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FATIGUE STRENGTH OF REINFORCED CONCRETE SLABS FAILING BY PUNCHING SHEAR

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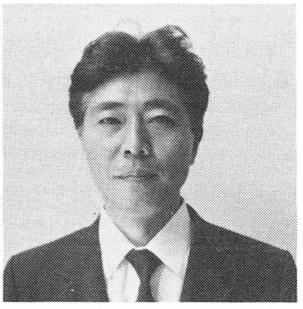

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SYNOPSIS

The purpose of this study is to investigate the fatigue strength of reinforced concrete slabs failing by punching shear. A total of 108 slabs were tested under constant or combined cyclic loads. The results of the tests show that the number of loading cycles to failure has a logarithmic normal distribution and the fatigue strength for one million cycles is about 55 % of the ultimate strength, and that Miner's hypothesis is applicable to the punching failure. Finally, the design equation of fatigue punching strengths is proposed.

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1. INTRODUCTION

The investigation regarding the fatigue of concrete structures started at the end of the 19th century [1]. Since then a number of reports have been presented, however, most of them are limited to the flexural or compression failure of concrete or the tension failure of reinforcing bars. Knowledge of fatigue strength for the other types of failure is quite short at the moment. Under the circumstances the recent investigations by Higai [2][3] on the fatigue shear strength of reinforced concrete beams are worthy of note.

The purpose of this study is to investigate the fatigue strength of reinforced concrete slabs in punching shear.

2. TEST SPECIMENS AND PROCEDURE

The tests were carried out by seven series A-G. The series A and B were preliminary tests and the series G was a supplementary one. The main parts of the tests were as follows;

Series C --- The specimens were 140 cm × 120 cm rectangular slabs of 10 cm thickness which were reinforced orthogonally by deformed bars with a diameter of 10 mm and a yield point of 3850 kg/cm^2 (377 N/mm^2). The reinforcement ratios in the direction of the span and the transverse direction to the span were 0.99 % and 0.80 %, respectively. High early strength cement, coast sand and river gravel were used as the materials of concrete. The slabs were wet-cured for seven days and then stored in the laboratory for more than two weeks before tests. The average compressive strength of concrete was 351 kg/cm^2 (34.4 N/mm^2) at the test.

The slabs were supported along the longer sides and a load was acted on the slab center through a square steel plate with the side length of 10 cm. The span length was 100 cm and the corners of the slabs were free to lift.

First four slabs were statically loaded up to failure. Each of them failed by punching shear and the average ultimate load was 13.7 tons(134 kN). Then a total of 39 slabs were tested under constant cyclic loads. The maximum load level, S_o, which was defined as the ratio of the maximum cyclic load to the ultimate load (13.7 tons), varied between 0.45 and 0.85 at intervals of 0.05. The minimum cyclic load was kept constant to be 1.0 ton(9.8 kN), which corresponded to S_u of 0.07. The rate of load repetition was 250 cycles per minute.

Series F --- The specimens of this series were the same as those of the series C, except for bar spacings. The reinforcement ratios in the longitudinal and transverse directions were 1.98 % and 0.68 %, respectively. The static tests on four slabs showed that the failure was caused by the punching shear and the average ultimate load was 15.7 tons(154 kN). The fatigue tests were carried out on 21 slabs with S_o of 0.65, 0.70 and 0.75. The average compressive strength of concrete was 345 kg/cm^2 (33.8 N/mm^2).

Series D and E --- In these series combinations of two or more different values of S_o were applied to 18 slabs. The dimensions of the slabs were the same as those of the series C. Fig. 1 shows the typical loading processes in the series E, in which Δn_1 and Δn_2, or Δn_3 and Δn_4 are a tenth or a fifth of the number of loading cycles to failure under constant cyclic loads. The average compressive strength of concrete was 380 kg/cm^2 (37.2 N/mm^2).

