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DESIGN METHOD FOR STRUCTURAL CONCRETE MEMBERS IN COMBINED TORSION AND BENDING

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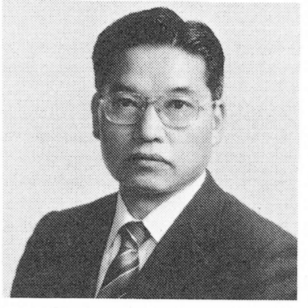

Mitsuaki IZUMI

SYNOPSIS

The object of the research described in this paper is to proposed the design method of ultimate strength for reinforced, prestressed and steel-reinforced concrete members of rectanglar and box section subjected to pure torsion, combined torsion and bending.
The approach is based on a space truss model in pure torsion and dominant torsional range, a skew-bending model in dominant bending range. The proposed equations are derived from considering both the equilibrium conditions and the compativility of deformation, and also the stress-strain characteristics of steel bars and concrete. The equations are capable of predicting the post cracking behavior of concrete members in pure torsion, combined torsion and bending.
Finally, the author proposes the design rule for structural concrete members in pure torsion, combined torsion and bending.

M.Izumi is a professor of civil engineering at Meijo University, Nagoya, Japan. He received his Doctor of Engineering Degree in 1981 from Tokyo Metropolitan University. He has the license of Japan Consulting Engineer.
His research includes the ultimate strength of structural concrete members in combined loads and the conservation of energy in constructions. He is a member of JSCE, JCI, JPCEA and ACI. He rewared JPCEA Prize in 1976, with "Torsional test on light-weight prestressed concrete members", and YOSHIDA Prize in 1983, with "Design method for structural concrete members under combined torsion and bending in ultimate state".

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1. INTRODUCTION

Combined bending with torsion may be considered as a general case of transverse bending when the plane of bending forces acting on the member is parallel to its longitudinal axis but does not pass through the shear center of cross section. Torsion arises as a result of the applied load acting normally to the longitudinal axis of the member exactly as eccentric compression arises from an eccentrically applied longitudinal force. Thus, axial compression is a particular case of eccentric compression, just as simple bending is a particular case of combined bending and torsion when the eccentricity of transverse forces to the shear center of the section is eqal to zero.

In the many researches, as changing of loading from pure torsion to pure bending the failure modes vary remarkably. Although, there are the design codes that the skew-bending theory(1) is applied to all loading cases. On the other hand, in CEB-Code, the space truss theory(2),(3) is applied to all loading cases. Design procedures which are based on rational models rather than empirical equations are enable to develop a better understanding of actual structural behavior to the engineers. In this research, from the results of tests, the modes of failure attempt to divide into two cases, i.e.,applying the space truss theory to the dominant torsional case and to the skew-bending theory to the dominant bending case. The autor shows the equation for the index of critical values of the dominant torsional range to the bending range of failure, and also the design method that in the dominant torsional range, the equations of space truss are applied and in the dominant bending range, the equations of skew-bending are applied to the concrete members in combined torsion and bending. Then, it is possible to design for concrete members reasonably to coincide with the modes of failure.

In this research, main themes,

a) The equation for Ko which is the index of critical values of the torsional failure to the bending failure at ultimate state is derived, and the applied theory to each failure mode is clarified.

b) In the dominant torsional range, the equations based on space truss, and in the dominant bending range, the equations based on skew-bending are given. The equations are derived from considering both the equilibrium conditions and the compatibility of deformations.

c) The equations for balanced reinforcement ratio in ultimate state, for the deformations at ultimate torsion, and for the stress of bars in over-reinforcement are given.

d) The interaction curve in combined torsion and bending at ultimate state is shown.

e) The design method for concrete members in combined torsion and bending is proposed.

2. EQUATIONS OF ULTIMATE STRENGTH IN COMBINED FORCES

The behavior of concrete members after cracking are divided into the two states, i.e., the first, the dominant torsional range, and the dominant bending range. In the first state, the equations based on space truss, and in the other, the equations based on skew-bending are derived.

2.1 Equations Based on Space Truss

The analysis of torsional behavior of concrete members after cracking, is based on the space truss(4) as shown in Fig.1.

In the truss, the diagonal concrete struts in compressive stresses are around the section of member at an angle α , and the reinforced bars are in tension. The tangential component of these stresses is provided the shear flow q which must be equilibrated the torsion. The normal components of the diagonal stress result in a longitudinal and transverse compression force which must be balanced by the bars.

