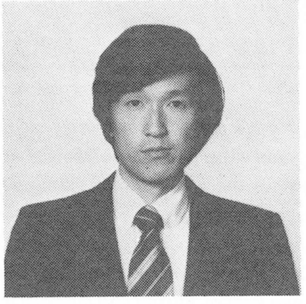
CONCRETE LIBRARY OF JSCE NO.2, DECEMBER 1983

BEHAVIOR IN SHEAR OF REINFORCED CONCRETE BEAMS UNDER FATIGUE LOADING

(Reprint from Journal of The Faculty of Engineering,
The University of Tokyo (B), Vol.37, No.1, 1983)



Tamon UEDA   Hajime OKAMURA

SYNOPSIS

This is the summary of two papers entitled as "Behaviors of Stirrups under Fatigue Loading" (Concrete Journal of JCI, Vol.19, No.5, 1981) and "Fatigue Strength in Shear of Beam without Web Reinforcement -- Influence of Load Range on Fatigue in Shear" (Concrete Journal of JCI, Vol.20, No.9, 1982).
An equation for the prediction of the fatigue strength in shear of beam without shear reinforcement is proposed. This equation includes the influence of load range.
Using this equation a procedure is derived to calculate the average strain of shear reinforcement under general variable loading. This procedure is based on a newly developed idea that the strains during the subsequent loading are essentially the same in spite of the difference of the previous loading history, if the stirrup strains produced by the same applied shear forces are same.

Tamon UEDA is a research associate of Civil Engineering at the University of Tokyo. He received his Doctor of Engineering in 1982 from the University of Tokyo. He was awarded for a part of this study in 1981 from the Japan Concrete Institute.

Hajime OKAMURA is a professor of Civil Engineering at the University of Tokyo. He was awarded JSCE Most Valuable Paper Prize in 1981 and JCI Valuable Paper Prize in 1981 and in 1982. He obtained his doctor at the Universitay of Tokyo in 1966 and has been a faculty member since then. He has written many scientific articles on fatigue and shear of reinforced concrete members, and authored a book 'Limit State Design of Concrete Structures'.
Prof. Okamura is a current chairman of JCI Committee on Shear and FEM Analysis, and a member of working commission III of IABSE since 1982.

JOURNAL OF THE FACULTY OF ENGINEERING, THE UNIVERSITY OF TOKYO (B)
Vol. XXXVII, No. 1 (1983)

Behavior in Shear of Reinforced Concrete Beams under Fatigue Loading

by
Tamon UEDA* and Hajime OKAMURA*

(Received September 29, 1982)

Fatigue tests of eleven T-beams with stirrups, four rectangular beams with bent-up bars and inclined stirrups and sixteen rectangular beams without shear reinforcement were carried out. Based on the test results, an equation for the prediction of the fatigue strength in shear of beam without shear reinforcement is proposed. This equation includes the influence of load range. Using this equation a procedure is derived to calculate the average strain of shear reinforcement under general variable loading. This procedure is based on a newly developed idea that the strains during the subsequent loading are essentially the same in spite of the difference of the previous loading history, if the stirrup strains produced by the same applied shear forces are same. The fatigue strength of a beam can be evaluated from the stress range calculated by this procedure.

Notation

a : shear span, distance between center of the load and support
A_b : area of bent-up bars within a distance s
A_i : area of inclined stirrups within a distance s
A_s : total area of tension reinforcement
A_v : area of vertical stirrups within a distance s
A_w : area of web reinforcement within a distance s
b : beam width
b_w : web width
d : effective depth (mm), distance from extreme compression fiber to centroid of tension reinforcement
E_s : Young's modulus of steel
E_w : Young's modulus of web reinforcement
f_c' : cylinder strength of concrete (MPa)
f_{wy} : yield strength of web reinforcement
M.S. : maximum size of coarse aggregate
N : number of loading cycles
N_c : number of loading cycles when stirrup strains begin to increase
N_{eq} : equivalent number of loading cycles
N_f : tested fatigue life
N_{f1} : number of loading cycles at the first fracture of stirrup
p : reinforcement ratio, $=A_s/(bd)$
p_w : reinforcement ratio, $=A_s/(b_w d)$
$r = V_{min}/V_{max}$
s : spacing of shear reinforcement
$\bar{s}$: average spacing of shear reinforcement

* Department of Civil Engineering.

