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DESIGN METHOD FOR STRUCTURAL CONCRETE MEMBERS IN ULTIMATE TORSION

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SYNOPSIS

The purpose of this paper is to propose the calculating method of ultimate torsional strength and deformation for the structural concrete members. In this study, it is assumed that the failure mechanism of the structural concrete members in torsion is based on the space truss. The study may applied to the members which have rectangular cross section and are comprised reinforced concrete, steel-reinforced concrete, and reinforced, prestressed concrete of lightweight concrete. The items are : (a) Equilibrium condition, (b) Geometrical condition, (c) Shear flow, (d) Ultimate torsional strength of prestressed concrete and steel-reinforced concrete member, (e) Relation between failure modes and reinforcement ratios, (f) Application of the equations for concrete members in combined torsion and bending.

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1. INTRODUCTION

Torsion is generally a secondary effect in the concrete structures. However, it becomes gradually important to investigate the torsional behavior of concrete members, because of the decreasing cross section with increasing of the strength of material used, the enlargement for the size of structures, the complication of structure, the advance of accuracy for calculating method and the use of limit design method. Therefore, the clarification of the behavior for concrete members in torsion is very important and basic design factor for the concrete structures.

Consequently, in this paper, as the begining for the solution concerning with the problem, the theoretical equations are given for the member of reinforced concrete, prestressed concrete and steel-reinforced concrete with rectangular section in pure torsion, comprising of the light-weight aggregate concrete. The equations are based on a space truss model, and derived from considering both the equilibrium conditions and the compatibility of deformation, and also the stress-strain characteristics of steel bars and concrete. The limitation of proposed equations are showed. Finaly, the author proposed the design procedures for structural concrete members in torsion.

2. CHARACTERISTICS OF THIS RESEARCH

In the characteristics of this research are as follows.
a) Application of space truss to the basic conception.
From the cracking mode at ultimate state, the mechanism of torsional resistance of reinforced concrete member is the space truss in which diagonal concrete struts with effective wall thickness (t) carry compression, and transverse and longitudinal bars carry only axial tension.
b) Equilibrium conditions and compativility of strain.
The equations for ultimate torsional strength are led by considering the balance of forces and the compatibility of strain, therefore these equations make it possible to estimate the balanced reinforcement and the stresses of steel.
c) Prediction of the failure modes.
By using the equations, the failure modes of reinforced and prestressed concrete member are able to predict in case of under-reinforcement, over-reinforcement and partially-over-reinforcement.
d) Estimation of torsional deformation.
At ultimate state, the torsional rigidity can be estimated by the equations from the theory of space truss.
e) Limit of application for the equations.
The author proposed the applicational limit of equations that based on the theory of space truss.
f) Tests for all kinds of structural members.
This research is applied to the members which are comprised reinforced, steeel-reinforced concrete, and reinforced, prestressed concrete of light-weight concrete with the square or rectangular cross section.
g) Proposing the design procedure of concrete members in torsion.
The flow chart of design procedure is shown.

3. ANALYSIS TO THE ULTIMATE STRENGTH OF CONCRETE MEMBERS AND THE BEHAVIOR OF CONCRETE MEMBERS.

It is generally that in reinforced concrete subject to torsion the reinforcement has no appreciable effect on the stiffness before cracking. Similaly, the longitudinal or transverse reinforcement acting alone provides little additional strength beyond the capacity of plain concrete. However, if the longitudinal and the transverse steels are combined, the torsional moment corresponding to first cracking is usually somewhat increased. After cracking, the stiffness is markedly reduced but considerable increase in strength and a large amount of plastic deformation are possible depending on the amount and disposition of the reinforcement. After cracking, the theory of elasticity is no longer applicable and we need another theoretical model. Shown in Fig.1 is the prediction of the cracked model which will be presented in this paper.