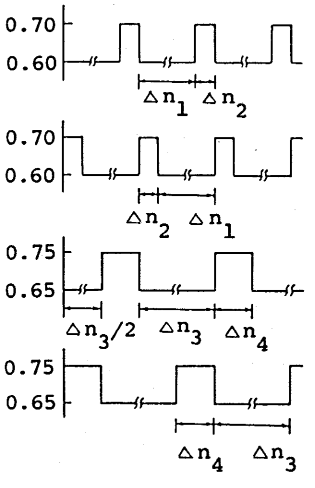

Fig.1 Typical loading processes in the series E

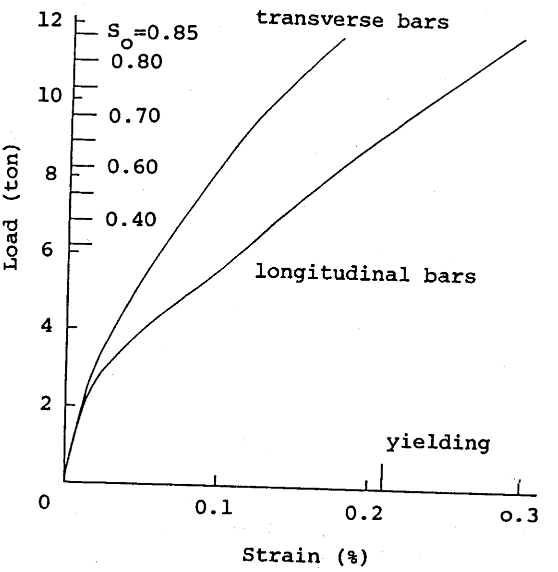

Fig.2 Strains of reinforcing bars at the first loading - Series C

3. TEST RESULTS AND DISCUSSION

3.1 Cracking and Strains

Each slab cracked at the load of 2 to 3 tons (20-30 kN) and orthogonal crack pattern was found in the central part of the slab at the first loading. Fig. 2 and 3 show the average strains of reinforcing bars near the slab center during the first loading. As seen in these figures the strains of longitudinal bars (bars in the direction of the span) were greater than those of transverse bars and reached yield point at about the load of 70 % of the ultimate load in the series C, while in the series F the strains of transverse bars were greater than those of longitudinal bars and reached yield point at about the load of 75 % of the ultimate load.

Fig. 4 - 7 show the strains of reinforcing bars and concrete, and crack widths under repeated loads. It is found that the strains and crack widths gradually increase with the number of loading cycles, n , however, these properties are stable in the range of large values of n.

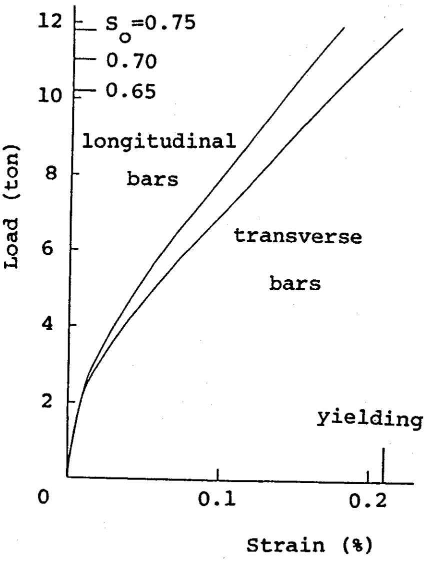

Fig.3 Strains of reinforcing bars at the first loading -Series F

3.2 Mode of Failure

All slabs in the series C and F failed by punching failure under repeated loads. However, in a part of the slabs in the series C fracture of longitudinal bars was also observed.

