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A STUDY ON COMPRESSIVE FATIGUE STRENGTH OF CONCRETE BOTH IN THE AIR AND IN THE WATER COSIDERED SURVIVAL PROBABILITY

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SYNOPSIS

In order to make clear the compressive fatigue strength of concrete subjected to repeated load, the fatigue test of cylinder specimens was carrried out. The test results were treated in terms of statistics, and the relationship between stress ratio,S and the mean of fatigue life,$\overline{N}$, that is, S-$\overline{N}$ curve was proposed. Still more, it was presumed that the scatter of fatigue life was not only due to characteristic property of fatigue failure but also due to the variance of static strength, and the magnitude of the scatter of fatigue life, D(logN) was proposed Consequently, S-N-P(N) curve which was the S-N curve considered survival probability P(N) was proposed. The discussion was carried on concrete not only in the air but also in the water, and the comparison of the fatigue behavior of both conditions was made.

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1. INTRODUCTION

Fatigue failure of concrete often occures under repeated load which is less than the static failure load. Recently, concrete structures subjected to repeated load have increased, so fatigue of concrete has been a significant factor when those are designed.

Many reports on fatigue of concrete have been presented. But the character of fatigue of concrete, for example, fatigue strength and deformation have not been encleared. Because, for instance, the method of repeated loading is a full of variety, the fatigue test needs a long time to be carried and still the test results scatter widely.

This study aims to examine the fatigue strength considered survival probability of concrete under constant repeated load, including test results of previous studies of others. That is, the distribution of fatigue life of concrete is clarified by treating the process of fatigue failure in terms of probabilities. And the relationship between the fatigue life and the maximum and minimum stress of repeated load, that is, S-N curve is proposed. Still more, S-N curves considered survival probability, that is, S-N-P(N) curves are suggested.

With the advance of marine concrete structure, it will be important to examine the behavior of concrete which is in a state placed not only in the air but also in the water. There are some reports which show that, even if concrete is casted and cured in the same condition, the static strength of concrete in wet condition at testing is less than that in dry condition. Therefore, it is anticipated that the fatigue strength of concrete in the water is different from that in the air. Then, in this study, the fatigue tests of concrete both in the air and in the water are conducted and a comparison of the test results of both is made.

In this study, the tests in the air and the tests in the water are respectively called as Series A and Series W.

2. PREVIOUS STUDIES ON THE STATISTICAL DISTRIBUTION OF FATIGUE LIFE

Many investigators have pointed out that the fatigue test results scatter widely so that, for considering the safety of structures subjected to repeated load, it is important problems to know what theoretical distribution the fatigue life approximates to, or how much the average and the deviation are estimated. The process of fatigue failure must be made clear at first, since it is anticipated that the fatigue test results scatter widely depending on it originally. For metal materials, though many studies on fatigue have been done for a long time, and some arguments over the process of fatigue failure have been evolved [1] , it is the situation that the distribution of the fatigue life is practically analized as the exponential distribution, the logarithmic normal distribution or Weibul [1] distribution.

Freudential [2] indicated that the process of fatigue failure of metal materials depended on the behavior of the condition of outer loads in the weak point of test specimen or in the narrow region distorted highly, and that the condition of outer loads varied statistically even the test under constant repeated stresses. And he led that the fatigue life was distributed as the logarithmic normal distribution. Sakata and the others [3] also reported that the distribution of fatigue life was allowed to treat as the logarithmic normal distribution for the concrete.

Yokobori [4] considered that the occurrence of the failure of metal materials was understood in terms of probability after certain repetition number. And he led the exponential distribution, and made clear that the probability of fatigue failure per one repetition was constant. And he proposed that the distribution of fatigue life was within the exponential distribution.

Weibull [5], Freudential and Gumbel [6] indicated that the test fracturing on actual fatigue test were the weakest ones among sufficiently large test materials sampled out of the population within the logarithmic normal distribution even if the distribution of fatigue life in the population would be within probability distribution, which was the logarithmic normal distribution or etc. Applying the probability theory of minimum values, they led the distribution of minimum values, that is Weibull distribution.

McCall [7] carried out bending tests on plain concrete, and he proposed his original mathematical model.

Nishimatsu and the others [8], also, confirmed that the distribution of fatigue life of rocks did not act on the exponential distribution different from the case of metal materials. After that, they regarded the process of fatigue failure as two processes which are called the intergranular fracture and the innergranular fracture, and showed that the survival probability against macroscopic fatigue failures is expressed as a liner combination of two Poisson processes of the first degree.

As described above, some theories of the distribution of fatigue life have been presented. When the fitness between these theories of the distribution and the fatigue test results on concrete is examined, if the total number of specimens is small, it is necessary to estimate the probability that failures will not happen until each measured fatigue life, N is attained, that is survival probality. In this study, by ranking in order of fatigue life N, the expectation of failure probability, $P_f(N_r)$ is expressed as follows.

$$P_f(N_r) = \frac{r}{n+1} \qquad (1)$$

where, Nr is the fatigue life of the rth and n is total number of specimens. Hence, the probability that the fatigue life is greater than Nr, that is the expectation of survival probability $P(N_r)$ is

$$P(N_r) = 1 - \frac{r}{n+1} \qquad (2)$$

Also in the case that fatigue failure of all specimens do not occur after the defined repetition number Nx (in general, Nx is 2×10^6 repetitions for concrete), the statistical treatment of survival probability is different from that mentioned above. Hamada and the others [9] showed the method that presume $P(N_r)$ by means of giving a reasonable interpretation to the statistical theory in above case. If m pieces of specimens, among n pieces of ones are not failed until the defined repetition number Nx, they consider that $(n+1)$ pieces of specimens were tested, and that the specimen at $(n-m+1)$ th is failed at Nx. And they indicate that the expectation of the survival probability $P(N_r)$ mesured value Nr can be calculated.

$$P(N_r) = 1 - \frac{r}{n+2} \qquad (3)$$

In this study, survival probability of the fatigue test results are analyzed by use of Eq. (2) and (3) .

3. OUTLINE OF EXPERIMENT

(1) Test Specimen

The fatigue tests were carried out both in the air and in the water. In all specimens, ordinary Portland cement, sea sand and crushed stone were used. Those properties of materials are shown in Table 1. For each test, three kinds of mix proportions were used to examine the influence of the static strength on the fatigue strength, as shown in Table 2. Series A consisted of a group of 13 separately mixed baches, and Series W consisted of a group of 8 ones.

Table 1 Properties of materials of concrete.

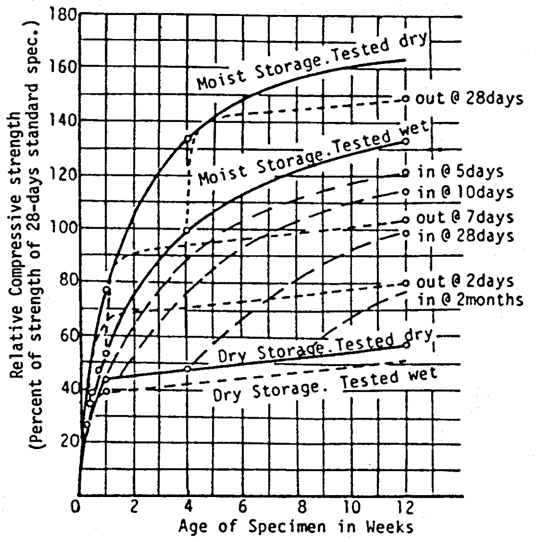
	Mix prop. NO.	Slump (cm)	M.S. of agg. (mm)	W/C (%)	s/a (%)	Unit content (kg/m^3) W	C	S	G
Series A	A-1	4±1	20	70	44	180	257	832	1218
	A-2	4±1	20	63	45	189	300	824	1162
	A-3	4±1	20	78	44	190	244	845	1192
Series W	W-1	6±1	20	70	44	180	259	832	1162
	W-2	6±1	20	63	45	189	300	824	1218
	W-3	6±1	20	50	42	180	360	742	1166

Table 2 Mix proportion of concrete.

		Specific gravity	Water absorption modulus (%)	Finess modulus
Ordinary Portland cement		3.17	—	—
Sea sand	Series A	2.56	1.8	2.45
	Series W	2.59	1.1	2.73
Crushed stone		2.95	0.8	6.60

The specimens were concrete cylinders, 7.5cm in diameter by 15cm high. They were cast in steel forms and cured in moist room maintained at 20±1 ℃ and more than 95% relative humidity. Age of specimens at testing was long enough to prevent the increase of the strength of concrete by hydration of cement during the period of fatigue tests. In Series A, all specimens were removed to storage in normal laboratory atmospher from 2 weeks before testing, because the water absorption of concrete had great influence on the strength, as Gilky [10] reported as shown in Fig.1.

On the other hand, in Series W, all specimens were removed into the water tank from 2 weeks before testing in order that void inside of concrete might be filled with water.



o Experimentally determined points。

out = Removed from moist to dry storage at the age indicated on horizontal scale

in = placed in moist storage at age indicated

Fig.1 Variance of static strength on the dry and wet condition。

(2) Testing Methods

Shortly before the fatigue test, the static test was conducted on from 5 to 26 randomly chosen specimens for each group, and the mean value of static strength of them was taken as the static strength for each group. The static test was conducted in 50ton universal testing machine, with the loading rate of 2 ~3 kg /㎠/sec.