s_b : spacing of bent-up bars
s_i : spacing of inclined stirrups
s_v : spacing of vertical stirrups
t : time (min)
V : shear force
V_c : shear force carried by concrete
V_{co} : shear force carried by concrete at the initial loading
V_{cu} : static strength in shear of beam without web reinforcement
V_f : shear force at calculated flexural failure
V_{max} : applied maximum shear force
$V_{max\,cal}$: calculated applied maximum shear force
$V_{max\,test}$: tested applied maximum shear force
V_{min} : applied minimum shear force
V_r : range of applied shear force, $=V_{max}-V_{min}$
V_s : shear force carried by the assumed truss with 45° diagonals
V_y : shear force at yielding of stirrups
z : arm length of the assumed truss, $=d/1.15$
α : angle between shear reinforcement and longitudinal axis of member
β_x : a coefficient for each stirrup to cover the influence of support and loading point [13]
$\bar{\beta}_x$: average of the coefficient β_x
$\bar{\varepsilon}_b$: average strain of bent-up bars
ε_c : uniform compressive strain in the assumed pure shear strain state
$\bar{\varepsilon}_i$: average strain of inclined stirrups
ε_t : uniform tensile strain in the assumed pure shear strain state
$\bar{\varepsilon}_v$: average strain of vertical stirrups
ε_w : strain of web reinforcement
$\varepsilon_{w\,max}$: strain of web reinforcement at applied maximum shear force
$\bar{\varepsilon}_{w\,max}$: average strain of web reinforcement at applied maximum shear force
$\bar{\varepsilon}_{wo}$: average residual strain of web reinforcement
$\bar{\varepsilon}_{wr}$: average strain range of web reinforcement
$\bar{\varepsilon}_{wr\,cal}$: calculated average strain range of web reinforcement
$\bar{\varepsilon}_{wr\,test}$: tested average strain range of web reinforcement
$\bar{\sigma}_{wr\,cal}$: calculated average stress range of web reinforcement
$\sigma_{wr\,test}$: tested stress range of the first fractured stirrup

1. Introduction

A reinforced concrete beam sometimes fails in shear under fatigue loading due to the fracture of web reinforcement even if the applied maximum shear force is much less than the ultimate static strength [1], [2]. Since the fatigue fracture of web reinforcement depends on the stress intensity, the characteristics of stress under fatigue loading are firstly to be investigated. Some previous reports pointed out that stirrup strains increased during fatigue loading [2]–[5], the fatigue fracture of stirrup occurred at bend [2], [6], [7] and the fatigue strength of stirrup was smaller than that of bar itself [2], [7]. Some of the reports proposed the equations for the calculation of stirrup strains under fatigue loading according to the observed relationship between applied shear forces and stirrup strains [2], [3], [5]. Recently, Sabry reported that stirrup strains increased in proportion to the logarithm of the cycles of repeated loading due to the decrease of shear force carried by concrete and proposed the equation for the calculation of stirrup strain by applying the fatigue strength in shear of the identical beam without shear reinforcement to the decrease of shear force carried by concrete [1].

This study is the extension of the Sabry's work. Fatigue tests of eleven T-beams with stirrups, four rectangular beams with bent-up bars and sixteen rectangular beams without shear reinforcement were carried out. In the tests of beams with shear reinforcement the maximum and/or the minimum load was changed for each specimen and all the strains of shear reinforcement were measured in detail to investigate the behavior of shear reinforcement under general variable loadings. The tests of beams without shear reinforcement were carried out to support the assumption for the explanation of the behavior of shear reinforcement under the fatigue loading. In the tests the load range was taken as a main parameter.

2. Outlines of Tests

Eleven T-beams had identical section as shown in Fig. 1(a). Loading points were determined to make shear span depth ratio 2.0 for the right span and 4.0 for the left, while the ratio was 2.5 for Sabry's tests. The spacings of stirrups were relatively small compared with the Sabry's beams. Stirrups in the both shear spans were so designed that all the stirrups except the one nearest to the loading point should yield when the main tensile bars yielded. The shear force at yielding of the vertical stirrups was calculated by eq. 1 (see Table 1 and Fig. 2).