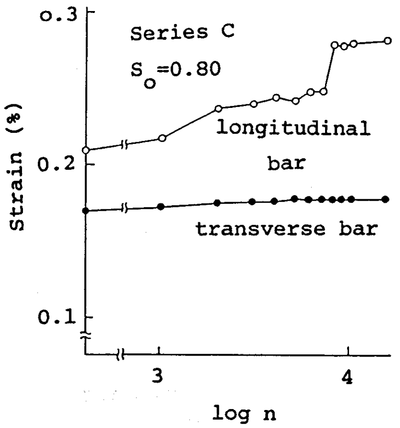

Fig.4 Strains of reinforcing bars under repeated loads

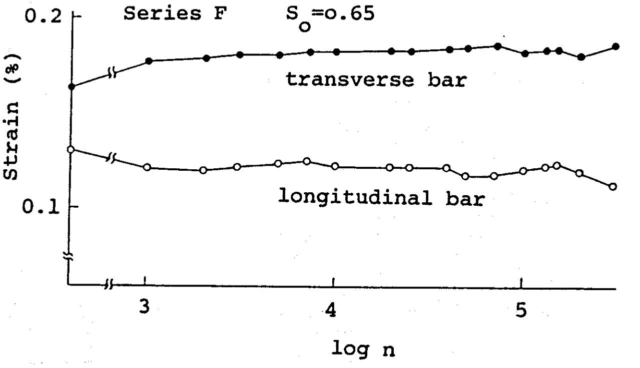

Fig.5 Strains of reinforcing bars under repeated loads

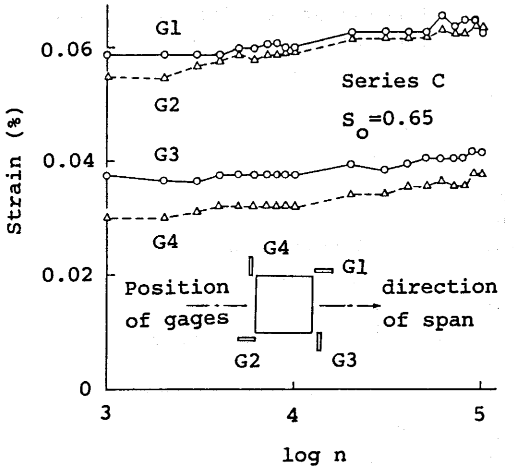

Fig.6 Strains of concrete under repeated loads

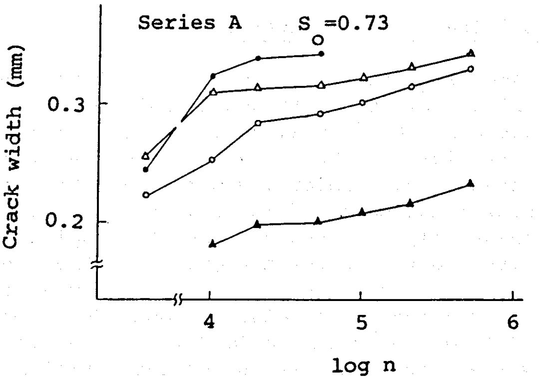

Fig.7 Crack widths under repeated loads

Fig. 8 and 9 show the relation between S_o and log N from the data of the series C and F, respectively, in which N represents the number of loading cycles to failure. From Fig. 8 it is found that fracture of bars occurred only in the case of low values of S_o and that the inclination of S_o-log N curve for the fracture of bars (broken line in the figure) clearly differs from the one for the punching failure. These results mean that the values of N in this case strongly associated with the fatigue strength of bars in spite of the apparent mode of punching failure. This can easily be understood because the stresses of longitudinal bars were high enough to cause fatigue failure even when the load was only about 50 % of the ultimate load, as seen in Fig. 2.

It can generally be said that the ultimate load of a slab is fairly larger than the load at which the yielding of reinforcing bars takes place. Therefore, the fatigue of bars may play an important part in the range of moderate values of S_o. It can also be said that even in the slabs designed to fail statically by the punching failure it should be necessary to check up safety against fatigue failure of reinforcing bars as well as fatigue punching failure under repeated loads.

3.3 Fatigue Strength under Constant Loads

The distributions of log N for each value of S_o are drawn on the normal distribution probability papers as shown in Fig. 10 and 11. According to these figures the inclination of the distribution curves varies with the variation of S_o, however, each of the curves is almost linear. It can be concluded that the distribution of fatigue life is approximately a logarithmic normal distribution.

Fig. 8 and 9 show almost linear relations between S_o and log N. A regression analysis of the data of S_o not less than 0.60 in the series C gives the following equation;