The fatigue test was conducted in universal testing machine using a pulsator, with applied 300 repetitions of load per minute. The attachment of loading plate had a hinge in order to avoid the eccentric compression. The tests of Series W were conducted in the vessel filled with water as shown in Photo 1.

The magnitude of the repeated stress was determined by the amplitude of stress ratio, S to the static strength. In Series A, the maximum stress ratio, S_1 was varied from 64 % to 90%, and the minimum stress ratio, S_2 was varied from 2% to 60%. On the other hand, in Series W, the maximum stress ratio, S_1 was varied from 55 to 85 %, and the minimum stress ratio, S_2 was varied from 10% to 55%, as shown in Table 3 and 4.

Table 3 Static strength and Repeated stress ratio in Series A.

Group	Mix prop. NO.	Age of specimens (months)	Total of specimens: Static test	Total of specimens: Fatigue test	Static strength: Mean (kg/cm^2)	Static strength: Coef. of variation (%)	Repeated stress ratio (%)	Repeated stress (kg/cm^2)
A1	A-1	13	26	46	297	5.4	8 ~ 75	24 ~ 223
A2	A-2	17	5	6	318	2.5	8 ~ 80	25 ~ 254
A3	A-2	1 week	6	8	193	4.1	8 ~ 80	15 ~ 154
A4	A-1	18	6	14	301	6.6	8 ~ 80	24 ~ 241
A5	A-3	16	5	6	164	3.7	8 ~ 80	13 ~ 131
A6	A-1	20	10	20	280	6.4	8 ~ 85	22 ~ 238
A7	A-1	8	10	7 4 5 5	301	5.6	2 ~ 64 2 ~ 68 2 ~ 73 2 ~ 80	6 ~ 193 6 ~ 205 6 ~ 220 6 ~ 241
A8	A-1	13	5	5 5	292	1.7	5 ~ 70 5 ~ 77	15 ~ 204 15 ~ 225
A9	A-1	2	5	5	285	5.3	30 ~ 77	86 ~ 219
A10	A-2	2	6	4	265	1.9	30 ~ 84	80 ~ 223
A11	A-3	2	5	6	192	3.6	30 ~ 84	58 ~ 161
A12	A-1	13	6	10	281	4.3	30 ~ 88	84 ~ 247
A13	A-3	5	6	10	186	5.4	60 ~ 90	112 ~ 167

Table 4 Static strength and Repeated stress ratio in Series W.

Group	Mix prop. NO.	Age of specimens (months)	Total of specimens: Static test	Total of specimens: Fatigue test	Static Strength: Mean (kg/cm^2)	Static Strength: Coef. of variation (%)	Repeated stress ratio (%)	Repeated stress (kg/cm^2)
W1	W-1	9	5	10	211	2.7	10 ~ 65	21 ~ 137
W2	W-2	3	5	2	275	5.2	10 ~ 65	28 ~ 179
W3	W-3	3	5	10	368	5.6	10 ~ 65	37 ~ 239
W4	W-1	10	5	6 7	233	6.0	10 ~ 75 30 ~ 80	24 ~ 179 71 ~ 190
W5	W-1	14	5	8	200	5.0	30 ~ 75	60 ~ 150
W6	W-1	1	5	8	175	5.3	50 ~ 85	88 ~ 150
W7	W-2	10	5	8 9 8	272	4.1	30 ~ 70 50 ~ 75 50 ~ 80	83 ~ 190 136 ~ 204 136 ~ 218
W8	W-1	13	5	10	245	6.2	10 ~ 55	25 ~ 135

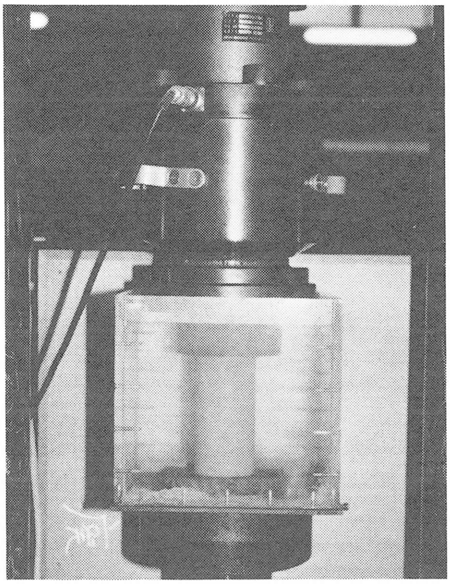


Photo 1

Loading apparatus of statistic and fatigue test in the water.

4. TEST RESULT AND DISCUSSION

(1) Distribution of Fatigue Life

The fatigue test results of group A1 and the values of $P(N_r)$ from Eq. (2) are shown in Table 5. It is indicated that the fatigue test results scatter widely,so the statistical treatment is to be used.

Fig. 2 shows the linear relationship between survival propability P(N) and the logarithmic of fatigue life,logN on normal probability paper, and that the distribution of fatigue life of concrete can be regarded as the logarithmic normal distribution. As discribed later, the distribution of fatigue life of other groups are also regarded as the logarithmic normal distribution. Consequently, in this study, the distribution of fatigue life is treated as the logarithmic normal distribution.

Now, using the method of least squares, the straight line equationbetween survival probability,P (N) and fatigue life,N is expressed as follows.

$$t = A \log N_r + B \qquad (4)$$

where, t is a distance from the symmetry axis of normal distribution curve, and given in the table of cumulative distribution, and A and B are experimental constant.

From Eq. (4) , the mean of fatigue life, $\overline{\log N}$ and the standerd deviation of fatigue life, D(logN) which indicates the amount of scatter of fatigue life, are given as follows ;

$$\overline{\log N} = -\frac{B}{A} \qquad (5)$$

$$D(\log N) = \frac{1}{A} \qquad (6)$$

Table 5 Fatigue test results of Group A1.

r	N	P(N) (%)	r	N	P(N) (%)
1	780	97.9	24	93 000	48.9
2	1 000	95.7	25	104 700	46.8
3	1 020	93.6	26	178 200	44.7
4	1 360	91.5	27	185 700	42.6
5	2 770	89.4	28	208 100	40.4
6	4 120	87.2	29	239 500	38.3
7	4 770	85.1	30	293 000	36.1
8	5 120	83.0	31	388 400	34.0
9	5 580	80.9	32	403 500	31.9
10	5 910	78.7	33	419 000	29.8
11	5 990	76.6	34	423 760	27.7
12	8 380	74.5	35	427 830	25.5
13	11 000	72.3	36	564 690	23.4
14	13 450	70.2	37	585 860	21.3
15	14 700	68.1	38	730 810	19.1
16	16 850	66.0	39	762 280	17.0
17	18 050	63.8	40	877 000	14.9
18	25 000	61.7	41	877 000	12.8
19	29 500	59.6	42	968 150	10.6
20	34 260	57.4	43	1 160 000	8.5
21	35 200	55.3	44	1 990 000	6.4
22	68 910	53.2	45	1 998 000	4.3
23	80 150	51.1	46	5 980 000	2.1

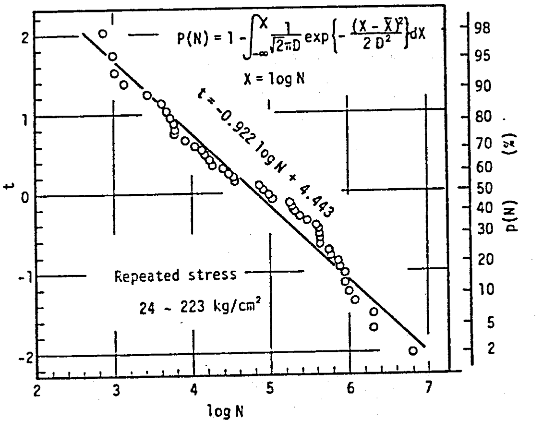


Fig.2 Relationship between the survival probability P(N) and logarithmic of fatigue life of Group A1.

(2) Influence of Static Strength

According to previous reports on compressive fatigue strength of concrete, the fatigue strength can often be expressed by the ratio to the static strength. If the fatigue strength can be represented by a certain definite ratio to the static strength varied by the mix proportion and testing age of concrete and if the amount of the scatter of fatigue life can be indipendent of mix proportion and testing age of concrete, the factor which has an effect on the static strength influences the fatigue one as much as on the static one.

i) Series A
Group A2~A5 of specimens which had different mix proportion and testing age were tested under the constant repeated stress ratio of from 8% to 80%. These test results and values of $P(N_r)$ from Eq. (2) are shown in Table 6, and they are plotted on the normal probability paper as shown in Fig. 3. Therefore, the fatigue strength can be represented by the ratio to the static strength.

P(N)-N line equation, $\overline{\log N}$ and D(logN) of each group is shown in Table 7. An analysis of variance of these data shows no significant difference at the 95% significant level as shown in Table 8. Therefore, the fatigue strength of concrete in the air can be represented by the ratio to the static strength.

ii) Series W
Group W1~W3 of specimens which had different mix proportion and testing age were tested under the constant repeated stress ratio of from 10% to 65%.

The test results of these three groups and values of P (Nr) from Eq. (2) are shown in Table 9, and they are plotted on the normal probability paper as shown in Fig.4. An analysis of variance of these data shows no significant difference at the 95% significant level as shown in Table 10. Therefore, the fatigue strength of concrete in the water can be also represented by the ratio to the static strength.

Table 6 Fatigue test results of Group A2,A3, A4 and A5.