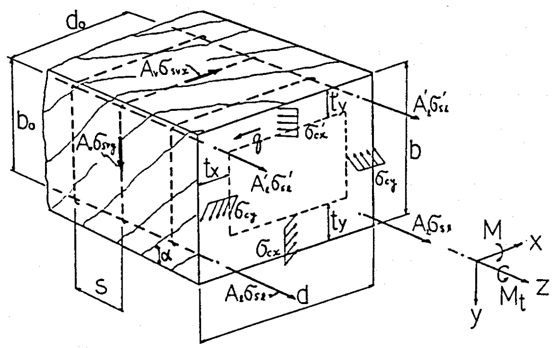


Fig.1 Equilibrium of cracked model

The theory presented is a development of the truss model for torsion and resembles the model proposed by Collins(1). In his research, the truss model have a box section with constant thickness of wall, and consists of closed hoops perpendicular to the axis of the member and longitudinal bars distributed symmetrically around the section.

The equations of equilibrium conditions are

$$q = \sigma_{cm} tx \sin\alpha \cos\alpha \tag{1}$$

$$Al\, \sigma_{sl} = \sigma_{cm} tx \cos^2\alpha\, Po \tag{2}$$

$$Av\, \sigma_{sv} = \sigma_{cm} tx \sin^2\alpha\, s \tag{3}$$

In the model, the torsion is resisted by diagonal concrete compressive stresses which spiral around the beam at a constant angle α. It is possible to calculate the twist of a beam in torsion if we know the strain in the longitudinal, transverse steel and the diagonal concrete strain, and the angle α,

$$\tan^2\alpha = \frac{\varepsilon_c + \varepsilon_l}{\varepsilon_c + \varepsilon_v (Pv/PO)} \tag{4}$$

An expression for the shear flow can be obtained

$$q = \sqrt{\frac{Al\,\sigma_{sl}}{Po} \frac{Av\,\sigma_{sv}}{s}} \tag{5}$$

The torsional moment on the beam is obtained from the fundamental equilibrium equation

$$Mt = 2\ Am\ q \tag{6}$$

In the prestressed concrete beams, the longitudinal prestressing steel will help to provide the longitudinal tension needed for equilibrium Eq(2) will thus become

$$Al\ \sigma_{sl} + Apl\ \sigma_{pl} = tx\, \sigma_c \cos^2\alpha\ Po \tag{7}$$

In author's research, as shown in Fig.1, the truss model have a box section with variable thickness of wall(tx, ty), and longitudinal bars distributed unsymmetrically around the section. Moreover, the torsional shearing forces are variable to the proportion to the length of the wall(2), and then the theory is suitable for the change of the sectional shape, also is applicable to the steel-reinforced concrete member. The author proposed the applicable limit of the theory.

3.1 Equilibrium Condition

The concrete member in torsion is resisted by the space truss in which diagonal concrete struts in compressive stresses are around the section of member at an angle α, and the reinforced bars are in tension, moreover, in this truss the tangential component of these stresses is provided the shear flow q which must be equilibrated the torsion. The normal components of the diagonal stresses result in a longitudinal and transverse compression forces which must be balanced by the tension in the bars.

The equations for equilibrium condition are
Axis-x and y:

$$\left.\begin{aligned} \sigma_{cx}' \, s. \ ty' \sin^2\alpha &= Av\,\sigma_{svx} \\ \sigma_{cx} \, s \ \ ty \ \sin^2\alpha &= Av\,\sigma_{svx} \\ \sigma_{cy} \, s \ \ tx \ \sin^2\alpha &= Av\,\sigma_{svy} \end{aligned}\right\} \qquad (8)$$

Axis-z

$$\left.\begin{aligned} 1/2 \left[\sigma_{cx} d \ tx + \sigma_{cy} b \ tx - tx \ ty(\sigma_{cx} + \sigma_{cy})\right] \cos^2\alpha &= Al \ \sigma_{sl} \\ 1/2 \left[\sigma_{cx} d \ tx' + \sigma_{cy} b \ tx - tx \ ty(\sigma_{cx}' + \sigma_{cy})\right] \cos^2\alpha &= Al' \sigma_{sl}' \end{aligned}\right\} \qquad (9)$$