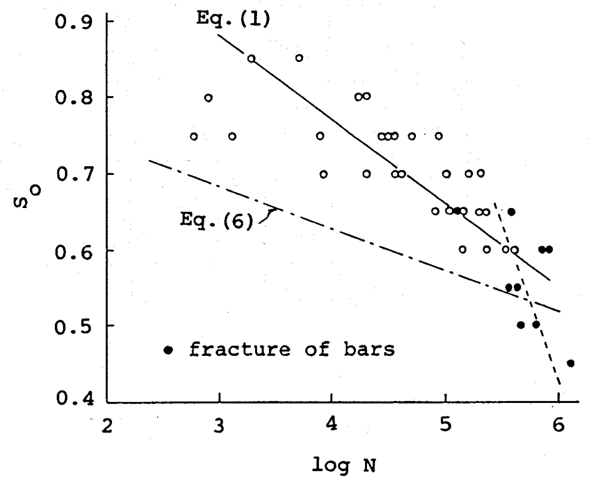

Fig.8 S_o-log N relation - Series C

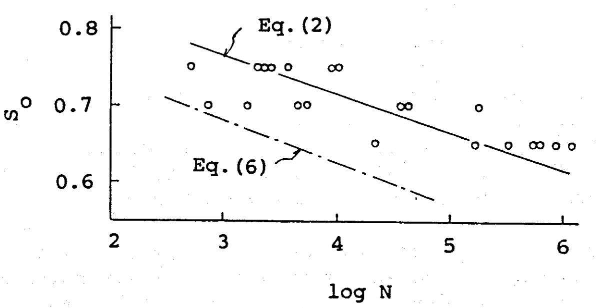

Fig.9 S_o-log N relation - Series F

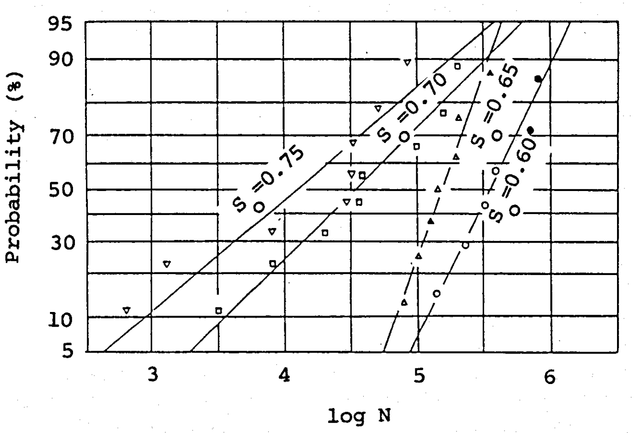

Fig.10 Distribution of log N - Series C

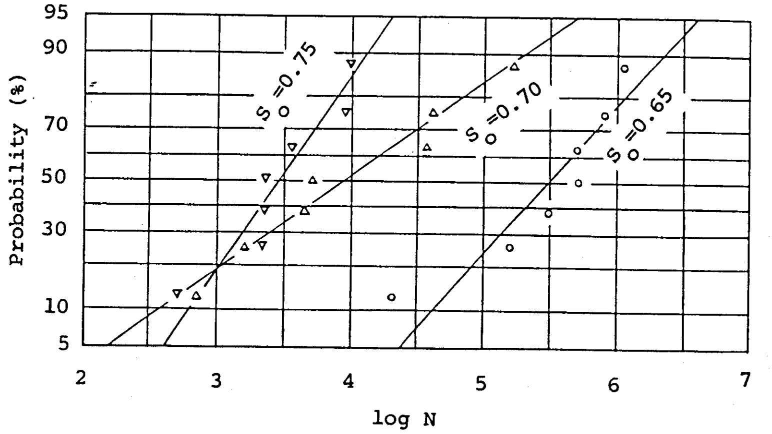

Fig.11 Distribution of log N - Series F

$$\log N = 10.90 - 8.94\ S_o \quad (1)$$

Similarly from the data of the series F the following equation is obtained;

$$\log N = 18.56 - 20.36\ S_o \quad (2)$$

According to Eq. (1) and (2) the fatigue strengths for one million cycles are calculated as 0.55 and 0.62 of S_o, respectively. These results are for the case of S_u=0.07 in which the tests were carried out. If the modified Goodman diagram is assumed, one million strengths for the case of S_u=0 are estimated at 0.51 and 0.59 for the slabs in the series C and F, respectively.

The results mentioned above are compared with the fatigue strength in shear of reinforced concrete beams without shear reinforcement, as follows. Chang and kesler [4] performed fatigue tests of beams in the case of about 0.02 of S_u and obtained S_o=0.63 as one million strength. Stelson and Cernica [5] obtained S_o=0.60 - 0.65 as 5×10^5 strength by their own tests with S_u=0.1 - 0.2. Their results correspond to about 0.56 of S_o for the case of S_u=0, by assuming modified Goodman diagram. Higai [3] conducted fatigue tests on the beams with lightweight aggregate concrete for the case of S_u=0.10 -0.15 and obtained S_o=0.55 -0.65 as one million strengths. His results correspond to about 0.55 of S_o for the case of S_u=0.

It can, therefore, be concluded that one million strengths of reinforced concrete slabs in punching shear is almost the same as those in shear of reinforced concrete beams without shear reinforcement and about 55 % of the ultimate strength.