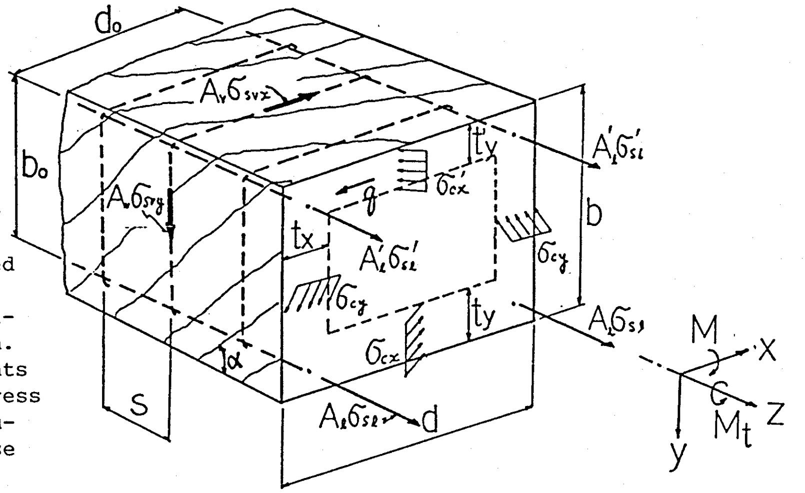


Fig.1 Space truss model and equilibrium of forces

The equations for geometrical condition, torsional moment and twist in ultimate state are,

Geometrical condition:

$$\tan \alpha = \sqrt{\frac{\varepsilon_l \, C4 + \varepsilon_c}{\varepsilon_v (Pv/Po) + \varepsilon_c}} \qquad (1)$$

Equation of depth of diagonal concrete strut(ab):

$$ab = \frac{1}{k1 \; k3 \; \sigma cu} \left\{ \frac{Av \; \sigma sv}{s \; C''1} + \frac{Al \; \sigma sl(1+C5)}{C2+C3} \right\} \qquad (2)$$

Effective torsional area(Am), and Path of shear flow(Po) are,

$$Am = (bo-ab)(do-ab)$$

$$Po = 2(bo+do-ab)$$

Shear flow(q) and torsional moment(Mt) are,

$$q = \sqrt{\frac{C''1 \; Av \; \sigma sv}{s} \; \frac{Al \; \sigma sl(1+C5)}{C2+C3}} \qquad (3)$$

$$Mt = 2Am \; q \qquad (4)$$

Stress of the longitudinal bar is given as follow,

$$\sigma sl = \frac{1}{2}\sqrt{\left\{\frac{\varepsilon cs\ Es(2-\beta c)}{2C4}\right\}^2 - \frac{2\ \varepsilon cs\ Es\ Am\ \sigma cu\ \beta c(C2+C3)k1\ k3}{C4\ Po\ Al(1+C5)}} - \frac{\varepsilon cs\ Es(2-\beta c)}{4\ C4} \tag{5}$$

Torsional stiffness of reinforced concrete member after cracking(GcKcu) is,

$$GcKcu = \frac{4Am}{\varepsilon cs\ k1\ k3\ \beta c\ \sigma cu}\left\{\frac{Av\ \sigma sv}{s}\ \frac{Al\ \sigma sl(1+C5)}{C2+C3}\right\} \tag{6}$$

In the dominant torsional range, as shown in Fig.1, at ultimate state the cracks occur on the four surfaces of member. Supposed that the bending moment(M') is resisted by the longitudinal bars only, in the case, neglecting the tensile strength of concrete, the tensile stress in longitudinal bars is,

$$\sigma sly = \frac{Mt}{\Sigma Al\ Dt} \pm \frac{M'}{Is}ysi \tag{7}$$

and then, from Eq(7), M' is,

$$M' = \frac{Is}{ysi}\left(\sigma sly - \frac{Mt}{\Sigma Al\ Dt}\right) \tag{8}$$

The equation above is the general form for bending moment(M'). It must be changed in accordance with the different arrangement of reinforced bars in the member.