Group	r	N	P(N) (%)
A2	1	170	85.7
	2	2 990	71.4
	3	3 510	57.1
	4	3 530	42.9
	5	4 880	28.6
	6	8 470	14.3
A3	1	580	88.9
	2	1 340	77.8
	3	2 090	66.7
	4	3 730	55.6
	5	4 120	44.4
	6	4 530	33.3
	7	4 470	22.2
	8	17 490	11.1
A4	1	260	93.3
	2	440	86.7
	3	600	80.0
	4	780	73.3
	5	870	66.7
	6	1 700	60.0
	7	2 000	53.3
	8	3 200	46.7
	9	4 700	40.0
	10	6 150	33.3
	11	12 500	26.7
	12	23 000	20.0
	13	84 200	13.3
	14	107 340	6.7
A5	1	460	85.7
	2	2 000	71.4
	3	2 430	57.1
	4	2 940	42.9
	5	58 350	28.6
	6	113 730	14.3

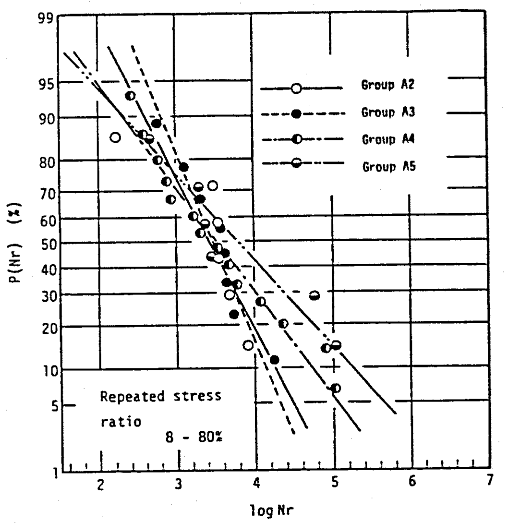


Fig.3 P(N)-N lines of Group A2,A3,A4 and A5.

Table 7 P(N)-N line equations, logN and D(logN) of Group A2,A3, A4 and A5.

Group	P(N)-N line equation	$\overline{\log N}$	D(log N)
A2	$t=-1.550\log N+5.279$	3.14	0.65
A3	$t=-1.883\log N+6.599$	3.50	0.53
A4	$t=-1.065\log N+3.773$	3.54	0.94
A5	$t=-0.879\log N+3.317$	3.78	1.14

Table 8 Analysis of variance table of Group A2, A3, A4 and A5

	Sums of Squares	Degrees of Freedom	Mean Squares	F ratio
Between	0.448	3	0.149	$F_0=\frac{0.149}{0.545}=0.274$
Within	16.353	30	0.545	
Total	16.801	33		$F_{95}(3,30)=2.92$

Table 9

Fatigue test results of Group W1,W2 and W3.

Group	r	N	P(N) (%)
W1	1	5 300	90.9
	2	6 500	81.8
	3	7 350	72.7
	4	10 660	63.6
	5	30 280	54.5
	6	51 500	45.5
	7	56 400	36.4
	8	76 260	27.3
	9	86 410	18.2
	10	313 100	9.1
W2	1	1 760	87.5
	2	5 180	75.0
	3	14 670	62.5
	4	15 400	50.0
	5	27 590	37.5
	6	33 580	25.0
	7	73 810	12.5
W3	1	2 700	90.9
	2	15 630	81.8
	3	19 410	72.7
	4	25 510	63.6
	5	36 040	54.5
	6	46 020	45.5
	7	50 750	36.4
	8	60 660	27.3
	9	64 220	18.2
	10	66 890	9.1

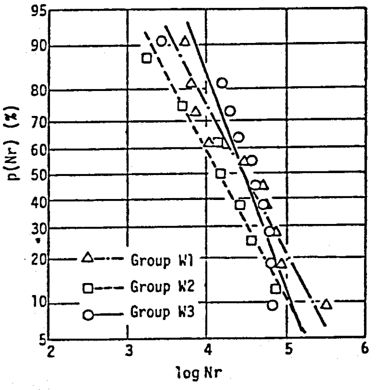


Fig.4 P(N)-N lines of W1,W2 and W3.

Table 10 Analysis of variance table of Group W1, W2 and W3.

	Sums of squarers	Degrees of freedom	Mean square	F ratio
Between	0.479	2	0.239	$F_0 = \frac{0.239}{0.271} = 0.884$
Within	6.503	24	0.271	
Total	6.982	26		$F_{95}(2,24) = 3.40$ F_0

(3) S_1-$\overline{N}$ Curve

i) Series A

The fatigue test results of group A2~A13 are shown in Table 11. And P(N)-N lines for each minimum stress ratio,S_2 are shown in Fig. 5. In any case, almost they can approximate to straight line on the normal probability paper, and it is re-confirmed that the distribution of fatigue life of concrete in the air is fitted to the logarithmic normal distribution.

Table 12 gives P(N)-N line equations, $\overline{\log N}$ and D(logN) calculated by Eq. (5) and (6) respectively, for each of stress ratio. It shows that the maximum and minimum stress ratio, S_1 and ,S_2 influence the mean of fatigue life, $\overline{\log N}$. And Fig. 6 shows S_1-$\overline{N}$ curves. On this figure, the maximum stress ratio,S_1 is protted against the logarithm of fatigue life, logN. A linear relationship between the maximum stress ratio, S_1 and $\overline{\log N}$ is recognized for each minimum stress ratio,S_2. The tendency, when S_2 is constant, the larger S_1 is, the smaller $\overline{\log N}$ is, and when S_1 is constant, the larger S_1 is, the larger $\overline{\log N}$ is. Consequently, the fatigue life is considerably dependent on the amplitude of stress ratio.

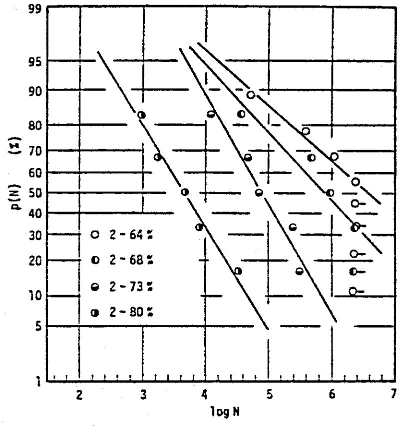
Table 13 shows S_1-$\overline{N}$ curve equations calculated by a least squares method and the fatigue strength at N repetitions, S_{1N}.

In Table 14, test conditions and test results reported by other investigators [3] and [11] ~ [14] are surmmarized. Fig.7 shows a comparison of S_1-$\overline{N}$ curve of this study with other investigators' at which tests the minimum stress ratios were less than 10%, so along with the test conditions of other investigators.

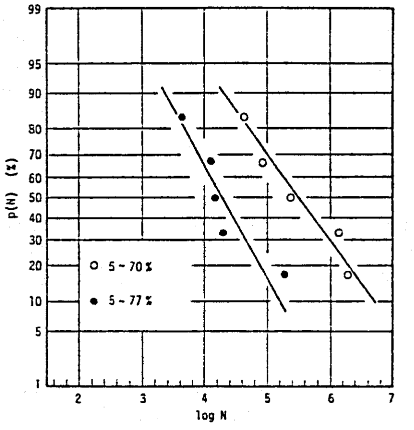
Although mix proportion of concrete and shape and size of specimen are different for each investigator's, S_1-$\overline{N}$ curves in the air obtained from this examine is similler to those of other investigators. That is to say, it is confirmed that the fatigue strength can be represented by the ratio to tatic strength, and expected that shape and size of specimen have an influence on the fatigue strength as much as on the static one.

Table 11 Fatigue test results in Series A. (Results of Group A1 are shown table 5.)