$$q = \sigma_{cm} tm \ \sin\alpha \cos\alpha \qquad (10)$$

Assuming as follows,

$$\left.\begin{aligned} & k1 = ty/tx, \quad k2 = ty'/tx, \quad k3 = \sigma_{cx}/\sigma_{cy}, \quad k4 = \sigma_{cx}/\sigma_{cy}, \\ & C1 = 1/2 \ (1 + k1 \ k3 \ b/d), \ C1' = 1/2 \ (1 + k2 \ k4 \ b/d) \\ & C1'' = 1/2 \ (C1 + C1') \\ & C2 = 1/2 \ \left[k1 \left(k3 \ d - tx \ (1 + k3)\right) + b\right] \\ & C3 = 1/2 \ \left[k2 \left(k4 \ d - tx \ (1 + k4)\right) + b\right] \end{aligned}\right\} \qquad (11)$$

and then, equations (8), (9) and (10) are

$$Av \ \sigma_{svy} = \sigma_{cy} \ tx \ s \ \sin^2\alpha \ C1'' \qquad (12)$$

$$Al \ \sigma_{sl} = \sigma_{cy} \ tx \ \cos^2\alpha \ C2 \qquad (13)$$

$$Al' \sigma_{sl}' = \sigma_{cy}' \ tx \ \cos^2\alpha \ C3 \qquad (14)$$

Assuming, $\sigma_{cm} = \sigma_{cy}$, tm = tx, the equation (10) yields

$$q = \sigma_{cy} \ tx \ \sin\alpha \cos\alpha \ C1'' \qquad (15)$$

3.2 Geometrical Condition

The expression between twist(Θ) and angle(α)is given from the torsional strain energy in member of unit length, and then

$$\Theta = \frac{Po}{2Am}\left(\varepsilon_l \frac{C4}{\tan\alpha} + \frac{\varepsilon_v \, Pv \, \tan\alpha}{Po} + \frac{2\varepsilon_c}{\sin 2\alpha} \right) \tag{16}$$

The internal energy will be minimum, if the minimum work done, and henc for a given load the external displacement is minimum, this means that $d\Theta/d\alpha = 0$, Solving this equation laeds to

$$\tan^2\alpha = \frac{\varepsilon_l C4 + \varepsilon_c}{\varepsilon_v (Pv/Po) + \varepsilon_c} \tag{17}$$

3.3 Depth of Diagonal Concrete Strut

If the concrete strain distribution (assumed linear) is known, the magnitude and position of the resultant compression can be calculated by using the stress-strain characteristics of the concrete. A convenient approch is to replace the true concrete stress distribution with an equivalent stress block of depth ab. For a given surface strain ε_{cs}, the rectangular stress block factors β and kc can be calculated from the stress-strain curve of the concrete, and then it can be seen that the resultant diagonal compression will act at a distance ab/2 below the surface. The position of this resultant defines the path of the shear flow and hence the terms Am and Po. An expression for the equivalent depth of compression ab can be obtained from equilibrium Eqs (12),(13) and (14), if the term σ_{cy} is replaced by kc σ_{cu} ab,

$$ab = \frac{1}{kc\,\sigma_{cu}}\left(\frac{Av\,\sigma_{svy}}{s\;C1''} + \frac{Al\,\sigma_{sl}\,(1+C5)}{C2 + C3} \right) \tag{18}$$

3.4 Shear Flow(q), Torsional Moment(Mt) and Twist(Θ)

The equation for q,Mt and Θ can be obtained from Eqs.(12),(13)and(15), and then Shear flow is

$$q = \sqrt{\frac{C1''\;Av\,\sigma_{svy}}{s} \quad \frac{Al\,\sigma_{sl}\,(1+C5)}{C2 + C3}} \tag{19}$$

Torsional moment is

$$Mt = 2Am\sqrt{\frac{C1''\;Av\,\sigma_{svy}}{s} \quad \frac{Al\,\sigma_{sl}\,(1+C5)}{C2 + C3}} \tag{20}$$

Assuming the following relation for members after cracking.

$$\Theta = \frac{Mt}{Gc\;Kcr} \tag{21}$$

From the relation between the bending deformation of diagonal compression strut and the twist of member,

$$\Theta = \frac{\varepsilon_{cs} kc\, \beta\, \sigma_{cu}}{2} \sqrt{\frac{C1\ s}{Av\, \sigma_{svy}} \quad \frac{C2 + C3}{Al\, \sigma_{sl}\, (1+C5)}} \tag{22}$$

