_t on e_2/h in columns under short time loads(f_c'=29.4MPa,f_y=343MPa)

4. NUMERICAL TESTS AND ADDITIONAL ECCENTRICITY

4.1 Numerical Tests

Numerical tests are carried out to study the effects of a wide range of variables on additional moments of slender reinforced concrete columns. The variables considered are:

1. ratio of column lengths to cross section heights; $\beta=\ell/h$=10,15,20,25,30,35,40
2. reinforcement ratio; $\rho_t=A_t/bh$=0.008,0.021,0.034,0.047,0.06
3. coefficient for location of reinforcement; $\gamma=(d-d')/h$=0.7,0.8,0.9
4. initial eccentricity; $\delta=e_1/h$=0.1,0.2,0.3,0.4
5. compressive strength of concrete; f_c'=19.6,29.4,39.2 MPa
6. yield strength of steel bar; f_y=235,343,392 MPa
7. ratio of sustained loads to their associated short time ultimate loads; 0.2, 0.3,0.4

A total of 453 columns under sort time loads and a total of 480 columns where loads are sustained for 25 years, have been analyzed to obtain additional eccentricity.

The calculations in this study are limited to the hinged columns bent in symmetrical single curvature.

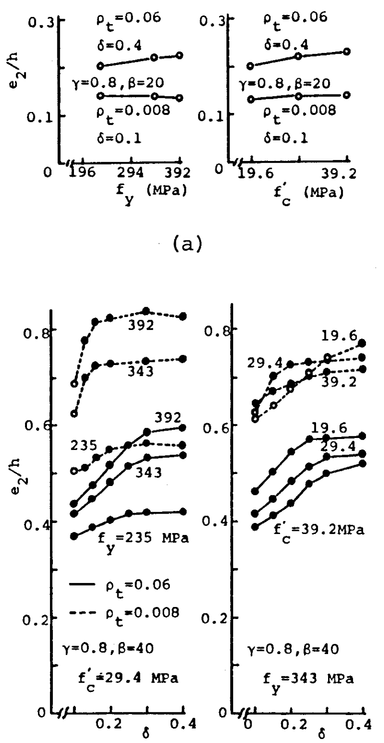

Fig. 8 Effect of material strengths on e_2/h in columns under short time loads

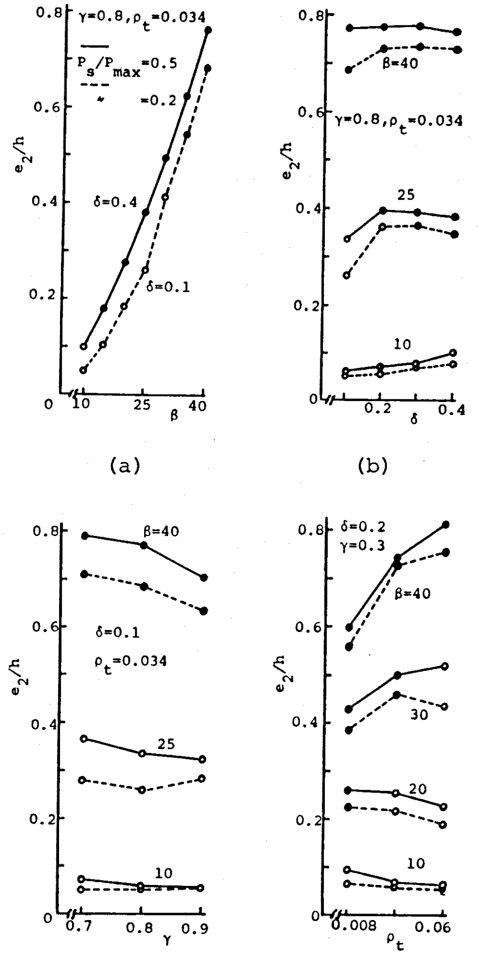

Fig. 9 Effect of β, δ, γ, and ρ_t on e_2/h in columns under short time and sustained loads

4.2 Additional Eccentricity

Figures.7, 8, and 9 show some examples of the effects of each variable on additional eccentricity, e_2/h. The calculated results in these figures have been divided into two cases, depending on whether the maximum load is greater than the balanced load or not. For convenience of subsequent explanation, the former is called CASE A and the latter CASE B.