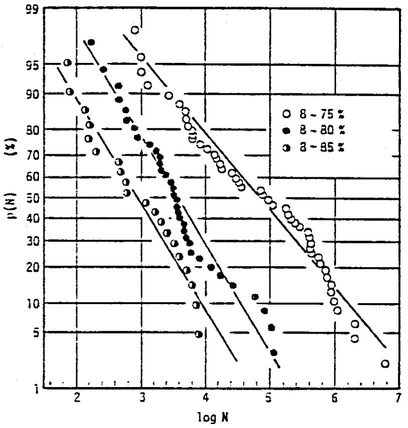
S (%)	r	N	P(N) (%)
2-64	1	51 800	88.9
	2	385 000	77.8
	3	1 080 000	66.7
	4	2 000 000→	
	5	2 000 000→	
	6	2 000 000→	
	7	2 000 000→	
2-68	1	35 750	83.3
	2	474 500	66.7
	3	899 000	50.0
	4	2 000 000→	
2-73	1	12 200	83.3
	2	46 400	66.7
	3	71 300	50.0
	4	246 000	33.3
	5	300 000	16.7
2-80	1	990	83.3
	2	1 670	66.7
	3	4 580	50.0
	4	7 940	33.3
	5	34 200	16.7
8-80	1	170	97.1
	2	260	94.3
	3	440	91.4
	4	460	88.6
	5	580	85.7
	6	600	82.9
	7	780	80.0
	8	870	77.1
	9	1 340	74.3
	10	1 700	71.4
	11	2 000	68.6
	12	2 000	65.7
	13	2 090	62.9
	14	2 430	60.0
	15	2 940	57.1
	16	2 990	54.3
	17	3 200	51.4
	18	3 510	48.6
	19	3 530	45.7
	20	3 730	42.9
8-80	21	4 120	40.0
	22	4 530	37.1
	23	4 700	34.3
	24	4 880	31.3
	25	5 470	28.6
	26	6 150	25.7
	27	8 470	22.9
	28	12 500	20.0
	29	17 490	17.1
	30	23 800	14.3
	31	58 350	11.4
	32	84 200	8.6
	33	107 340	5.7
	34	113 730	2.9
5-70	1	44 700	83.3
	2	85 200	66.7
	3	243 200	50.0
	4	1 375 800	33.3
	5	1 909 500	16.7
5-77	1	4 460	83.3
	2	13 040	66.7
	3	15 200	50.0
	4	19 720	33.3
	5	192 600	16.7
8-85	1	50	95.2
	2	60	90.5
	3	130	85.7
	4	150	81.0
	5	150	76.2
	6	200	71.4
	7	450	66.7
	8	470	61.9
	9	580	57.1
	10	600	52.4
	11	1 190	47.6
	12	1 650	42.9
	13	2 100	38.1
	14	2 400	33.3
	15	3 300	28.6
	16	3 990	23.8
	17	4 920	19.0
	18	6 100	14.3
	19	7 230	9.5
	20	7 650	4.8
30-77	1	99 800	85.7
	2	133 000	71.4
	3	319 000	57.1
	4	2 000 000→	
	5	2 000 000→	
30-84	1	1 100	90.9
	2	1 620	81.8
	3	2 120	72.7
	4	6 100	63.6
	5	9 700	54.5
	6	16 500	45.5
	7	19 600	36.4
	8	28 300	27.3
	9	247 200	18.2
	10	421 840	9.1
30-88	1	680	90.9
	2	1 120	81.8
	3	1 520	72.7
	4	1 670	63.6
	5	2 600	54.5
	6	2 900	45.5
	7	2 980	36.4
	8	14 100	27.3
	9	48 350	18.2
	10	93 000	9.1
60-90	1	1 600	90.9
	2	1 750	81.3
	3	2 920	72.7
	4	4 370	63.6
	5	20 350	54.5
	6	42 630	45.5
	7	74 900	36.4
	8	212 860	27.3
	9	492 420	18.2
	10	772 040	9.1



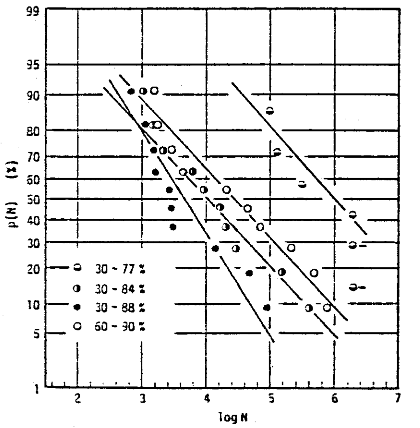
(a) Minimum stress ratio S_2=2%



(b) S_2=5%



(c) S_2=8%



(d) S_2=30%

Fig.5 P(N)-N lines in Series A.

Table 12 P(N)-N line equation, $\overline{\log N}$ and D(logN) in Series A for each repeated stress ratio.

Repeated stress ratio (%)	P(N)-N line equation	$\overline{\log N}$	D(log N)
2 ~ 64	$t=-0.676 \log N + 4.459$	6.60	1.48
2 ~ 68	$t=-0.826 \log N + 4.884$	5.91	1.21
2 ~ 73	$t=-1.351 \log N + 6.622$	4.90	0.74
2 ~ 80	$t=-1.266 \log N + 4.633$	3.66	0.79
5 ~ 70	$t=-1.075 \log N + 5.892$	5.48	0.93
5 ~ 77	$t=-1.370 \log N + 5.890$	4.30	0.73
8 ~ 75	$t=-0.922 \log N + 4.443$	4.82	1.08
8 ~ 80	$t=-1.316 \log N + 4.671$	3.55	0.76
8 ~ 85	$t=-1.282 \log N + 3.744$	2.92	0.78
30 ~ 77	$t=-1.000 \log N + 5.890$	5.89	1.00
30 ~ 84	$t=-0.990 \log N + 4.089$	4.13	1.01
30 ~ 88	$t=-1.235 \log N + 4.494$	3.64	0.81
60 ~ 90	$t=-0.840 \log N + 3.723$	4.43	1.19

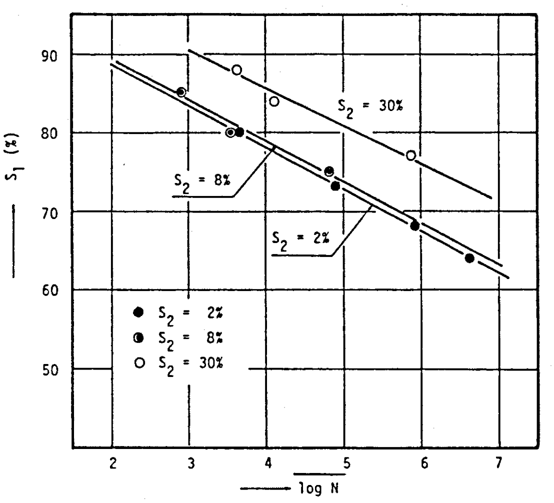


Fig.6 S_1-$\overline{N}$ curves in Series A when the minimum stress ratio S_2 is 2%, 8% and 30%.

Table 13 S_1-$\overline{N}$ curve equation and fatigue strength at N repetitions when S_2 is 2%, 8% and 30% in Series A.

S_2 (%)	S_1-$\overline{N}$ curve equation	Fatigue strength at N repetitions, S_1N (%)		
		$N=10^6$	$N=2*10^6$	$N=10^7$
2	$\overline{\log N} = -0.185 S_1 + 18.5$	67.6	65.9	62.2
8	$\overline{\log N} = -0.190 S_1 + 19.0$	68.4	66.8	63.2
30	$\overline{\log N} = -0.210 S_1 + 22.0$	76.2	74.8	71.4

Table 14 Test condition, S-$\overline{N}$ curve and fatigue strength at 2×10^6 repetitions of concrete in the air of other investigation.

Investigator	W/C (%)	Specimen form	Static strength (kg/cm^2)	Min.stress ratio (%)	S-$\overline{N}$ curve equation	Fatigue strength at $N=2*10^6$ (%)
N.K. Raju	50	prism	420	5	$\overline{\log N}=16.1-0.157S$	62.4
F.S. Ople	58 ~ 67	prism	422	10	$\overline{\log N}=18.2-0.182S$	65.4
J.C. Antrim	61	cylinder	288	2	$\overline{\log N}=20.5-0.214S$	66.3
E.W. Bennet	35 ~ 50	prism	400~650	14 ~ 22	$\overline{\log N}=21.6-0.233S$	65.7
K. Sakata	56	cylinder	233	8.6	$\overline{\log N}=19.4-0.191S$	68.3

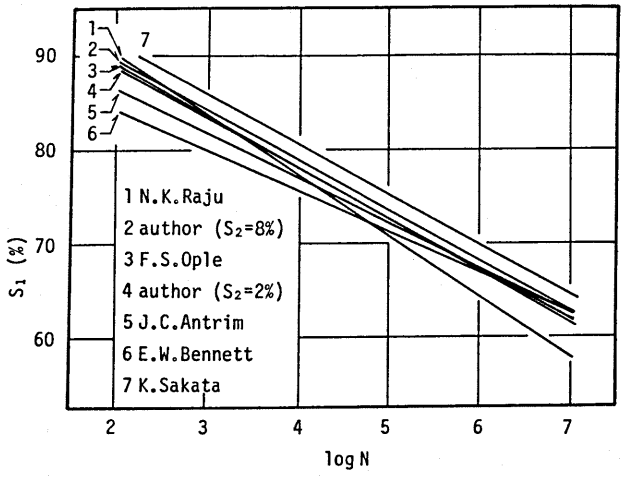


Fig.7 S_1-$\overline{N}$ curves of concrete in the air suggested by other investigators and author.

ii) Series W

The fatigue test results of Group W4~W8 are shown in Table 15. P(N)-N lines are shown in Fig. 8. Table 16 gives P(N)-N line equations, $\overline{\log N}$ and D(logN) . Fig. 9 shows S_1-$\overline{N}$ curves. It also shows that a linear relationship between the maximum stress ratio, S_1 and logN is recognized for each constant minimum stress ratio, S_2 under the same tendency with that in the air.

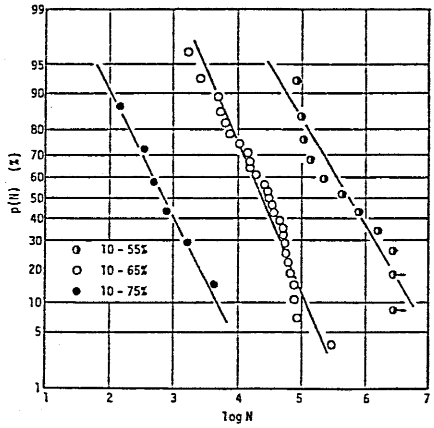
Table 17 shows S_1-$\overline{N}$ curve equation and the fatigue strength at N repetitions, S_{1N}. It indicates that the fatigue strength of concrete in the water is considerably smaller than that in the air. For example, the fatigue strength (S_2=30%) at 10^6, 2×10^6 and 10^7 repetitions in the water is 67.7 % , 66.3 % and 63.0 %respectively, on the other hand, in the air is 76.2%, 74.8% and 71.4% re-respectively.