2.2 Skew-Bending Equation

The concrete members with rectangular section under combined torsion and bending in dominant bending range, fail in the mode of skew-bending at ultimate state. Assuming the failure surface of skew-bending as shown in Fig.2, in formulating

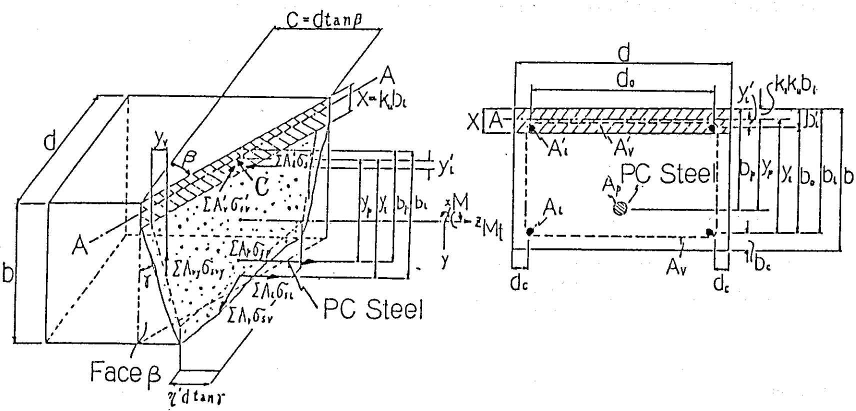


Fig.2 Skew-bending model and equilibrium of forces.

the condition of equilibrium in the general case in question, the same assumption have been made as for the particular case of simple bending, i.e., the tensile strength of concrete is neglected , also, it is assumed that all of the reinforcing bars crossing the crack reach their yield points with the formation of the plastic hinge. It is, furthermore, supposed that within the failure region, there are no local loads and there is no change in the reinforcement. The torsional center is unchanged in all stages of loading.

The derivation is based on equating the moments due to external forces and those due to internal forces about the axis of rotation which is inclined at an angle to the axis of the member. The following assumption are made,

1) when cracks open, the compression hinge is formed at an angle to the longitudinal axis of the member.
2) Concrete is completely ignored in tension.
3) The transverse reinforcement is uniform over the length under consideration.
4) There are no concentrated loads within the failure zone.
5) All reinforcements passing through a failure crack reach yield at failure.
6) The reinforcements are consisted by the longitudinal and transverse bars.
7) The ultimate moment is reached by yielding of the tension bars and or by crushing of compressive concrete.
8) The dowel action of longitudinal and transverse bars are neglected.

Equation for equlibrium of forces at the failure surface, from Fig.2, is,

$$C + \sum Al' \sigma sl' \cos\beta + \sum Av' \sigma sv' \sin\beta = \sum Ap \sigma sp \cos\beta + \sum Al \sigma sl \cos\beta + \sum Av \sigma sv \sin\beta \quad (9)$$

The equilibrium condition of moments can be deduced from the external moment equal to the internal moment around the axis running through the center of compression zone, and then bending and torsional moment can be calculated by,

$$\left.\begin{aligned} M &= \frac{Mp + Ml + Mvs - Ml' + (Mv - Mv')\tan\beta}{1 + K \tan\beta} \\ Mt &= \frac{(Mp + Ml - Ml' - Mvs)\cot\beta + Mv - Mv'}{1 + 1/K \cot\beta} \end{aligned}\right\} \quad (10)$$

The coefficient concerning to the hight of compression zone of concrete section is,

$$ku = \frac{\sum Ap \sigma sp + \sum Al \sigma sl - \sum Al' \sigma sl' + (nv\ Av \sigma sv - nv'\ Av' \sigma sv')\tan\beta}{k1\ k3 \sigma cu\ bl\ d \sec^2\beta} \quad (11)$$

The reinforcement ratios for the supposed surface shown in Fig.2 are the longitudinal ratio(pbl) and the transverse ratio(pbv). The strains of reinforcement are generally given by Eq(12), taking γ an angle between the tensile crack and the face perpendicular to the longitudinal axis of member. For simplicity, the strain of concrete is neglect, Eq(1) is,

$$\cot^2\gamma = \frac{\varepsilon 1}{\varepsilon v} \ \frac{C4}{Pv/Po} \quad (12)$$

The strain at the position of tensile longitudinal bar ($\varepsilon 1\beta$) along to the direction angle β (5) is,

$$\varepsilon 1\beta = \varepsilon 1 \cos^2\beta + \varepsilon v \sin^2\beta + \tau 1b \sin 2\beta \tag{13}$$

and then, the relation to $\varepsilon 1\beta$ and $\varepsilon 1$ are,

$$\varepsilon 1\beta = \varepsilon 1 \cos^2\beta \left\{1 + C4\ C10\ kp \tan^2\beta\ (2 + C10 \tan^2\beta)\right\} \tag{14}$$