$$V_y = V_{co} + A_w f_{wy} (z/s)/\beta_x \tag{1}$$

where

$$V_{co} = 0.2 f_c'^{1/3}(1+\beta_p+\beta_d) b_w d \tag{2}$$

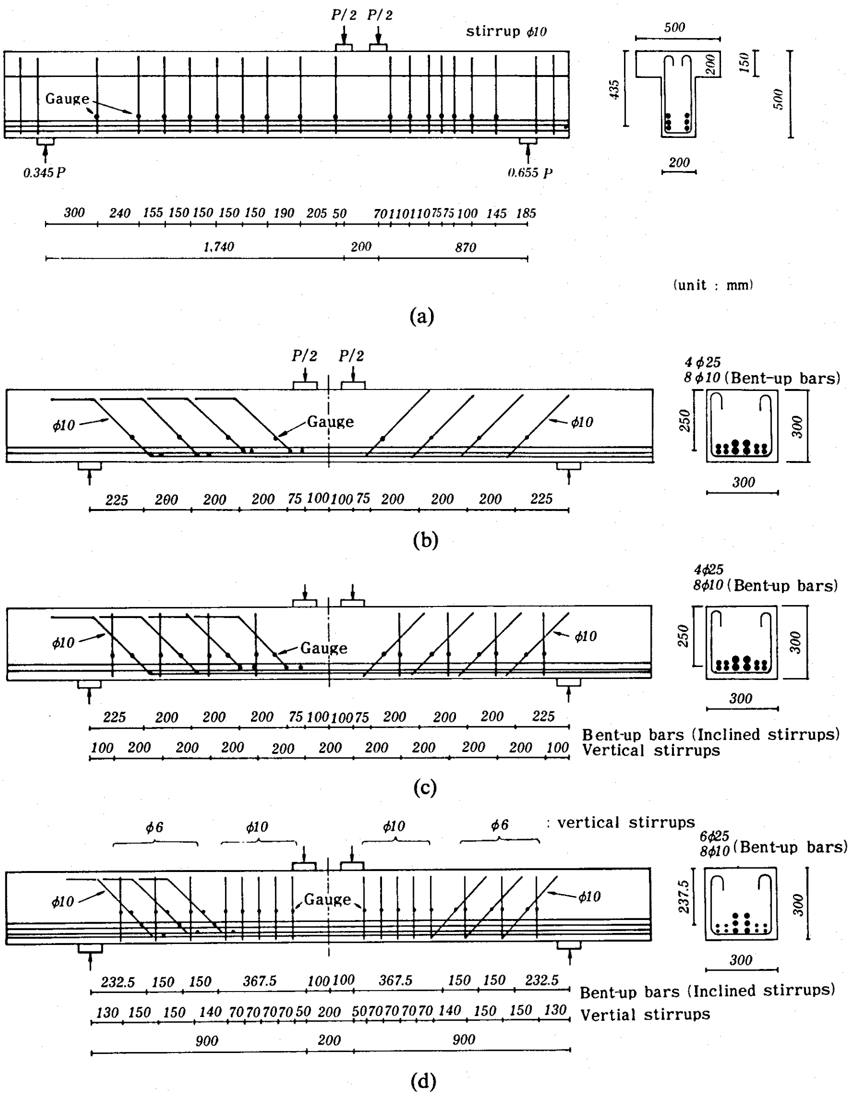
In Table 1, the tested values of V_{co} obtained from shear-strain curve of stirrup together with the one calculated by eq. 2, and the shear force at flexural failure, V_f, calculated by eq. 3 are also shown.

$$V_f = A_s f_y (1-0.6 p f_y / f_c')(d/a) \tag{3}$$

The stirrups were bent around the longitudinal bars and all the specimens were divided into two groups, called as FS series and FL series respectively, according to the radius of bend. The radius was 1.25 times the diameter of the stirrup in FS series and 2.5 times in FL series. Loading history was one of the main parameters, and the details were shown in Fig. 3.

The details of four rectangular beams with bent-up bars are shown in Figs. 1(b) (c) (d) and in Table 2. All the specimens had bent-up bars in the left shear span and inclined stirrups in the right. The diameters and the spacings were same for the bent-up bars and the inclined stirrups. Vertical stirrups were used together in the two specimens. The details of loading history were shown in Fig. 4. Sixteen rectangular beams without shear reinforcement consisted of eight different types of beams as shown in Table 3 and Fig. 1(e).