3.4 Fatigue Strength under Combined Loads

The fatigue strength of a material for combined cyclic loads is an extremely difficult problem. Various approximate methods for this problem have been proposed as a cumulative damage theory. Miner's hypothesis is one of the most practical theories in which the cumulative damage, D , is defined by Eq.(3) and the fatigue failure is assumed to occur when D reaches 1.0.

$$D = \sum (n_i/N_i) \quad (3)$$

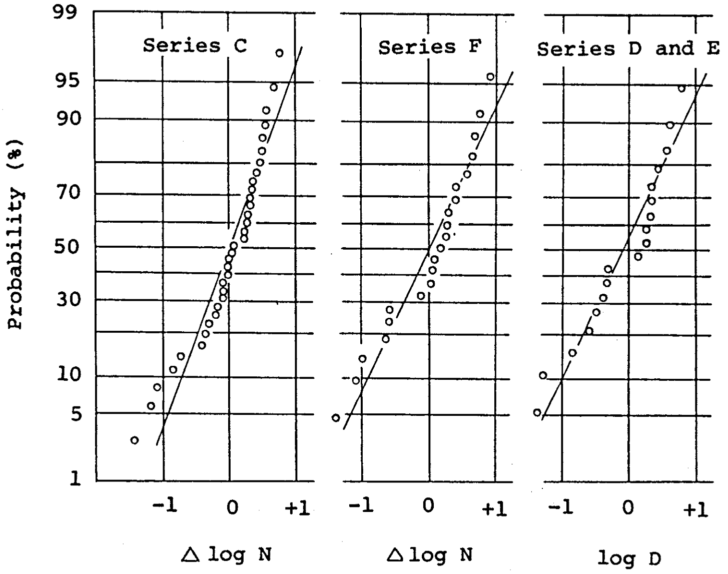

Fig.12 Distribution of △ log N and log D

in which n_i represents the number of cycles of the load level S_{oi} of the combined loads and N_i the number of cycles to failure under a constant load level of S_{oi}

The values of D were calculated from the results of tests on a total of 18 slabs of the series D and E. 10 slabs had D greater than 1.0 and the remaining 8 had D less than 1.0.

When the fatigue life N has a logarithmic normal distribution as mentioned earlier, it can be considered that the cumulative damage D also has a logarithmic normal distribution. Fig. 12 shows the distribution of log D on the normal distribution probability paper in comparison with the distributions of the deviations, △log N, of the observed values of log N in the series C and F from the values calculated by Eq. (1) and (2), respectively. No difference can be seen between the distribution of log D for combined loads and those of △log N for constant loads. The average value of log D is -0.10, which corresponds to D=0.79. From these results it can be concluded that the Miner's hypothesis is applicable to the punching failure of reinforced concrete slabs.

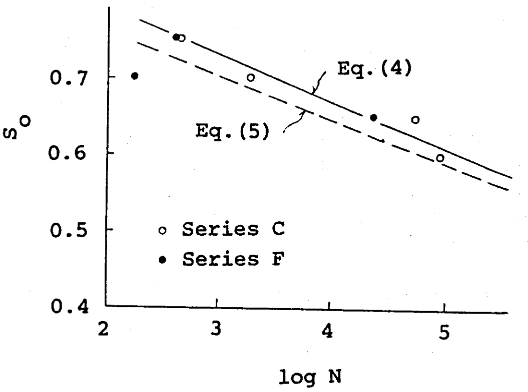

Fig.13 S_o-log N relations with 5 % failure probability

3.5 Proposed Design Fatigue Strength

Eq. (1) and (2) give the fatigue lives with 50 % failure probability.

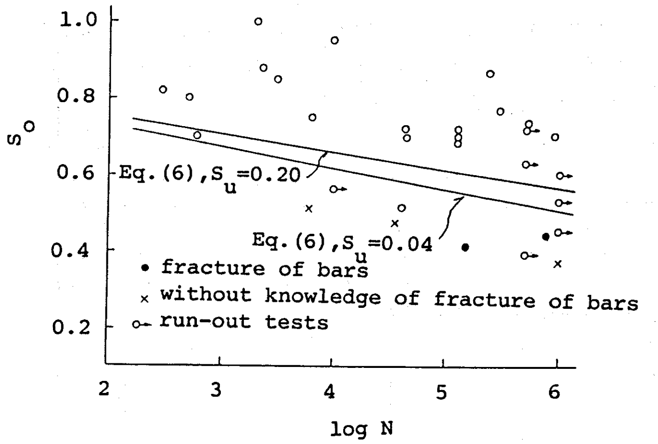

Fig.14 Test data of the series A, B and G

However, the equation with less probability is necessary for the desin of slabs in practical use.