Table 15

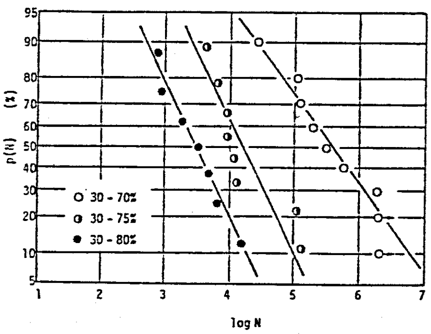
Fatigue test results of concrete in Series W.

S(%)	r	N	P(N) (%)
10-75	1	170	85.7
	2	380	71.4
	3	540	57.1
	4	890	42.9
	5	1 650	28.6
	6	3 800	14.3
30-80	1	770	87.5
	2	920	75.0
	3	1 830	62.5
	4	3 400	50.0
	5	4 830	37.5
	6	6 580	25.0
	7	15 520	12.5
30-75	1	4 220	88.9
	2	6 240	77.8
	3	8 650	66.7
	4	8 980	55.6
	5	11 580	44.4
	6	12 400	33.3
	7	109 540	22.2
	8	132 650	11.1
50-85	1	380	88.9
	2	1 080	77.8
	3	1 220	66.7
	4	2 000	55.6
	5	2 580	44.4
	6	4 320	33.3
	7	4 490	22.2
	8	5 430	11.1
30-70	1	27 600	90.0
	2	110 540	80.0
	3	117 630	70.0
	4	196 000	60.0
	5	317 770	50.0
	6	585 670	40.0
	7	2 000 000→	30.0
	8	2 000 000→	

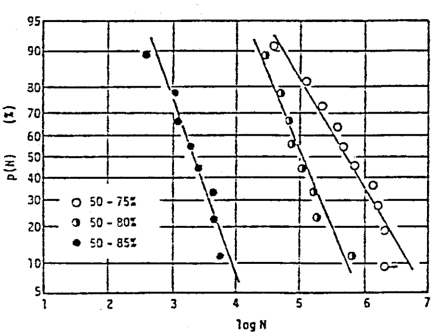
S(%)	r	N	P(N) (%)
50-75	1	38 600	90.9
	2	128 200	81.8
	3	221 500	72.7
	4	382 570	63.6
	5	467 760	54.5
	6	667 500	45.5
	7	1 302 760	36.4
	8	1 548 360	27.3
	9	2 000 000→	18.2
50-80	1	27 900	88.9
	2	50 500	77.8
	3	70 300	66.7
	4	75 710	55.6
	5	108 740	44.4
	6	165 830	33.3
	7	185 550	22.2
	8	726 950	11.1
10-55	1	83 950	91.7
	2	91 690	83.3
	3	97 800	75.0
	4	126 200	66.7
	5	193 080	58.3
	6	350 850	50.0
	7	596 950	41.7
	8	1 285 630	33.3
	9	2 000 000→	25.0
	10	2 000 000→	



(a) Minimum repeated stress ratio, S_2=10%



(b) S_2=30%



(c) S_2=50%

Fig.8 P(N)-N lines of concrete in Series W.

Table 16 P(N)-N line equation, $\overline{\log N}$ and D(logN) in Series W for each repeated stress ratio.

S_2	S_1	P(N)-N line equation	$\overline{\log N}$	D(log N)
10	55	$t=-1.38\log N+7.78$	5.63	0.72
10	65	$t=-1.73\log N+7.61$	4.39	0.56
10	75	$t=-1.62\log N+4.66$	2.88	0.62
30	70	$t=-1.08\log N+6.10$	5.62	0.92
30	75	$t=-1.60\log N+6.74$	4.20	0.62
30	80	$t=-1.70\log N+5.91$	3.48	0.59
50	75	$t=-1.33\log N+7.62$	5.75	0.75
50	80	$t=-1.95\log N+9.83$	5.03	0.51
50	85	$t=-2.14\log N+7.08$	3.30	0.47

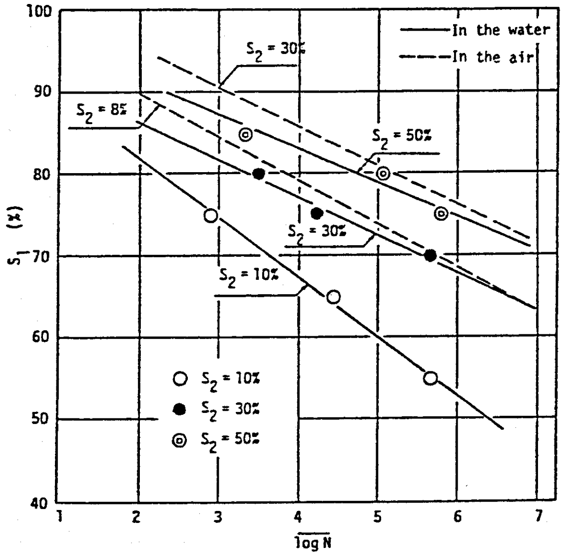


Fig.9 S_1-$\overline{N}$ curves in Series W for each minimum repeated stress ratio S_2.

Table 17 S_1-$\overline{N}$ curve equation and fatigue strength at N repetitions in Series W.

S_2 (%)	S_1 - $\overline{N}$ curve equation	Fatigue strength at N repetitions S_{1N} (%)				
		10^4	10^5	10^6	$2*10^6$	10^7
10	$\log N=-0.138S_1+13.24$	67.0	59.7	52.5	50.3	45.2
30	$\log N=-0.241S_1+20.48$	77.0	72.3	67.7	66.3	63.0
50	$\log N=-0.245S_1+24.29$	82.8	78.7	74 7	73 4	70.6

(4) S_1-$\overline{N}$ Curve Considered S_2

The S_1-$\overline{N}$ curve changes in accordance with the minimum stress ratio, S_2, that is, the larger the minimum stress ratio, S_2 is, the longer fatigue life is. Therefore the influence of the maximum stress ratio, S_1 can not be ignored. The diagram shown relationship between the maximum stress ratio, S_1 and the minimum stress ratio, S_2 at fatigue limit of materials is called endurance diagram. As most of metal materials have the fatigue limits, and endurance diagrams can be drawn. But, as concrete doesn't have the fatigue limit untill 10^7repetitions, time endurance diagram, the relationship between the fatigue strength at N repetions, S_{1N} and the minimum stress ratio, S_2 should be obtained. Only one report of time endurance diagram of compressive fatigue of concrete is presented, the time endurance diagram of the fatigue strength at 2×10^6 repetitions by Graf (15) . In this study, time endurance diagrams are obtained from S_1-$\overline{N}$ curves for several different kinds of the minimum stress ratio, S_2, and the mean of fatigue life, logN is proposed by considering not only the maximum stress ratio, S_1, but also the minimum stress ratio, S_2.

i) Series A
Fig. 10 shows the time endurance diagram at 10^4, 10^5, 10^6, 2×10^6 and 10^7repetitions calculated from O.Graf's and author's S_1-$\overline{N}$ curves. It shows that there is the linear relationship between the fatigue strength ratio, S_{1N} and the minimum stress ratio, S_2 at any repetition. Therefore, it can be concluded that the modified Goodman's reratioship comes into existence on time endurance diagram of concrete in the air at optinal repetitions. For concrete structures, the ratio of the dead load to the maximum load is large, so it is practical that S_1-$\overline{N}$ curves considered the minimum stress ratio, S_2 is used.

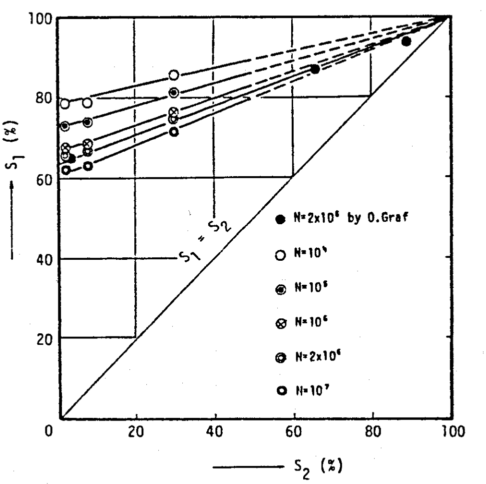


Fig.10 Time endurance diagrams of concrete in the air for each repetition.

When the maximum stress ratio, S_1 and the minimum stress ratio, S_2 are expressed in percent to the static strength, the $S-\overline{N}$ curve equation can be obtained by using the assumption described as follows.

I) When the minimum stress ratio, S_2 is constant, the relationship between the maximum stress ratio, S_1 and $\overline{\log N}$ is linear.

$$\overline{\log N} = A(S_2) \cdot (100 - S_1) + B(S_2) \qquad (7)$$

A, B : function of only S_2 obtained experimentally

II) When $\overline{\log N}$ is constant, the rerationship between the minimum stress ratio, S_2 and the fatigue strength at N repetitions, S_{1N} is linear, and the straight lines pass on the point (100, 100) on time endurance diagram.

$$(100 - S_1) = C(\overline{\log N}) \cdot (100 - S_2) \qquad (8)$$

C : function of only $\overline{\log N}$ obtained experimentally

From Eq. (7) and Eq. (8), the mean of fatigue life, $\overline{\log N}$ can be represented as follows.

$$\overline{\log N} = K_1 \frac{100 - S_1}{100 - S_2} + K_2 \qquad (9)$$

K_1, K_2 : experimental constant which is independent on the repeated stress ratio and fatigue life

The relationship between $(100 - S_1) / (100 - S_2)$ and $\overline{\log N}$ obtained from the test results of author and other investigators is almost linear as shown in Fig. 11. Plotted points in Fig.11 are based on the mean value of fatigue life obtained from more than 5 specimens. It can be read from this figure that Eq. (9) is suitable to representation of $S-\overline{N}$ curve.