3.5 Calculation of Steel Stresses

From geometric Eqs(16),(17)and (20), and the equilibrium equation, the following expression for the steel strain can be derived

$$\varepsilon_l = \frac{\varepsilon_{cs}}{2C4} \left[\frac{Am\, \sigma_{cu}\, \beta (C2+C3)\, kc}{Po\ Al\, \sigma_{sl}\, (1+C5)} - (2-\beta) \right] \tag{23}$$

$$\varepsilon_v = \varepsilon_{cs} \left[\frac{Am\, \sigma_{cu} kc\, \beta\, s\, C1''}{2 \sigma_{svy} Po\ Av} - \frac{Po}{Pv} \left(1 - \frac{\beta}{2} \right) \right] \tag{24}$$

If the stress-strain relationship of the longitudinal and transverse steel is known, the Eqs (23)and (24) can be solved for σ_{sl} and σ_{sv},

$$\sigma_{sl} = \frac{1}{2} \sqrt{ \left(\frac{\varepsilon_{cs} Es\ (2-\beta)}{2\ C4} \right)^2 + \frac{2 \varepsilon_{cs} Es\ Am\, \sigma_{cu}\, \beta (C2+C3)\ kc}{C4\ Po\ Al\ (1+C5)} } - \frac{\varepsilon_{cs} Es (2-\beta)}{4\ C4} \tag{25}$$

$$\sigma_{sv} = \frac{1}{2} \sqrt{ \left(\frac{\varepsilon_{cs} Es\ Po}{Pv} \left(1 - \frac{\beta}{2}\right) \right)^2 + \frac{2 \varepsilon_{cs} Es\ Am\, \sigma_{cu}\, \beta C1'' kc\ s}{Pv\ Av} } - \frac{\varepsilon_{cs} Es\ Po}{2\ Pv} \left(1 - \frac{\beta}{2} \right) \tag{26}$$

3.6 Analysis of Prestressed Concrete Beam

If the beam is prestressed, its torsional cracking strength is increased with the average prestress. After cracking, the longitudinal prestressing steel will help to provide the longitudinal tension needed for equilibrium. The left side of Eq (13) will thus become, see Fig.2

$$\left. \begin{array}{l} Al' \sigma_{sl}' + Ap' \sigma_{pl}' \\ Al\ \sigma_{sl} + Ap\ \sigma_{pl} \end{array} \right\} \tag{27}$$

This means that in Eqs(12),(13) and (14) the term $Al \sigma_{sl}$ is replaced by ($Al\, \sigma_{sl} + Ap\, \sigma_{pl}$) and C5 by Cp, and then

$$abp = \frac{1}{kc\, \sigma_{cu}} \left(\frac{Av\, \sigma_{svy}}{s\ C1''} + \frac{(Al \sigma_{sl} + Ap \sigma_{sp})(1+Cp)}{C2 + C3} \right) \tag{28}$$

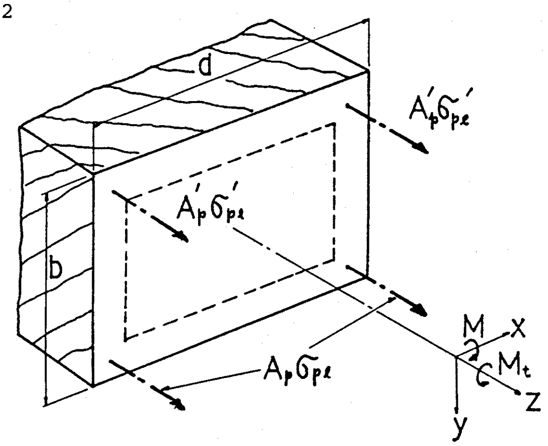


Fig.2 Longitudinal prestressing steel force

$$\varepsilon_{pl} = \frac{\varepsilon_{cs}}{2\ C4} \left(\frac{Am\, \sigma_{cu}\, \beta\, (C2+C3)\ kc}{Po(Al \sigma_{sl} + Ap \sigma_{pl})(1+Cp)} \quad (2-\beta) \right) \tag{29}$$

$$qp = \sqrt{\frac{C1''Av\,\sigma_{svy}}{s}\,\frac{(al\,\sigma_{sl} + Ap\,\sigma_{pl})(1+Cp)}{C2 + C3}} \tag{30}$$