With respect to β and γ, there is little qualitative difference between CASE A and CASE B. With respect to ρ_t, however, CASE A and CASE B indicate different trends. In CASE A e_2/h increases monotonically and in CASE B the increase rate of e_2/h becomes small gradually with increasing δ. From the results of these numerical test, it can be concluded that the ratio β has the most important effect on the additional eccentricity and other variables are secondary. The results of the various examinations found that the effect of β could be expressed by a cubic equation. The following expression was finally selected to calculate additional eccentricity:

$$\frac{e_2}{h}=(\beta+\beta_1)^3\left(\beta_2+\beta_3\delta+\beta_4\gamma+\beta_5\rho_t+\beta_6\rho_t\frac{f_y}{f'_c}+\beta_7\frac{f_y}{f'_c}+\beta_8\rho_t\frac{E_s}{f'_c}+\beta_9\frac{P_s}{P_{s,max}}+\beta_{10}\rho_t\frac{P_s}{P_{s,max}}\right)+\beta_{11} \qquad (2)$$

where P_s and $P_{s,max}$ are sustained loads and maximum loads, respectively. From a multiple non-linear regression analysis, the regression coefficients β_1 through β_{11} of CASE A and CASE B were determined. Table.2 shows the results.

Table. 2 Regression coefficients

Coefficient	CASE A	CASE B
β_1	14.76	46.65
β_2	5.48	0.966
β_3	6.44	0.313
β_4	-2.69	-0.435
β_5	-1.83	8.28
β_6	--	1.11
β_7	--	0.0176
β_8	--	-0.00206
β_9	5.13	0.331
β_{10}	-50.1	-2.36
β_{11}	-0.0217	-0.100
Standard Deviation	0.1082	0.06873
Coefficient of Variation	0.1074	0.06894
Square of Correlation Coefficient	0.9868	0.9794
Mean Value	1.007	0.9970

$\beta_2 \sim \beta_9$ ($\times 10^{-6}$)

4.3 Additional Eccentricity for Design

Equation (2) was obtained under the conditions that the age at the application of sustained load was 28 days and it continued for 25 years. To apply Eq.(2) to any age at loading and any loading period, creep coefficient ϕ is introduced. Complementary calculations for the columns which were first loaded at 35 days and 1 year, were carried out and the values corresponding to creep coefficients were calculated. The values ranged from 3.51 to 2.31 at 28 days, from 2.67 to 1.50 at 35 days, and from 1.05 to 0.66 at 1 year. From the results, ϕ=3 for 28 days and ϕ=1 for 1 year were approximately taken and the terms associated with sustained loading were multiplied by $(\phi-1)/2$. This means that the influence of sustained loading is neglected when ϕ=1.

The following equations on additional eccentricity for design, in which about 5% of the numerical test results are not conservative for CASE A and CASE B and f_y= 392 MPa and f'_c=19.6 MPa are assumed for CASE B, are proposed:

(a) $N_d > N_{bal}$ (CASE A)

$$e_2/h=(\ell_e/h+16)^3(5.48+6.4e_1/h-2.7\gamma-1.8\rho_t+C_N)\times10^{-6}-0.02 \qquad (3)$$

$$C_N=2.5(\phi-1)(1-10\rho_t)N_c/N_d$$

(b) $N_d \leqq N_{bal}$ (CASE B)

$$e_2/h=(\ell_e/h+48)^3(1.32+0.31e_1/h-0.44\gamma+8.9\rho_t+C_M)\ 10^{-6}-0.1 \qquad (4)$$

$$C_M=0.17(\phi-1)(1-7.3\rho_t)N_c/N_d$$

where ℓ_e is effective length of columns, N_d is design load, N_{bal} is balanced axial load, and N_c is design sustained load.

5. COMPARISON WITH TESTS

5.1 Previous Tests

An accumulation of test data has already been made to examine the validity of design formulas for slender reinforced concrete columns. In order to demonstrate the general validity of the proposed additional eccentricity, comparisons are made with a wide range of test data. A total of 626 tests have been examined, of which 381 tests were assembled by Cranston. Table.3 shows the test data adopted in this study.

5.2 Test Data

The types of concrete and steel used, age of first application of loads, and

Table. 3 Test data

Researcher	Date	Number of Tests	Kind of Columns			Loading Methods					
			P	F	B	S	L	L-S	L-C	L-C -S	C-S
Baumann[30]	1934	43	30	13		43					
Thomas[31]	1939	14	14			14					
Ramboll[32]	1951	38	38			38					
Ernst, et al.[33]	1952	8	8			8					
Gehler and Hutter[34]	1954	50	50			50					
Gaede[35]	1958	16	16			8	8				
Kordina[36]	1960	4	4			4					
Aas-Jakobsen[37]	1960	20	20			20					
Saenz and Martin[38]	1963	52		52		52					
Chang and Ferguson[4]	1963	6	6			6					
Breen and Ferguson[39]	1964	6		6		5		1			
Ramamurthy[40]	1965	55			55	55					
Martin and Olivieri[41]	1966	8	8			8					
MacGregor and Barter[42]	1966	8	4	4		8					
Furlong and Ferguson[43]	1966	7		7		6		1			
Ferguson and Breen[44]	1966	8		8		7	1				
Pannell and Robinson[45]	1968	16	9		7	16					
Green and Breen[46]	1969	5	5			1	4				
Mehmel, et al.[47]	1969	16	14	2		16					
Breen and Ferguson[11]	1969	10	10			10					
Ramu, et al.[48]	1969	37	37			6		19	12		
Cranston and Sturrock[49]	1971	8	3		5	8					