The regression equation of the S-N curve of concrete in the air obtained only from author's test results is

$$\overline{\log N} = 17.0 \frac{100 - S_1}{100 - S_2} + 0.23 \qquad (10)$$

The regression equation obtained from the results of both author and other instigators is

$$\overline{\log N} = 16.4 \frac{100 - S_1}{100 - S_2} + 0.29 \qquad (11)$$

There is few difference between Eq. (10) and Eq. (11) as shown in Fig.11. Now considering the wide scatter of fatigue life, an equation obtained by eliminating the constant term of Eq. (9) is proposed as follows.

$$\overline{\log N} = 17.5 \frac{100 - S_1}{100 - S_2} \qquad (12)$$

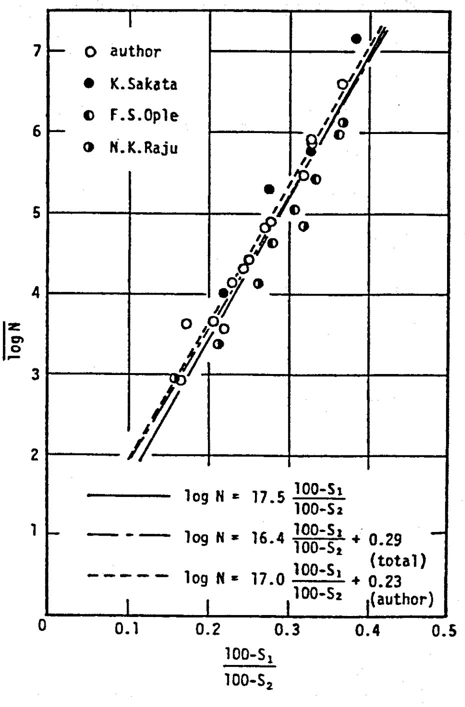


Fig.11 S-$\overline{N}$ curves considered the influence of the maximum repeated stress ratio S_1 and the minimum ratio S_2 of concrete in the air.

ii) Series W

Fig.12 shows the time endurance diagram at 10^4, 10^5, 10^6, 2×10^6 and 10^7 repetitions, and gives that the modified Goodman's relationship comes into existence on time endurance diagram of concrete even in the water

Fig.13 shows the relationship between $(100-S_1)/(100-S_2)$ and $\overline{\log N}$ obtained from author's fatigue test results, and the regression equation of the S-$\overline{N}$ curve of concrete in the water is,

$$\overline{\log N} = 12.4\ \frac{100-S_1}{100-S_2} - 0.22 \qquad (13)$$

and the equation obtained by eliminating the constant term is,

$$\overline{\log N} = 11.7\ \frac{100-S_1}{100-S_2} \qquad (14)$$

Comparing Eq. (14) with Eq. (12) , it is clear that the fatigue life in the water is fairly smaller than that in the air as shown in Fig.13.

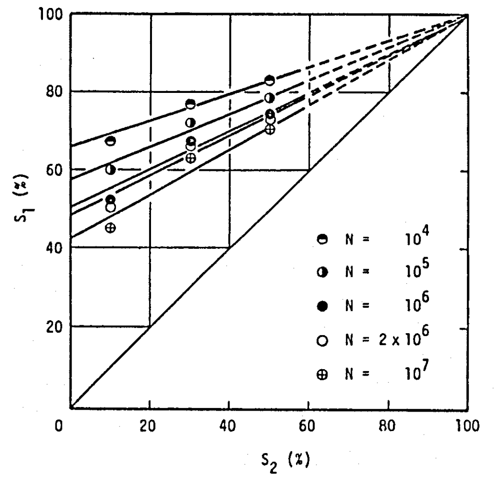


Fig.12 Time endurance diagram of concrete in the water for each repetitions.

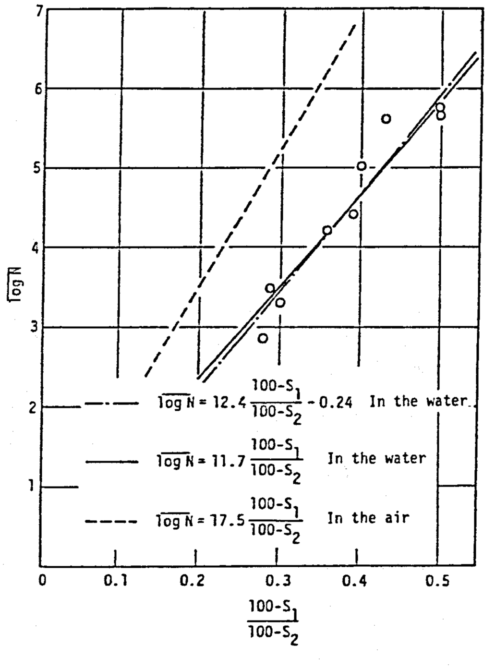


Fig.13 S-$\overline{N}$ curves considered the influence of the maximum repeated stress ratio S_1 and the minimum ratio S_2 of concrete in the water.

(5) Scatter of Fatigue Life

When the allowable stress of concrete subjected to the repeated load is determined, it is proper to use the S-N curve considered with survival probability, P(N), because fatigue life scatters widely. However, it is expected that the sca -tter of fatigue life is due to complicated factors, variance of static strength, accuracy of actual fatigue loading, characteristic property of fatigue phenomenon of concrete and so on.

If the scatter of fatigue life of concrete depended on only the variance of static strength, it should be recognized that there is a correlation between coefficient of variation of static strength, V (%) and that of fatigue life, $\eta = D(\log N) / \overline{\log N}$. But the correlation was not observed evidently as shown Fig.14.

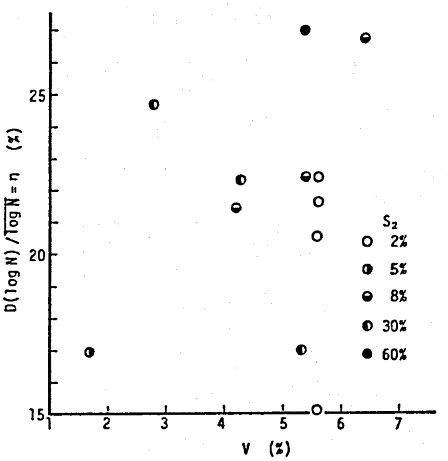


Fig.14 Relationship between the coefficient of variation of static strength, V(%) and that of the logarithmic value of the fatigue life, $\eta(\%) = D(\log N)/\log N$

Next, the magnitude of repeated load was checked to be constant by load cell during the period of fatigue tests. Therefore, considering the slope of S-N curves, the variance of the measured fatigue life can not be explained by that of repeated load.

After all, it is natural to suppose that the scatter of fatigue life depends on the characteristic property of fatigue failure, though the variance of the static strength is considered, too.

Generally, it has been expected that the scateer of fatigue life of metal materials is also a characteristic property of fatigue.

Sinclair [16] and Yokobori [17] reported that D(logN) was depended on the magnitude of the repeated stress, but Weibull [5] reported independent. In this study, it can be read from Table 12 that D(logN) is dependent on the magnitude of repeated stress ratio. When the minimum stress ratio, S_2 is constant, the larger the maximum stress ratio, S_1 is, the smaller D(logN) is, and when the maximum stress ratio, S_1 is constant, the larger the minimum stress ratio, S_2 is, the larger D(logN) is.

When the rerationship between D(logN) and the maximum and minimum stress ratio, S_1 and S_2 regarded to be linear, and there was no variance in static strength, using D_0 (logN) for D(logN) , D_0 (logN) could be expressed as follows ;

$$D_0\ (\log N) = AS_1 + BS_2 + C \qquad (15)$$

A,B,C : constant, $A<0$, $B>0$

Still more, the scatter of fatigue life increases by the variance of static strength and the increment of D(logN) by the variance static strength, ΔD(logN) is calculated as follows.

$$\begin{aligned}\Delta D(\log N) &= \frac{\partial\ (D_0 \log N)}{\partial \sigma_S}\Delta\sigma_S \\ &= \frac{\partial}{\partial\sigma_S}\ (AS_1 + BS_2 + C)\ \Delta\sigma_S \\ &= -\ \left(A\frac{\sigma_1}{\sigma_S^2} + B\frac{\sigma_2}{\sigma_S^2}\right)\Delta\sigma_S \\ &= -\ (AS_1 + BS_2)\ \frac{V}{100} \end{aligned} \qquad (16)$$

σ_S : Compressive static strength
$\Delta\sigma_S$: Variation of σ_S
V : Coefficient of variation of compressive static strength
$\{=\Delta\sigma_S / \sigma_S\ (\%)\}$

Cosequently, D(logN) can be represented as ;

$$\begin{aligned} D(\log N) &= D_0\ (\log N) + \Delta D(\log N) \\ &= (ES_1 + FS_2)\ \left(1 - \frac{V}{100}\right) + G \end{aligned} \qquad (17)$$

E,F,G : experimental constants

i) Series A

A regression analysis of Eq. (17) is meaningful at the 99 % significant level as shown in Table 18, and the regression line equation obtined by using the least squares method is,

$$D(\log N) = (-0.0301\ S_1 + 0.0137 S_2)\cdot\left(1 - \frac{V}{100}\right) + 3.00 \qquad (18)$$

Coefficient of determination, R^2 is 0.71.