The values of log N corresponding to the 5 % failure probability are obtained from Fig. 10 and 11 for each value of S_o as plotted in Fig. 13 and the regression lines are as follows;
For the series C

$$\log N = 15.24 - 16.80\, S_o \tag{4}$$

For the series F

$$\log N = 15.51 - 17.77\, S_o \tag{5}$$

Assuming modified Goodman diagram and enveloping the above two equations on the safe side, the following equation is proposed for the fatigue design of reinforced concrete slabs in punching shear.

$$\log N = 14\,\frac{0.85 - S_o}{0.85 - S_u} \tag{6}$$

The comparisons between the above equation and the test data of the series C and F are shown in Fig. 8 and 9. Fig. 14 shows the test results of the series A, B and G, in which the tests were carried out for S_u = 0.04 - 0.20. It is found that most of the data, except for the data of fracture of reinforcing bars, are on the safe side which Eq. (4) covers.

It has also been found that the values of the cumulative damage, D , are greater than 1.0 for all of the slabs of the series D and E, by applying Eq. (6) to the calculation of N_i in Eq. (3).

4. CONCLUSION

In the present study the fatigue strength of reinforced concrete slabs in punching shear are investigated experimentally. The results obtained are summarized as follows;

(1) Strains and crack widths gradually increase under repeated loads, however, their properties are stable till just before failure.

(2) Even if a slab fails by punching failure under static loads, fatigue failure of reinforcement as well as fatigue punching failure can take place under repeated loads.

(3) The number of loading cycles to failure by punching shear, N , has almost a logarithmic normal distribution.

(4) A linear relationship between the maximum load level and log N may be assumed.

(5) Fatigue strength in punching shear for one million cycles is about 55 % of the ultimate strength in the case of the minimum load level being zero.

(6) Miner's hypothesis may be applied to the fatigue punching failure due to combined loads.

(7) The following equation is proposed for the fatigue design of reinforced concrete slabs in punching shear;

$$\log N = 14 \frac{1 - P_o/0.85\ P_s}{1 - P_u/0.85\ P_s}$$

in which P_s represents the ultimate static load, P_o and P_u the maximum and minimum cyclic loads, respectively.

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APPENDICES

Tab.1 Number of cycles to failure - Series C

S_o	N					
0.85	1 900	520				
0.80	800	17 700	20 000			
0.75	600 50 400	1 300 85 800	8 000	29 300	31 400	32 100
0.70	3 100 156 900	8 100 204 300	19 500	35 800	37 000	100 000
0.65	78 600 365 400*	105 100	123 700*	137 800	201 800	204 700
0.60	137 200	228 500	325 500	379 600	716 400*	788 300*
0.55	368 500*	409 900*				
0.50	445 600*	601 000*				
0.45	1 241 000*					

* fracture of bars

Tab.2 Number of cycles to failure - Series F

S_o	N					
0.75	500 9 400	2 100	2 200	2 300	3 500	9 000
0.70	700 167 800	1 600	4 400	5 100	35 700	40 000
0.65	21 000 1 150 000	164 000	310 000	526 100	527 000	823 400

Tab.3 Test results of the series D and E

No.	$n_i - S_{oi}$			D
105	63 600 - 0.60	9 500 - 0.70		0.403
106	0 - 0.60	6 300 - 0.70		0.144
140	65 000 - 0.60 2 700 - 0.85	9 500 - 0.70	1 400 - 0.80	2.015
141	65 000 - 0.60 10 400 - 0.85	9 500 - 0.70	1 400 - 0.80	2.884
142	130 000 - 0.60 1 200 - 0.85	24 000 - 0.70	2 800 - 0.80	2.034
161	477 200 - 0.55	60 200 - 0.65	3 800 - 0.80	1.671
162	238 000 - 0.60	28 000 - 0.70		1.335
163*	683 000 - 0.60	85 300 - 0.70		3.944
164*	442 000 - 0.60	56 800 - 0.70		2.589
165	612 000 - 0.60	79 200 - 0.70		3.597
166	132 000 - 0.65	15 800 - 0.75		2.091
167	0 - 0.65	800 - 0.75		0.051
168	5 000 - 0.65	0 - 0.75		0.041
169	24 000 - 0.65	4 400 - 0.75		0.478
170	12 000 - 0.65	2 500 - 0.75		0.258
171	86 600 - 0.60	8 600 - 0.70		0.449
172	306 000 - 0.60	39 900 - 0.70		1.806
173	16 900 - 0.65	3 000 - 0.75		0.330

* fracture of bars