Continued

Researcher	Date	Number of Tests	Kind of Columns			Loading Methods					
			P	F	B	S	L	L-S	L-C	L-C-S	C-S
Drysdale and Huggins[50]	1971	57	8		49	26	16	15			
Goyal and Jackson[15]	1971	46	46			26		20			
Hellesland and Green[51]	1971	6	6							6	
Hirasawa[52]	1974	55	11		44	35		10	2	3	5
Kordina[53]	1975	12	12					12			
Blomeier and Breen[54]	1975	3		3		3					
Green and Hellesland[55]	1975	8	8			2			2		4
Gruber and Menn[56]	1978	4	4			4					
Total		626	371	95	160	485	48	71	4	9	9

P = Hinged, F = Constrained, B = Biaxially Loaded,

S = Short Time Loading, L = Sustained Loading, C = Cyclic Loading

curing conditions are different in each test. To avoid unnecessary complications in calculations, however, test data are uniformly treated under the assumptions described in the following. The same stress-strain relationships as those assumed in CHAPTER 3 are used for concrete and steel. Cube strengths are assumed to be 1.25 times cylinder strengths. Effective lengths, ℓ_e, are calculated using Eq.(20) through Eq.(23) of CP110. In the case of biaxial bending, ℓ_e/h is always considered under $b \geq h$. The initial eccentricity in the case of unequal eccentricity is determined by

$$e_{eq}=0.4e_{11}+0.6e_{12} \geqq 0.4e_{12} \qquad (5)$$

which has been adopted by ACI, CP110, and CEB-FIP. The initial eccentricity in columns of frame structure is a nominal value to be determined from moments and axial forces which are calculated by ordinary elastic analysis. Creep coefficients are evaluated by the following well-known equation:

$$\phi(t,t_0)=0.4\beta_d(t-t_0)+\phi_f(\beta_f(t)-\beta_f(t_0)) \qquad (6)$$

For all tests including sustained loads, 40% relative humidity is assumed. Creep coefficients calculated using Eq.(6) with 50% relative humidity, t_0=28 days, and t= 25 years, which have been considered in the numerical tests, were compared with those based on the numerical tests. From the results, it could be concluded that the effects of creep were evaluated by Eq.(6).

In this study the problem of biaxial bending is approximately considered as uniaxial bending. Namely, bending on weak axis is always considered and the reduction of load-carrying capacity by biaxial bending is evaluated by introducing an equivalent initial eccentricity. Due to the results of various examinations, the followimg equation was adopted as equivalent eccentricity($b \geq h$):

$$e_1/h=e_x/h+0.6\sin\theta \; e_y/b \qquad (7)$$

$$\tan\theta=e_y h/e_x b \qquad (8)$$

where e_x and e_y express the initial eccentricity of the direction of weak axis and strong axis, respectively.

5.3 Comparison with Tests

Figure.10 shows the histograms of the ratio of $N_{u,test}$ to $N_{u,cal}$ calculated using Eqs.(3) and (4). The histograms have been divided into the four categories of hinged, constrained, biaxially loaded, and creep failure columns.

Most of the results on creep failure columns are not conservative. Since it is rare in design to consider high sustained load by which creep fialure occurs in columns, creep failure columns have been excluded from the application of the design formula proposed in this study. Therefore, due consideration is required in the design of possible creep failure columns.

The histogram of $N_{u,test}/N_{u,cal}$ on constrained columns is similar to that on hinged columns. This means that the use of effective length is valid.

In the case of biaxially loaded columns, large values of $N_{u,test}/N_{u,cal}$ occur. Most of them are the test results obtained by Sturrock, et al. and Pannel, et al., in which ℓ_e/h is 50 and 41.6, and b/h is 4.0 and 1.48, respectively. Figure.11 shows the histograms on biaxially loaded columns excluding the data.