Table 18 Analysis of variance table in Series A.

	Sums of Squares	Degrees of Freedom	Mean Square	F ratio
due to regression	0.2375	2	0.1188	$F_0=\frac{0.1188}{0.0119}=9.98$
from regression	0.0952	8	0.0119	
Total	0.3327	10		$F_{99}(2,8)=8.65$

ii) Series W

Table 19 shows that a regression analysis of Eq. (17) is meaningful at the 90 % significant level and the regression equation calculated by the least squares method is,

$$D(\log N) = (-0.0115\,S_1 + 0.0026\,S_2) \cdot \left(1 - \frac{V}{100}\right) + 1.35 \qquad (19)$$

Coefficient of determination, R^2 is 0.71.

Table 19 Analysis of variance table in Series W.

	Sums of squares	Degrees of freedom	Mean square	F ratio
Due to regression	0.04014	2	0.02007	$F_0=\frac{0.02007}{0.00400}=5.0175$
From regression	0.01600	4	0.00400	$F_{90}(2,4)=4.324$
Total	0.05614	6		

Comparing the magnitude of D(logN) of concrete in Series A with that in Series W, the rerationship between the maximum stress ratio, S_1 and D(logN), when coefficient of variation of static strength, V is 15%, is shown in Fig.15. It is clear that the scatter of fatigue life of concrete in the water is fairly smaller than that in the air.

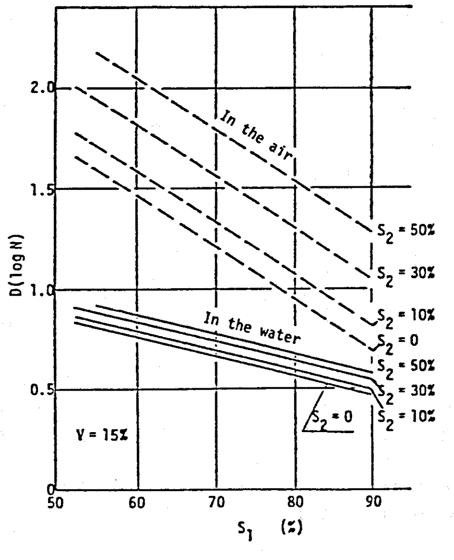


Fig.15 Comparison of both D(logN) of concrete in the air and that in the water at V=15%.

(6) S-N-P(N) Curve

$\overline{\mathrm{LogN}}$ and D(logN) are obtained from Eq. (9) and (17) respectively, so S-N curve considered survival probability, P(N), that is, S-N-P(N) curve an be proposed. Let $t_{P(N)}$ is the value of t corresponding to the optional survival probability, P(N), and t is given in the table of cumulative distribution function of normal distribution.
S-N-P(N) curve can be expressed as follows.

$$\log N = \overline{\log N} - t_{P(N)} \cdot D(\log N) \qquad (20)$$

Considering the variance of static strength of concrete, the maximum stress ratio, S_1 is required to satisfy a follow condition.

$$S_1 \leq 100 - t_{P(N)} \cdot V \qquad (21)$$

i) Series A
From Eq. (12), (18) and (21), S-N-P(N) curve at constant minimum stress ratio, S_2 can be drawn. S_1-N curves at minimum stress ratio, S_2= 0%, 10%, 30% with the coefficient of variation of static strength V = 0%, 15 % andsurvival proba bility, P(N)=50%, 90%, 95% are shown in Fig.16. And from theseequations, ti me endurance diagrams at 10^3, 10^5 and 10^7 repetitions consideredsurvival proba bility, P(N) can be drawn as shown in Fig.17.

When the minimum stress ratio, S_2 is 0%, the fatigue strength of concrete in the air at 10^7 repetitions is 60% at survival probability, P(N)=50%, and is 48.7 % and 46.8 % with V =0 % and 15 % respectively at survival probability, P(N) =90%, and is 44.3 and 41.8% with V =0 % and 15 % respectively at survival probability, P(N)=95%. In general, the coefficient of variation of compressive static strength of sprcimens which are made carefully in labratories is under 6 ~7 %, on the other hand, that in fields is about 15 %. When the coefficient of variation of static strength is large and survival probability, P(N)is high, it is presumed that the maximum stress can be determined by not only the scatter of fatigue life but also the variance of static strength.

ii) Series W
S-N-P(N) curves at the minimum stress ratio, S_2= 0%, 10%, 30% with V = 0%, 15% and at the survival probability, P(N)=50%, 90%, 95% are shown in Fig.18. And time endurance diagrams at 10^3, 10^5 and 10^7 repetitions with survival probability, P (N)=50%, 90% are shown in Fig.19.

When the minimum stress ratio, S_2 is 0%, the fatigue strength at 10^7 repetitions is 40 % at survival probability, P(N)=50%, and is 29 % and 28 % with V =0 % and 15% respectively at survival probability, P(N)=90%, and is 26 % and 24 % with V= 0% and 15 % respectively at survival probability, P(N)=95%.

It is clear that the static strength of concrete in the water is 10 ~20% lower than that in the air, and this examination shows that the fatigue strength of concrete in the water is about 20 % lower than that in the air.

Consequently, it is more necessary to take care of the safety for fatigue failure of concrete in the water than that in the air.

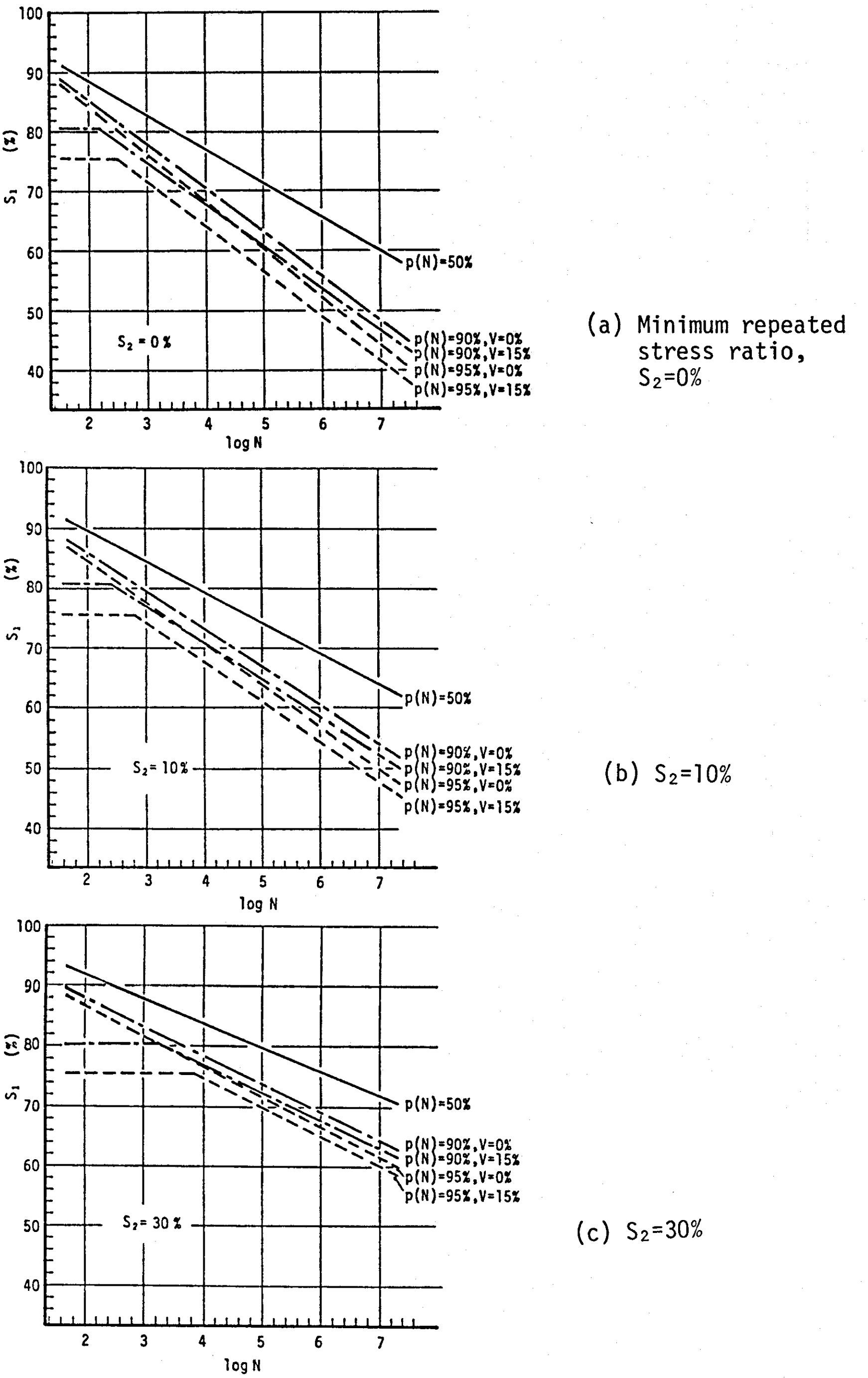
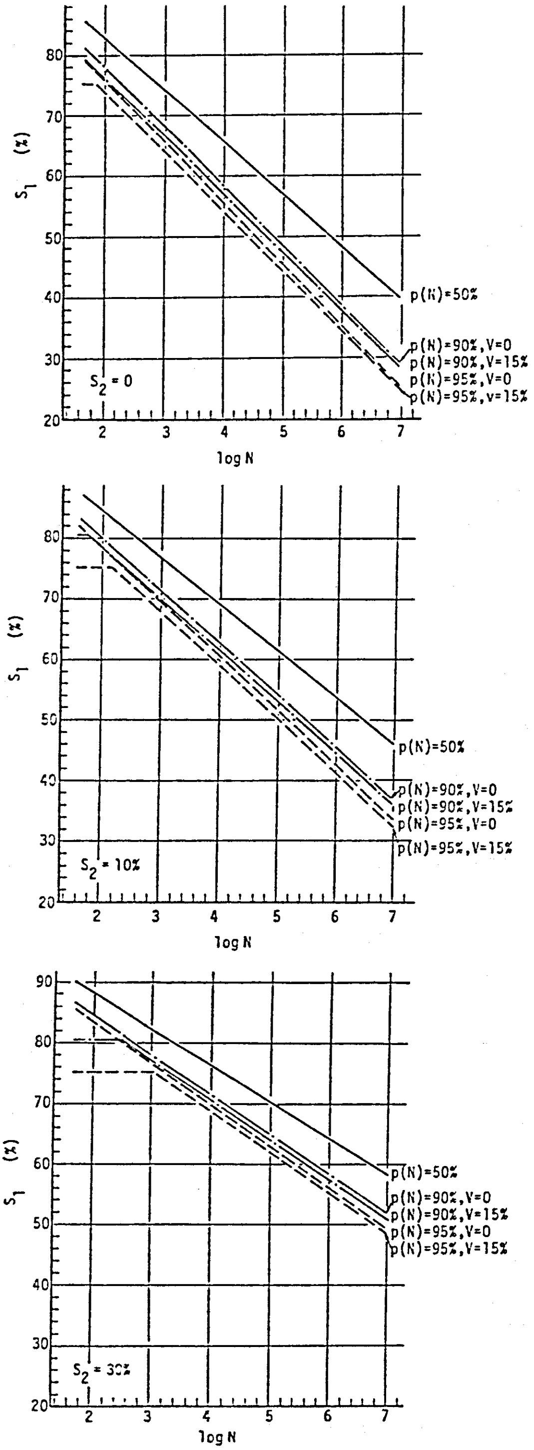