From the balance of forces, the balanced reinforcement ratio for surface β is,

$$p\beta b = \frac{k1\ k3}{k\sigma 1} \frac{\varepsilon c\beta u'}{\varepsilon c\beta u' + \varepsilon 1\beta y} + p\beta' \frac{k\sigma 1'}{k\sigma 1}\left(\frac{\varepsilon 1\beta u'}{\varepsilon 1\beta y} - \frac{bl'}{bl}\frac{\varepsilon 1\beta u \pm \varepsilon 1\beta y}{\varepsilon 1\beta y}\right) + p\beta 1p \frac{k\sigma p}{k\sigma 1}\left(\frac{\varepsilon c\beta u'}{\varepsilon 1\beta y} - \frac{bp}{bl}\frac{\varepsilon c\beta u' + \varepsilon 1\beta y}{\varepsilon 1\beta y}\right) \tag{15}$$

From Eq(14) and equation for equilibrium of forces, the balanced reinforcement ratio for longitudinal direction to the member(plb) is given. When the member is over-reinfoced, it is necessary to know the stress of bars for the calculation of the ultimate torsional strength of members with concrete strain at ultimate state. The stress of bar is given by,

$$\sigma s1^3 + a''\sigma s1^2 + b''\sigma s1 + c'' = 0 \tag{16}$$

The equation for the angle β is,

$$\tan\beta = -\frac{M}{Mt} + \sqrt{\left(\frac{M}{Mt}\right)^2 + \frac{Fl}{Fv}} \tag{17}$$

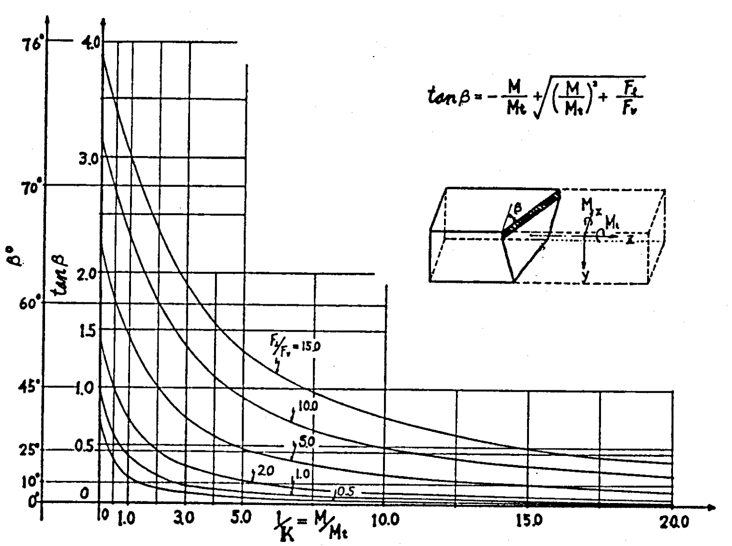
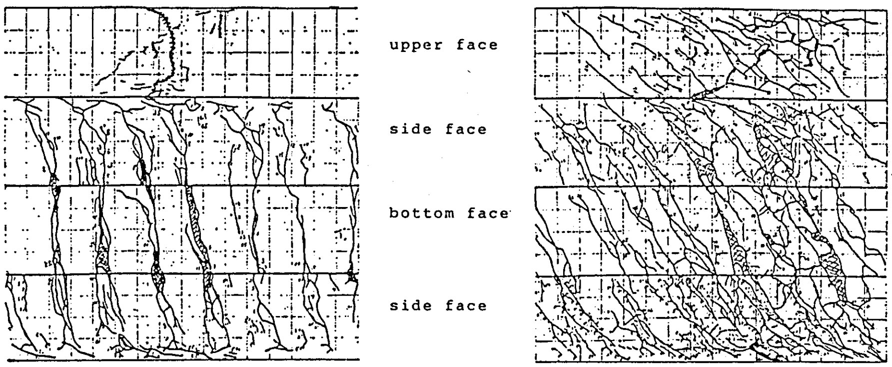


Fig.3 Relations between β and 1/K, Fl/Fv.

The relations between angle β to Mt/M, Fl/Fv are shown in Fig.3.

3. EQUATION FOR Ko WHICH DIVIDE INTO THE TORSIONAL RANGE AND BENDING RANGE

In the dominant torsional range, the cracks occur in all surface of concrete member at ultimate state, it is resonable to apply the space truss theory to the member. In the dominant bending range the cracks occur in three surfaces without compression side, it can apply the theory of skew-bending for the member. See, Fig.4.