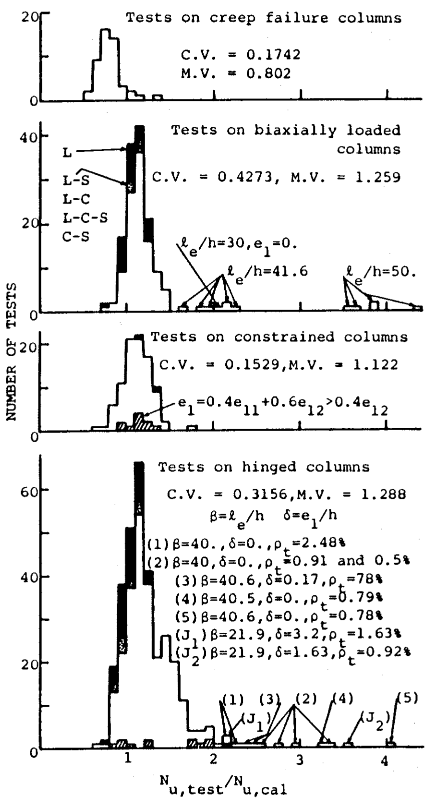

Fig. 10 $N_{u,test}/N_{u,cal}$ for various types of test

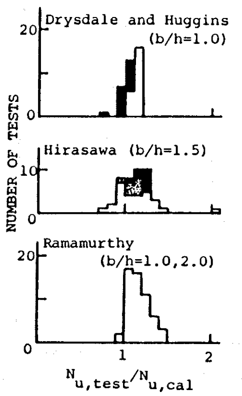

Fig. 11 $N_{u,test}/N_{u,cal}$ for biaxially loaded columns

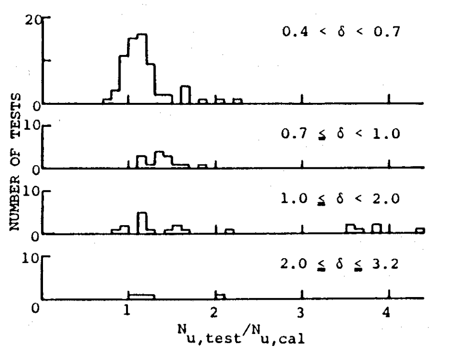

Fig. 12 $N_{u,test}/N_{u,cal}$ for δ>0.4

Similarly, large values of $N_{u,test}/N_{u,cal}$ occur in the case of hinged columns. Most of them are the columns of large slenderness, small reinforcement ratio, and/or no initial eccentricity. From this result, it may be said that in such cases the proposed design formula give a considerably conservative result.

There is no significant trend or variation of $N_{u,test}/N_{u,cal}$ for the test cases which include the sustained loadings and the cyclic loadings. Therefore, it seems that the proposed design formula is applicable to these cases.

Figure.10 also shows the results of cases subjected to unequal eccentric loads. Some hinged columns indicate relatively large values of $N_{u,test}/N_{u,cal}$. These results are test data cases carried by Martin, et al.. The other two cases in test data by Martin, et al., in which the use of equivalent eccentricity was unnecessary, also showed large values of $N_{u,test}/N_{u,cal}$. Therefore, it can be said that the use of equivalent eccentricity is not the cause. Thus, it seems that the equivalent eccentricity used are reasonable.

The range of δ considered in the numerical tests was between 0.1 and 0.4. Figure.12 shows the histograms used to examine the applicability to the cases of δ greater than 0.4. There is no significant difference between the histograms. The cases of $N_{u,test}/N_{u,cal}$ greater than 2 have already been explained. From these results, it can be said that the proposed design formula is at least applicable up to δ=3.

Figure.13 shows the histogram for all tests except for creep failure columns. There are 100 results of $N_{u,test}/N_{u,cal}$ less than 1. The values of $N_{u,test}/N_{u,cal}$ less than 0.85, of which there are 16, are less than 3% of the total data and the values of $N_{u,test}/N_{u,cal}$ less than 0.9, of which there are 34, are approximately 6% of the total data. In the calculation of $N_{u,cal}$, cylinder strengths have been used as concrete strengths. This means that the design formula have been evaluated under the condition which is not conservative with respect to concrete strengths, since real concrete strengths are usually less than cylinder strengths.