Fig.16 S-N-P(N) curves of concrete in the air



(a) Minimum repeated stress ratio, S_2=0%

(b) S_2=10 %

(c) S_2=30 %

Fig.18 S-N-P(N) curves of concrete in the water

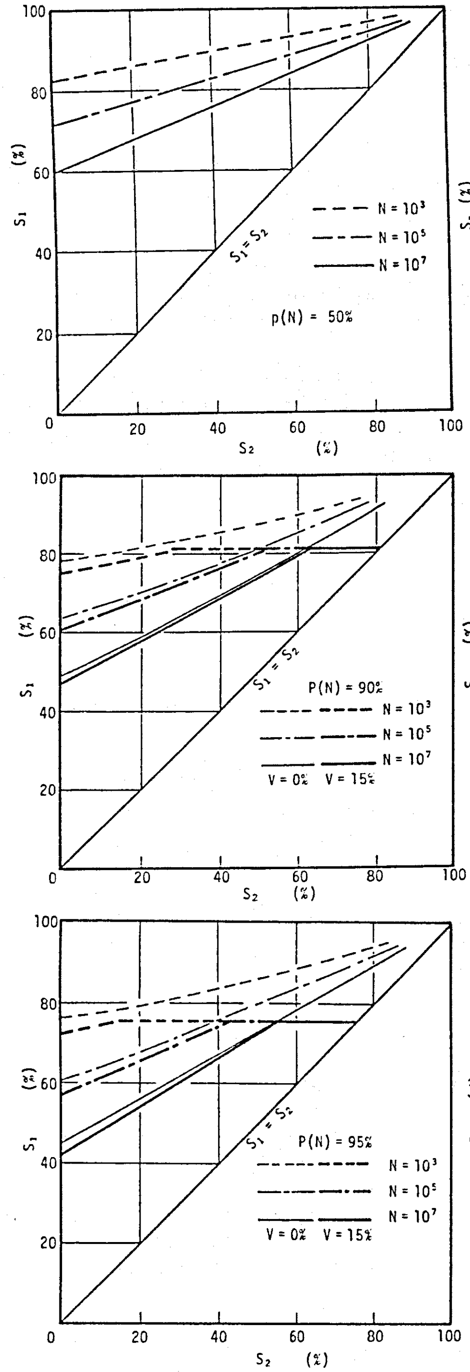


Fig.17 Time endurance diagrams of concrete in the air.

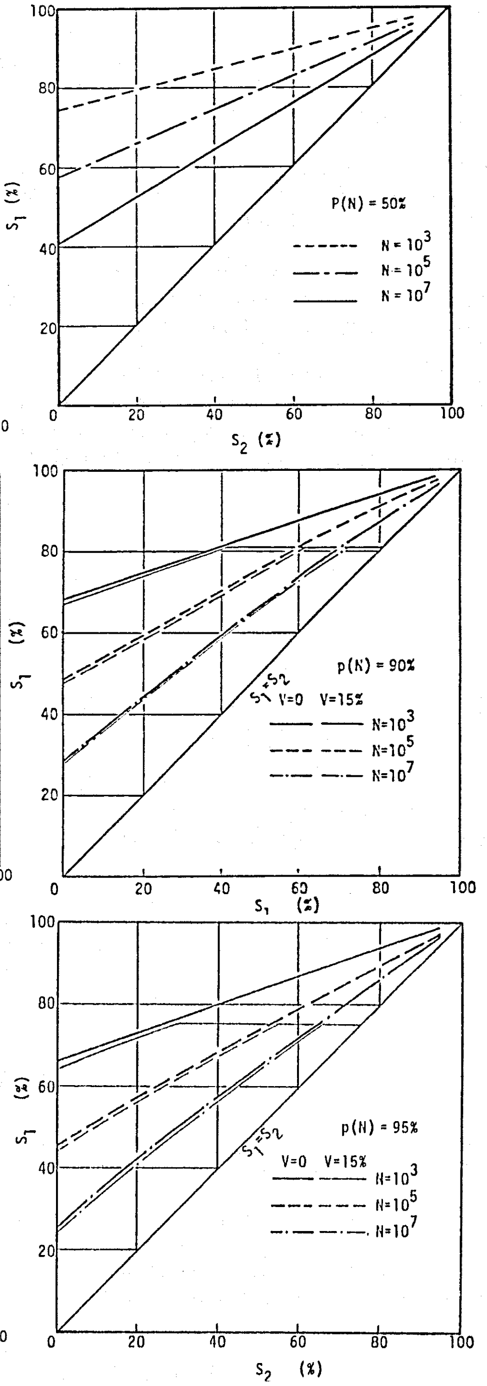


Fig.19 Time endurance diagrams of concrete in the water.

5. CONCLUSION

In this study, the following became clear after the investigation of the compressive fatigue failure of concrete subjected to constant repeated load both in the air and in the water.

(1) The fatigue life of concrete subjected to constant compressive repeated stress scatters widely, and the distribution of fatigue life can be practically regarded as the logarithmic normal distribution.

(2) The fatigue strength of concrete can be expressed by the ratio to the static strength. If the repeated stress ratio is constant, there is no significant difference among the fatigue lives of concrete having different static strength.

(3) The mean of the fatigue life of concrete is affected not only by the maximum stress ratio, S_1 but also by the minimum stress ratio, S_2. On time endurance diagram, the modified Goodman's relationship comes in existence. Consequently, the mean of fatigue life, $\overline{\log N}$ is estimated as follows,

in the air,

$$\overline{\log N} = 17.5 \frac{100 - S_1}{100 - S_2}$$

in the water,

$$\overline{\log N} = 11.7 \frac{100 - S_1}{100 - S_2}$$

The equation in the air agrees with some equation calculated from the other investigators' test results.

(4) The amount of the scatter of fatigue life of concrete, D(logN) is represented by linear equation of the maximum and minimum stress ratio, S_1 and S_2 and the coefficient of variation of static strength, V as follows.

in the air,

$$D(\log N) = (-0.0301\, S_1 + 0.0137\, S_2) \cdot \left(1 - \frac{V}{100}\right) + 3.00$$

in the water,

$$D(\log N) = (-0.0115\, S_1 + 0.0026\, S_2) \cdot \left(1 - \frac{V}{100}\right) + 1.35$$

D(logN) of concrete in the water is fairly smaller than that in the air.

(5) Estimating the fatigue strength of concrete, S-N curves considered survival probability, P(N), that is, S-N-P(N) curves are proposed.

For concrete in the air, when the minimum stress ratio, S_2 is 0% at survival probability, P(N)=50%, the fatigue strength at 10^7 repetitions is 60% to the static strength. Those values are 48.7% and 46.8 % with V =0 % and 15 % respectively at the survival probability, P(N)=95%.

On the other hand, for concrete in the water, when the minimum stress ratio, S_2 is 0%, the fatigue strength at 10^7 repetitions is 40% at survival probability, P (N)=50%, and is 29% and 28 % with V =0 % and 15 % respectiveiy at survival probability, P(N)=90%, and is 26% and 24 % with V =0 % and 15 % respectively at survival probability, P(N)=95%.

When the coefficient of variation of static strength is large and survival probability, P(N) is high, the maximum repeated stress can be determined by not only the scatter of fatigue life but also that of static strength.

And the fatigue strength of concrete at the minimum stress ratio, $S_2 = 0\%$ in the water is about 20 % lower than that in the air.

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