Torsional moment of prestressed concrete beam (Mtup)

$$Mtup = 2Am\sqrt{\frac{C1''Av\,\sigma_{svy}}{s}\,\frac{(Al\,\sigma_{sl} + Ap\,\sigma_{pl})(1+Cp)}{C2 + C3}} \tag{31}$$

Torsional stiffness of prestressed concrete bean (GcKup)

$$GcKup = \frac{4\ Am}{\varepsilon_{cs}\,kc\,\beta\,\sigma_{cu}}\left(\frac{Av\,\sigma_{svy}}{s}\,\frac{(Al\,\sigma_{sl} + Ap\,\sigma_{pl})(1+Cp)}{C2 + C3}\right) \tag{32}$$

3.7 Ultimate Torsional Strength of Steel-Reinforced Concrete Member

On the basis of test results, the behavior of steel-reinforced concrete members are approximately as same as that of the reinforced concrete member. If the steel joist in the member convert into the bars, the expressions for reinforced concrete can be applied to the analysis for the behaviors of steel-reinforced concrete member.

For the steel joist shown in Fig.3

$$\left.\begin{aligned} Avt &= bt\ t\ \frac{s\,\sigma_{ssy}}{st\,\sigma_{sy}} \\ \sum Alt &= 4\ Als\ \frac{\sigma_{ssy}}{\sigma_{sy}} \end{aligned}\right\} \tag{33}$$

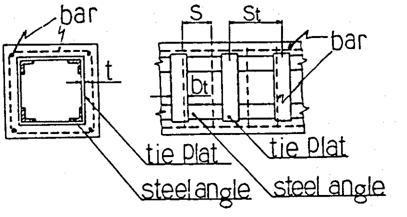


Fig.3 Steel-reinforced concrete and steel joist

The total area of the converted steel bars are

$$\left.\begin{aligned} \sum Avs &= Av + Avt \\ \sum Als &= \sum Alt + \sum Al \end{aligned}\right\} \tag{34}$$

It is necessary to calculate the converted width and height for the estimation of the Po and Am, and then, see Fig.4.

$$\left.\begin{aligned} bos &= \frac{Av\ bo + Avt\ bot}{Av + Avt} \\ dos &= \frac{Av\ do + Avt\ dot}{Av + Avt} \end{aligned}\right\} \tag{35}$$

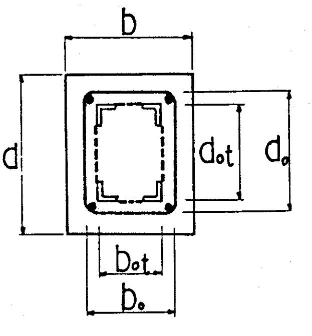


Fig.4 Cross section of steel-reinforced concrete

3.8 Relation between Failure Modes and Reinforcement Ratios

In the prediction of failure modes, it is convenient to define the following reinforcement ratios

$$\left.\begin{aligned} pl &= \frac{\sum Al\ s}{Am\ s} = \frac{\sum Al}{Am} \\ pv &= \frac{Av\ Pv}{Am\ s} \end{aligned}\right\} \qquad (36)$$

The value of the longitudinal and transverse reinforcement ratio which will produce balanced conditions at failure can be obtained from the equations of strain for steel bars.

$$plb = \frac{\sigma_{cu}}{\sigma_{sly}} \frac{\beta\ kc\ (C2 + C3)}{Po(1+C5)\{2\ \varepsilon_l / \varepsilon_{cs}\ C4 + (2-\beta)\}} \qquad (37)$$

$$pvb = \frac{\sigma_{cu}}{\sigma_{svyy}} \frac{\beta\ kc\ C1''}{2\{\varepsilon_v / \varepsilon_{cs} + Po/Pv\ (1-\beta/2)\}} \qquad (38)$$

The balanced values of plb and pvb for the range of material properties and ratio bo/do usualy used in design are given in Fig.5. The two balanced conditions which are given by Eqs (37) and (38) define five failure modes(neither steel yields, only transverse steel or only longitudinal steel yield, both steels yield and balanced failure), and then we can predict the failure modes examining the reinforcement ratios of beams by Eqs(37) and (38).