Table.4 indicates the number of data and the mean value and coefficient of variation of $N_{u,test}/N_{u,cal}$ calculated using the design formulas by DIN1045, CEB-FIP, CP110, and the authors. The small number of data in DIN1045 is due to its narrow range of application. In the case of CEB-FIP, $e_2=\ell_e^2/10\times\psi$ was

Table. 4 Reliability of various design formula

Design Equation	Number of Data	Coefficient of Variation	Mean Value
DIN1045	240	0.172	1.18
CEB-FIP	447	0.582	1.51
CP110	578	0.515 (o.493)	1.23 (1.27)
Author	578	0.337 (0.324)	1.25 (1.34)

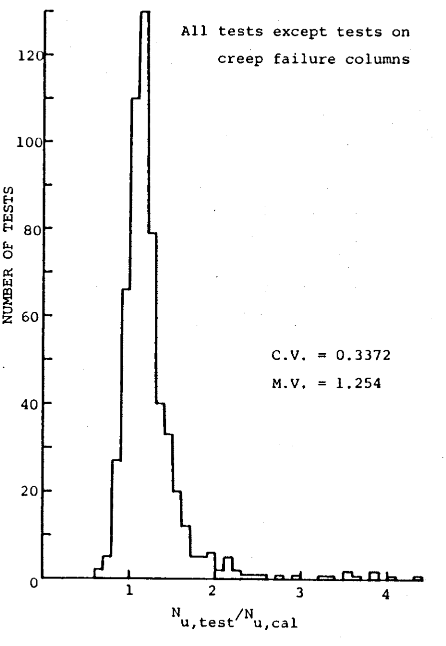

Fig. 13 $N_{u,test}/N_{u,cal}$ for all tests except for creep failure columns used as additional eccentricity, in which $\psi=5\times10^{-3}/d$ was taken according to the commentary. The smaller number of data in CEB-FIP than those of CP110 and the authors, is due to the exclusion of data other than $|e_{Y1}h/e_{X1}b|\leq0.2$, since there is no method to evaluate such cases.

The result of CP110 was calculated without considering adjustment factor, K, which is permissible to use when design load is smaller than balanced axial load. The values in parentheses are the cases in which the adjustment factor was considered and the concrete strength 0.838 times the cylinder strength was used. Generally, the calculation of $N_{u,cal}$ using small concrete strength gives a conservative histogram of $N_{u,test}/N_{u,cal}$. To the contrary, the consideration of K gives larger $N_{u,cal}$. Due to an offset of both effects, there is little change in the mean value. The authors' values in parentheses were calculated using the concrete strength 0.85 times the cylinder strength. In this case, the number of $N_{u,test}/N_{u,cal}$ less than 0.9 and 0.85 are 12 and 6, respectively.

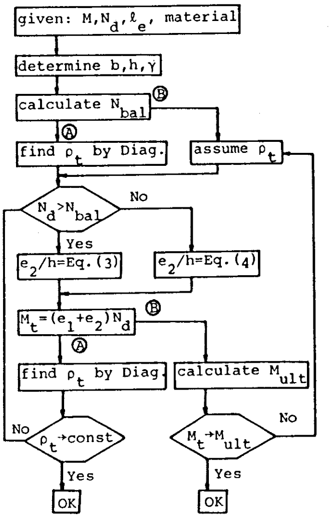

Fig. 14 Design procedure

From these comparisons with test data of a wide range, it can be concluded that the proposed additional eccentricity for design is adequately effective for practical use.

6. DESIGN METHOD

Figure.14 gives the design procedure of slender reinforced concrete columns subjected to combined bending and axial forces. If the diagrams of the relationships among moments, axial forces, and reinforcement ratio exist, FLOW A in the figure should be used. If this is not the case, FLOW B should be used. The outline of the design procedure is described in the following:

(a) determine section properties,
(b) calculate balanced axial force, N_{bal},
(c) in FLOW A find ρ_t from given M and N_d and in FLOW B assume ρ_t,
(d) calculate e_2 using Eqs.(3) or (4),
(e) calculate total moment, M_t, including additional moment,
(f) in FLOW A find ρ_t to M_t and N and repeat this procedure until it becomes constant, and in FLOW B calculate ultimate moments, M_{ult}, and repeat this procedure until M_t is less than or equal to M_{ult}.

7. SUMMARY AND CONCLUSIONS

The purpose of this study was to present an approximate design method on slender reinforced concrete columns. To accomplish it, numerical tests were carried out on slender reinforced concrete columns subjected to short time and sustained loads using an iterative procedure. Based on the regression equation introduced from the results of numerical tests, additional eccentricity for design was proposed. In order to examine the validity of the proposed design formula, comparisons were made with 626 test data cases and the results showed that the proposed design formula was adequately effective for practical use including the cases of constrained columns and biaxially loaded columns if effective lengths and equivalent eccentricities were appropriately considered. It could be concluded that the proposed design formula gave considerably good accuracy, compared with those of CP110 and CEB-FIP. Although the design formula of DIN1045 shows better result than those of the authors, the range of application is narrow.

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