K=o.27 Bending failure

K=1.52 Torsional failure

Fig.4 Cracking after failure.

Therefore, it is necessary to find the value Ko which indicate the critical point of different failure modes for concrete members. The value of Mt/M which is characterized by the condition of zero stress in the upper longitudinal bars, is the index of critical point of different failure modes, as shown in Fig.4.

The compression stress ($\sigma slb'$) in upper reinforced bars by bending moment is,

$$\sigma slb' = \frac{M}{\Sigma Al'\ Db'} \qquad (18)$$

The tensile stress ($\sigma slt'$) in upper reinforced bars which is given by the space truss theory is,

$$\sigma slt' = \frac{Mt}{\Sigma Al'Dt'} \qquad (19)$$

and then, the stress in upper bars is zero,

$$\sigma sl' = \sigma slt' - \sigma slb' = 0 \qquad (20)$$

From relation to equations of (18),(19) and (20), Ko which indicate the stress in upper bars being zero, is,

$$Ko = Mt/M = Dt'/Db' \quad (21)$$

In accordance with the equation (21), the application of the equation(4)and(10) are as follows.

The equations of space truss, when;

$$Ko < Mt/M \quad (22)$$

The equations of skew-bending, when;

$$Ko \geqq Mt/M \quad (23)$$

The relationship between Ko and the failure modes of concrete member in combined moments is shown in Fig.5. The values of Dt' and Db' are given by the shape of cross-section of member, by the contents of reinforcement, by the displacement of bars, etc.. According to avove, Ko is mainly determined as the values characteristics of the member.

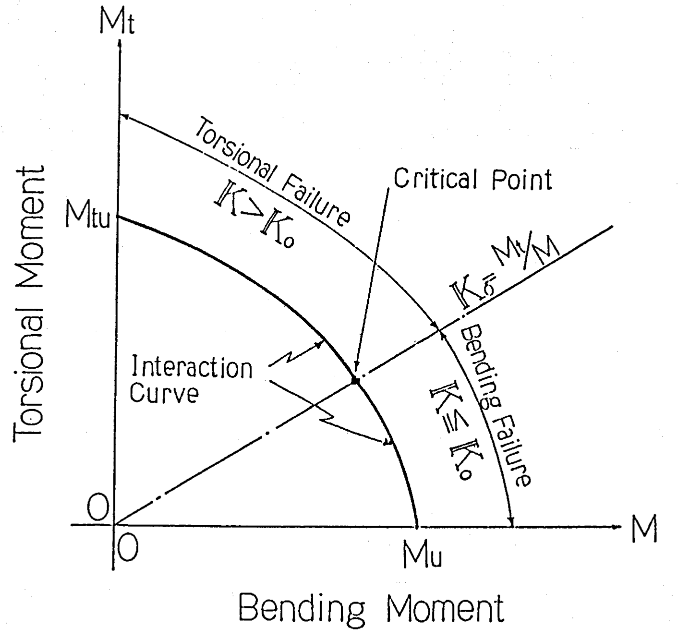


Fig.5 Failure modes in combined torsion and bending.

4. INTERACTION CURVE FOR TORSION-BENDING IN THE ULTIMAT STATE

The interaction at ultimate state of concrete member subject to combined torsion and bending has been studied by Elfren, Thürlimann, Kuyt and Collins. Author proposed the equation of interaction curve for combined torsion and bending based on the stress of longitudinal bars and equation (4).

$$\frac{1}{A''}\left(\frac{Mt}{Mtu}\right)^2 + \frac{B''}{A''}\,\frac{M}{Mu} = 1.0 \quad (24)$$

A graphical representation of the equation (24) is shown in Fig.6.