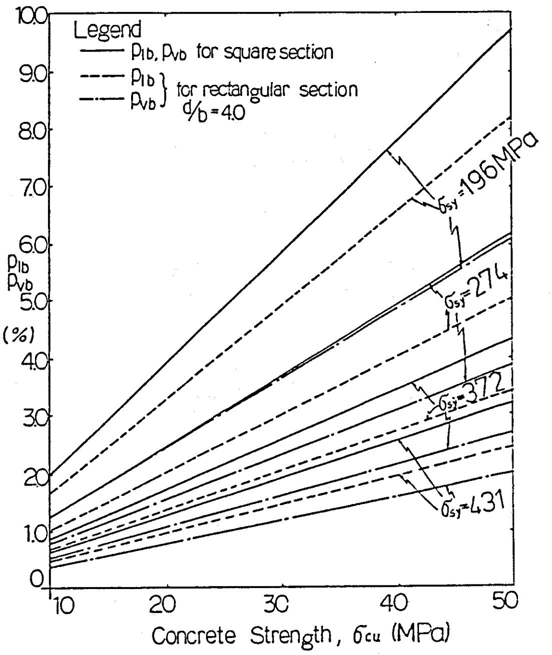


Fig.5 Balanced reinforcement ratio

3.9 Application of the Equation for Concrete Members in Combined Torsion and Bending

In combined torsion and bending, the equations for concrete members which are given by the analogy of space truss can apply the analysis of behavior of concrete member within the stress on the upper surface of member in tension. Here, it takes for convenience that the limit Ko = Mt/M is characterized by the condition of zero stress in the top longitudinal bars(3). From the equilibrium condition

$$Al'\ \sigma_{sl}' = \frac{Mt\ Po}{8bo\ do\ \tan\alpha\ C1''} - \frac{M}{4bo} \qquad (39)$$

In the equation(39), $\sigma'_{sl} = 0$ and then

$$Mt/M = \frac{2\ do\ \tan\alpha\ Ci''}{Po} = Ko \tag{40}$$

The equations being based on the space truss are reasonable in the range Ko $<$ Mt/M, in the case of Ko $\geqq$ Mt/M, different theory must be applied.

4. TESTS OF CONCRETE MEMBERS AND RESULTS

4.1 Outline of Tests

To study the behavior of reinforced, steel-reinforced concrete beams, and pre-stressed concrete beams of light-weight concrete(4),with square and rectangular cross-section under pure torsion, 19 beams were tested, involving the following 4 major variables:

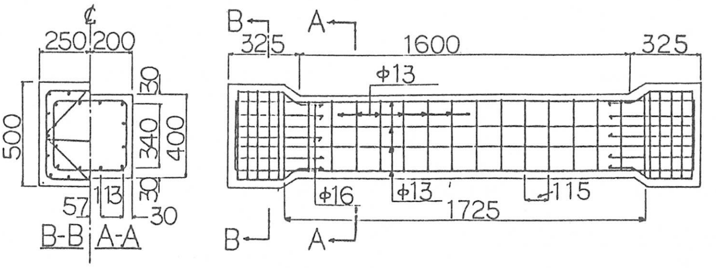
1) Amount of reinforcement
2) Concrete strength
3) Concrete quality
4) Cross-section

Five groups of beams were tested as outlined in Table.1.