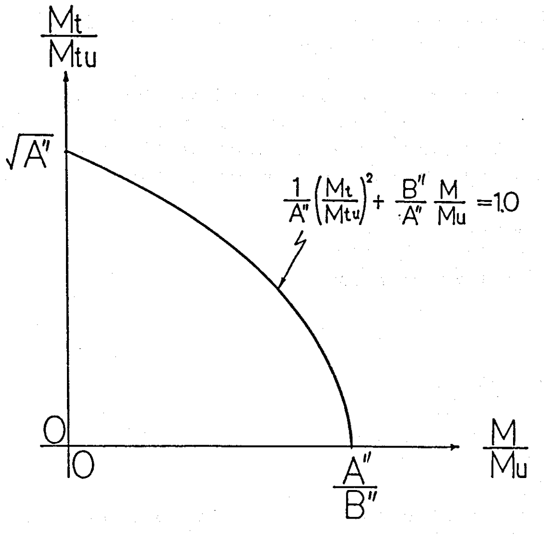
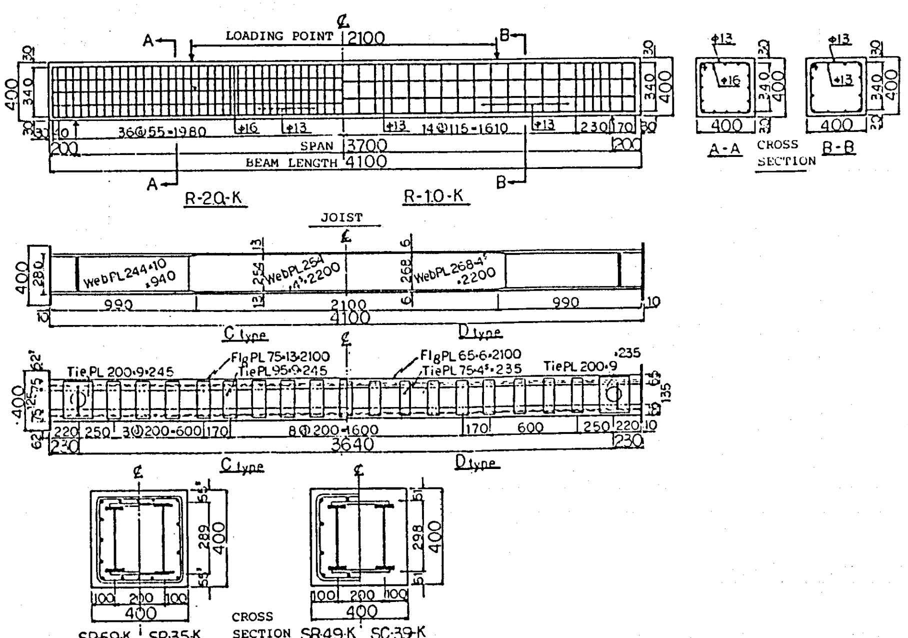


Fig.6 Interaction of combined torsion and bending at failure. (members having equal top and bottom bars).

5. EXPERIMENTAL WORK(6)

The 15 beams with square section (40cm x 40cm, 3m70cm. span) were divided into two series (reinforced concrete, steel-reinforced concrete), the beams within each series being similar. Details of the reinforcement are given in Fig.7.



(Unit, mm)

Fig.7 Details of beams of reinforced and steel-reinforced concrete.

For the concrete used in this investigation σ_{28} = 27.4 MPa and all reinforced steel was SS41 grade, round bars and steel plate for joist having yield strength of most steel being closed to 293.5 MPa.

All of the beams were tested by loading arms as shown in Fig.8. They were simply supported over a span of 3.70m. on a special type of saddle which allowed free rotation of the ends. Two cantilevered loads at the two points center to center 2.10m applied the combined torsion and bending moment.

The results of the experiments are summarized in Table.1. In Fig.4, the crack pattern after failure of beams are shown.

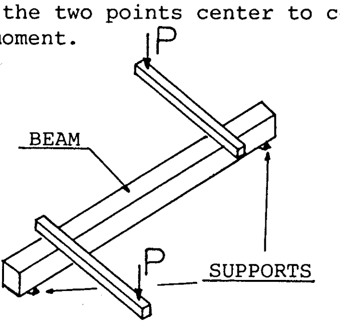


Fig.8 Loading system for combined torsion and bending.

6. COMPARISON OF RESULTS

The comparison of test results and theory are given in Table.1. From author's tests results, there are the discrepancy between the tests results and theory, but it is able to calculate the Ko, M', Mt and M at ultimate state clearly.