Table.1. Tests and Results

Beam	Group	section	steel ratio (%)	Ult' moment (kN-m) test (1)	calcul (2)	(1)/(2)	mean	Note
R-2.o-∞	RC	square	2.0	110 131	121	0.90 1.08	0.99	
R-1.0-∞	RC	square	1.0	77 82	72	1.07 1.13	1.10	
R-0.5-∞	RC	square	0.5	50	49.5	1.01	1.01	
SR-5.9-∞	SRC	square	5.9	194 214	198	0.98 1.08	1.03	Steel-reinforced concrete
SR-4.5-∞	SRC	square	4.5	148	190	0.78	0.78	Steel-reinforced concrete
SR-4.9-∞	SRC	square	4.9	209 199	212	0.98 0.94	0.96	Steel-reinforced concrete
SR-3.5-∞	SRC	square	3.5	168	182	0.92	0.92	Steel-reinforced concrete
Rr-2.0-∞	RC	rect'	2.0	52 51	48.5	1.07 1.05	1.06	
RL-40-2.0-∞	LRC	square	2.0	124	144	0.86	0.86	Light-weight reinforced concrete
RL-40-1.0-∞	LRC	square	1.0	95	84	1.13	1.13	Light-weight reinforced concrete
RL-40-0.5-∞	LRC	square	0.5	47	42	1.11	1.11	Light-weight reinforced concrete
PL-40-2.0-∞	LPC	square	2.0	157	153	1.02	1.02	Light-weight prestressed concrete
PL-40-1.0-∞	LPC	square	1.0	125	123	1.01	1.01	Light-weight prestressed concrete
PL-40-0.5-∞	LPC	square	0.5	86	69	1.25	1.25	Light-weight prestressed concrete
			Average				1.02	

A typical test beam is shown in Fig.6. The length of all beams was 2250mm., The clear span subjected to torsion was 1600mm. To avoid local failure close to the clamping heads due to stress concentration, a length of 325mm. at each end of the beams was reinforced with about 50°/₀ additional stirrups. Seventeen beams had a nominal cross-section of 400x400mm., twobeams beams had a nominal cross-section of 400x250mm., and contained both longitudinal and transverse steel.



cross-section

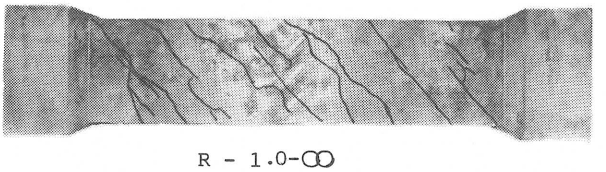
R-1.0-∞

Fig.6 Test beam of reinforced concrete

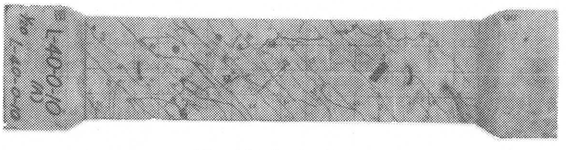
4.2 Tests results

The analytical and test results for 19 beams tested in torsion are given in Table.1. the analytical strengths have been computed using the analysis proposed befor.

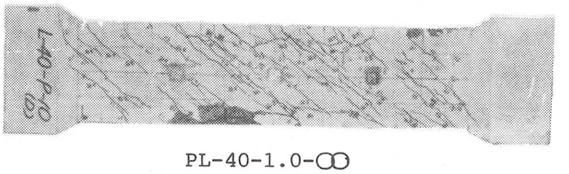
The general pattern of cracks is shown in Fig.7. The patterns of cracks for other beams were similar except that the inclination of the cracks with the axis of the beam in the zone of axial compression stresses and space of cracks are different between reinforced concrete beams and steel-reinforced concrete beams.



R - 1.0-∞



RL-40-1.0-∞



PL-40-1.0-∞

Fig.7 Cracking pattern after failure

5. DESIGN PROCEDURES

Design procedures for structural concrete members subject to torsion are not yet fully developed and codified. this paper sets out what is believed to be a reasonably conservative design procedure for concrete members subject to torsion. It is based on the author's evaluation of studies of torsional behavior so far completed, and the equations proposed by author. In Fig.8., the outline of flow chart for design procedures are shown.

6. CONCLUDING REMARKS

The space truss model is capable of predicting the post-cracking torsional behavior of concrete members, especially in ultimate torsion. Some of the capabilities of the theory are listed below;

1. The theory can be applied to concerte members(reinforced, steel-reinforced and prestressed concrete) having a rectangular and box section.
2. The torsional strength of under-reinforced, partially-over-reinforced and completely-over-reinforced members can be predicted.
3. The theory can predict the behavior of concrete membres in over-reinforcement. The theory presented in the paper is applicable only to St'Venant torsion and hence can not predict the response where warping torsion dominates.

The section subjected to combined torsion and bending , and unsymmetrically reinforced section can be treated by the procedures.