Table.1 Tests and results

BEAM	GROUP	SEC-TION	STEEL RATIO	K=Mt/M Ko	K	MODE of FAILURE	ULT' MOMENT TEST (1)	CALCUL (2)	(1)/(2)	NOTE
R-1.0	R C	Squ-are	1.0	0.72	0	bending	75.4	78.4	0.95	
					0.27	"	66.6	74.6	0.89	bending
							24.1	20.1	1.19	torsion
					0.81	"	65.6	60.1	1.09	
							53.4	48.7	1.10	
					1.52	torsion	38.2	35.5	1.08	
							58.2	45.3	1.28	
					∞	"	79.6	71.9	1.10	
SR-4.9	SRC	Squ-are	4.86	o.49	0	bending	235	232	1.01	
					0.21	"	324	337	0.98	
							69.1	70.5	o.98	
					0.56	torsion	222	108	2.04	
							124	142	0.88	
							245	102	2.40	
							137	149	0.92	
					1.02	"	151	109	1.38	
							155	141	1.09	
							155	107	1.44	
							159	143	1.10	
					∞	"	204	211	0.96	
							Average		1.19	

7. DESIGN PROCEDURES

Design procedures for structural concrete members subject to combined torsion and bending moment are not yet fully developed and codified. This paper sets out what is believed to be a reasonably conservative design procedure for concrete members subject to combined torsion and bending moment. It is based on the author's evaluation of studies for combined moments behavior so far completed, and the equations proposed by author. In Fig.9, the outline of flow chart for design procedures are shown.

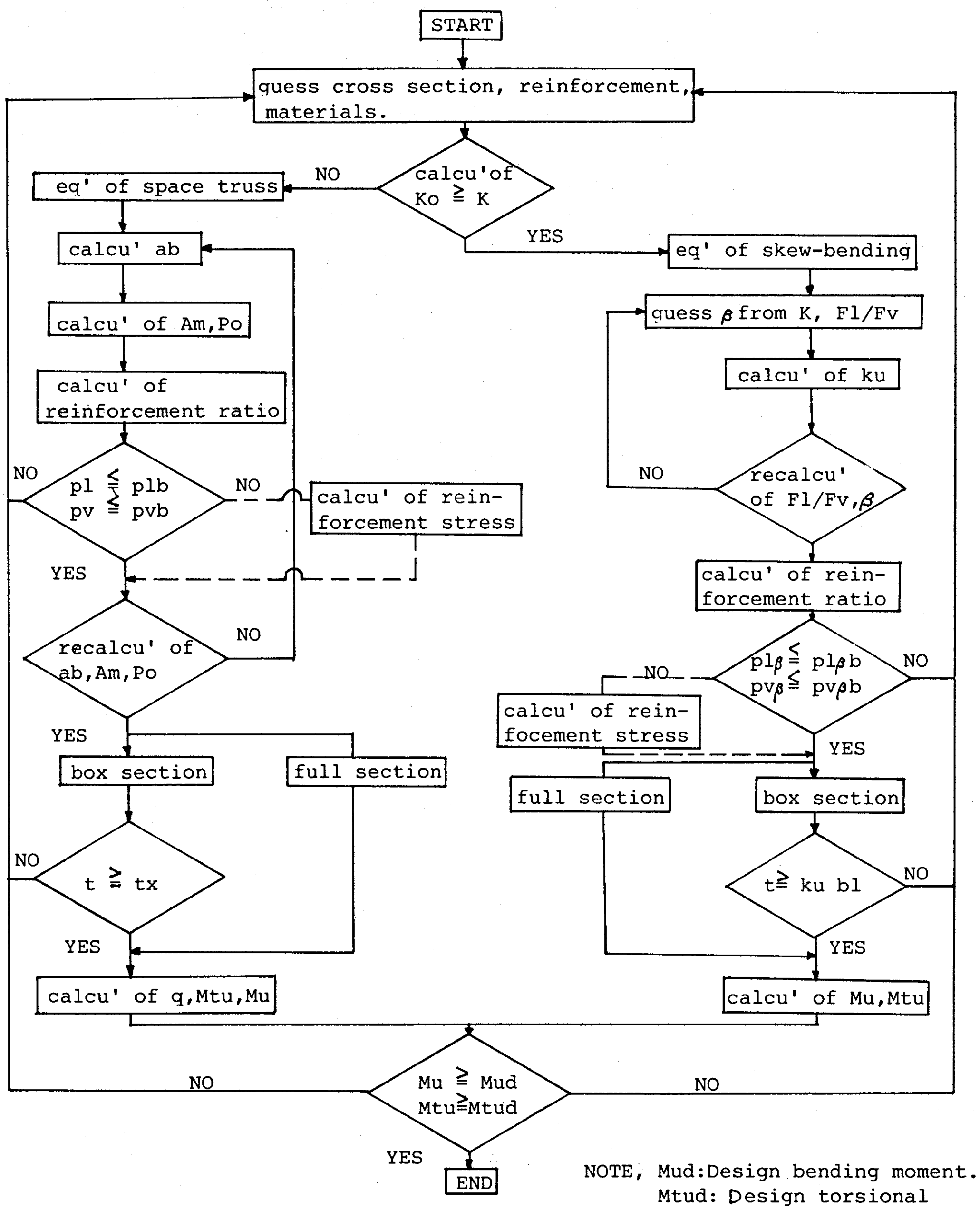


Fig.9 Flow chart of design procedure.