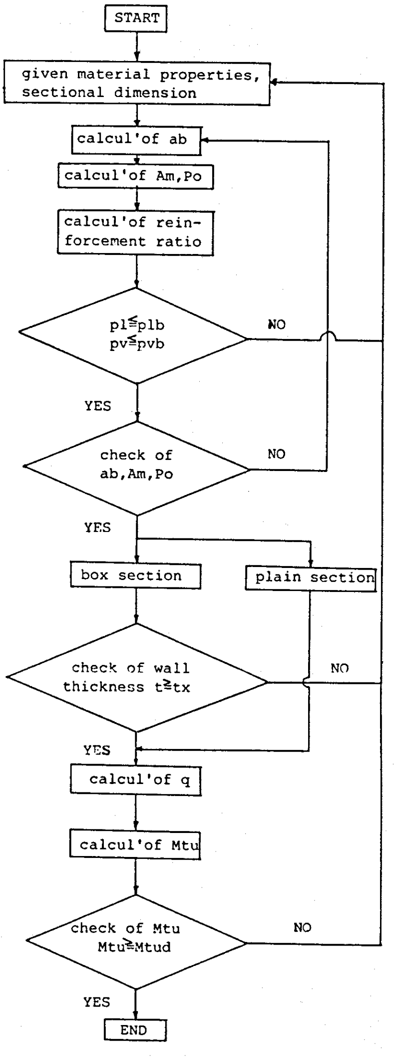


Fig.8 Design flow chart

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NOTATION

Al = area of bottom longitudinal reinforcing steel
Al'= area of upper longitudinal reinforcing steel
Alt = area of longitudinal steel angle of joist
Als = total area of the converted steel bars
Am = area enclosed by shear flow
Ap = area of bottom longitudinal prestressing steel
Ap'= area of upper longitudinal prestressing steel
Av = area of one transvrese steel

Avs = area of the converted transverse steel
Avt = area of tie plate converted into transverse steel
ab = equivalent depth of compression strut
abp = equivalent depth of compression strut in prestressed concrete
b = width of cross-section
bo = smaller center-to-center dimension of closed rectangular stirrup
bot = smaller dimension of steel joist
C4 = 2(C2+k_ε C3)/(po C1")
C5 = $(Al' \sigma_{sl}')/(Al\ \sigma_{sl})$
Cp = $(Al' \sigma_{sl}' + Ap' \sigma_{pl}')/(Al\ \sigma_{sl} + Ap\ \sigma_{pl})$
d = depth of cross-section
do = larger center-to-center dimension of closed rectangular stirrup
dot = larger dimension of steel joist
Es = modulus of elasticity of steel
Gc = modulus of rigidity of concrete
Ko = Mt/M
Ku = torsional stiffness of reinforced concrete at ultimate torsion
Kup = torsional stiffness of prestressed concrete at ultimate torsion
kc = an equivalent rectangular stress block factor
$k_\varepsilon = \varepsilon_l'/\varepsilon_l$
M = bending moment
Mt = torsional moment
Mtu = ultimate torsional moment
Po = perimeter of shear flow path
Pv = transverse steel (stirrup) perimeter
s = transverse steel (stirrup) spacing
tm = mean thickness of compression struts
α = angle of diagonal cracking
β = an equivalent rectangular stress block factor
ε_c = concrete diagonal strain at the position of the resultant shear flow
ε_{cs} = concrete diagonal strain at the surface
ε_l = strain in longitudinal steel
ε_v = strain in transverse steel (stirrup)
ε_{lp} = strain in longitudinal steel of prestressed concrete
σ_{cm} = average compressive stress in concrete strut
σ_{cu} = compressive strength of concrete
σ_{pl}, σ_{pl}' = stresses of prestressing steel in bottom, in upper, respectively
σ_{sl}, σ_{sl}' = stresses of longitudinal steel in bottom, in upper, respectively
σ_{sv} = stresses in transverse steel (stirrup)
σ_{ssy}, σ_{sy} = yield points of steel plate, bar, respectively
σ_{sly}, σ_{svyy} = yield points of longitudinal, transverse steel, respectively
θ = twist per unit length
θ_u = twist per unit length at ultimate