The design equations above are complicated with many parameter, and then, it is convienient to use the computor for the design of strucures.

8. CONCLUDING REMARKS

The space truss and skew-bending models are capable of predicting the post-cracking behavior of concrete members in combined torsion and bending in the ultimate state. Some of the capabilities of the study are listed below;

1. The modes of failure predicted by the proposed equation(21) are in agreement with those observed in author's tests, and then, the equation Ko shows the application range of the equation(4),(8) and equation(10).
2. The combined torsion and bending strength of under-reinforced, partially-over-reinforced and completely-over-reinforced concrete members can predicted by the equations proposed by author.
3. The equations can be applied to concrete members(reinforced, steel-reinforced and prestressed concrete) having a rectangular and box section.
4. The stresses of steels between post-cracking and ultimate state can be calculated by the equation(5) and (16).

The reasonable and integrated design of concrete members in combined torsion and bending moment are carried out by applying the equations proposed by author. The equations presented in this paper are applicable only to St'Venant torsion and hence can be treated the response of sections where warping torsion dominant by approximate procedures.

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NOTATION

A" = (σsv σsl)/(σsvy σsly)
Al,Al'= area of longitudinal reinforcing steel in bottom, in upper, respectively
Am = area enclosed by shear flow.
Ap, Ap'= area of longitudinal prestressing steel in bottom, in upper, respectively.
Av, Av'= area of one transverese steel in bottom, in upper, respectively.
Avs = area of one transverse steel in sides.
ab = equivalent depth of compression strut.
B" = σsv/(σsvy(1+C5)).
b = width of cross-section
bo = smaller center-to-center dimension of closed rectangular stirrup.

$C"1 = 1+b/2d(k11\ k13 + k12\ k14)$
$C2 = 1/2\ (k11\ (k13d-tx(1+k13))+b)$
$C3 = 1/2\ (k12\ (k14d\ tx(1+k14))+b)$
$C4 = 2(C2+k\varepsilon\ C3)/(po\ C1")$
$C5 = (Al'\ \sigma sl')/(Al\ \sigma sl)$
$C10 = do/(2(b-x)+d)$
$Cp" = Ap\ \sigma sp/\ Al'\ \sigma sl'$
d = depth of cross-section
do = larger center-to -center dimention of closed rectangular stirrup

$$Db' = Cp"(\frac{bp}{bl} - k2\ ku)\ bl + \frac{1}{C5}(1-k2\ ku)bl-(\frac{bl'}{bl} - k2\ ku)bl+ \frac{C1\ \tan^2\beta\ \tan^2\alpha}{2C3}$$

$(d-2dc- \eta d')Dv$
Dv = coefficient of reinfocement in sides

$$Dt' = Am\ \tan\alpha\sqrt{\frac{C1\ C"1(1+C5)}{C3\ C5\ (C2+C3)}}$$

Es = modulus of elasticity of steel
Gc = modulus of rigidity of concrete
K = Mt/M
Kcu = torsional stiffness of reinforced concrete at ultimate torsion
$k\varepsilon = \varepsilon l'/\varepsilon l$
s = transverse steel (stirrup) spacing
α = angle of diagonal cracking
β = an equivalent rectangular stress block factor, or angle between the compression face and the face perpendicular to the longitudinal axis of member
γ = angle between the tensile crack and the face perpendicular to the longitudinal axis of member
ε_c= concrete diagonal strain at the position of the resultant shear flow
ε_{cs}= concrete diagonal strain at the surface
ε_l= strain in longitudinal steel
ε_v= strain in transverse steel (stirrup)
σcu = compressive strength of concrete
σsl, $\sigma sl'$ = stresses of longitudinal steel in bottom, in upper, respectively
σsv = stresses in transverse steel (stirrup)
σsly, σsvy = yield point of longitudinal, transverse steel, respectively
a", b", c" = coefficients of the equation of σsl