All the specimens were loaded statically during the first hundred cycles, and after that,



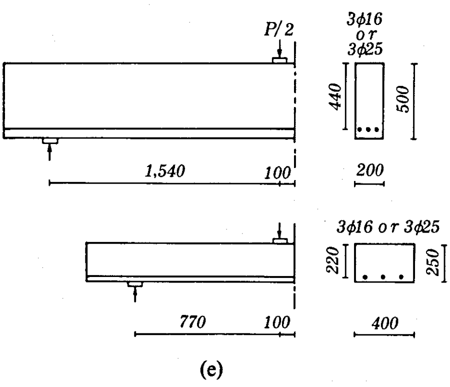
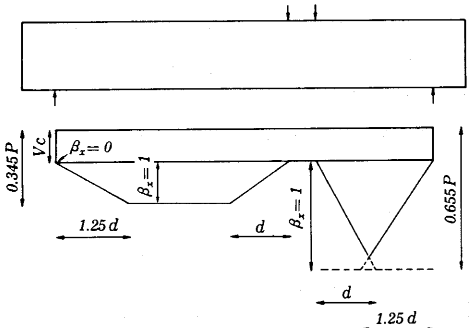


Fig. 1. Specimens: (a) T-beam with stirrups; (b) T-beam with bent-up bars, BI1 and BI3; (c) T-beam with bent-up bars, BI2; (d) T-beam with bent-up bars, BI4; (e) rectangular beams without web reinforcement.

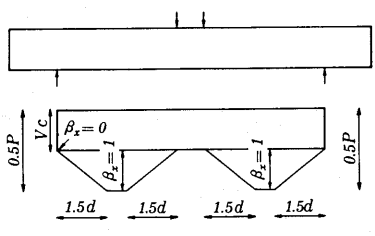
Table 1. Properties of T-beams with stirrups.

Specimens	V_{co} (1) kN	V_{co} (2) kN	V_y $a/d=4$ kN	V_y $a/d=2$ kN	V_f $a/d=4$ kN	V_f $a/d=2$ kN
FS1, FL2, FS3, FL4	96	99	229	428	229	459
FS5, FL6, FS7, FL8	97	101	230	429	231	462
FS9, FL10, FS11	106	106	239	438	235	470

(1) Tested value obtained from shear-strain curve of stirrup.
(2) Calculated value by eq. 2.
(3) $\bar{\beta}_x\, s$ is 155.5 mm for $a/d=4$ and 62.2 mm for $a/d=2$.



(a)



(b)

Fig. 2. Shear diagrams: (a) T-beams with stirrups; (b) rectangular beams with bent-up bars.

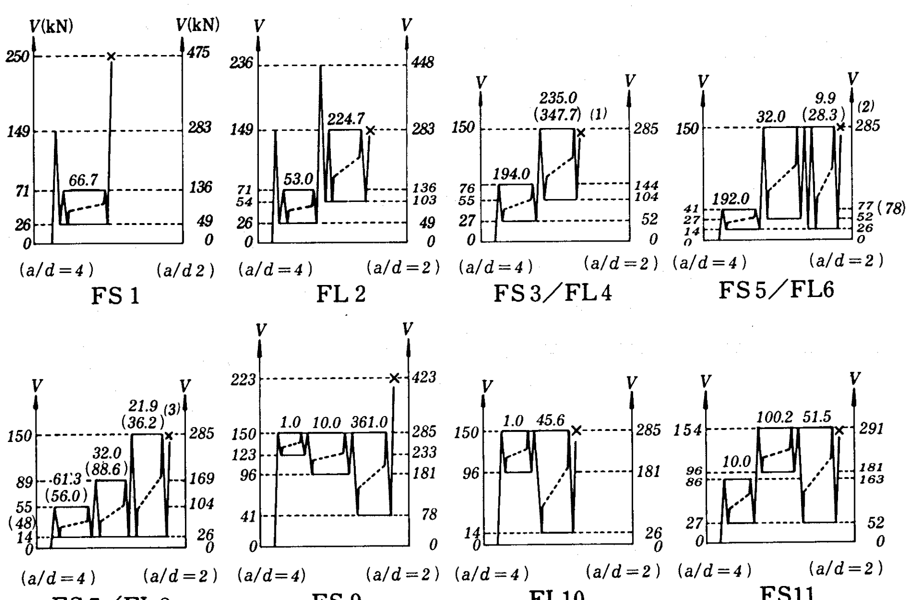


Fig. 3. Loading histories of T-beams with stirrups (numerals indicate loading cycles $\times 10^4$).

Table 2. Properties of beams with bent-up bars.

Specimen	Bent-up bars V_{co} (1) kN	Bent-up bars V_{co} (2) kN	Bent-up bars V_y kN	Inclined stirrups V_{co} (1) kN	Inclined stirrups V_{co} (2) kN	Inclined stirrups V_y kN	V_f kN
BI1	98	92	220	104	97	225	155
BI2	90	95	225	96	101	231	172
BI3	98	97	326	104	103	332	179
BI4	134	124	362	138	128	365	245

(1) and (2) see Table 1.

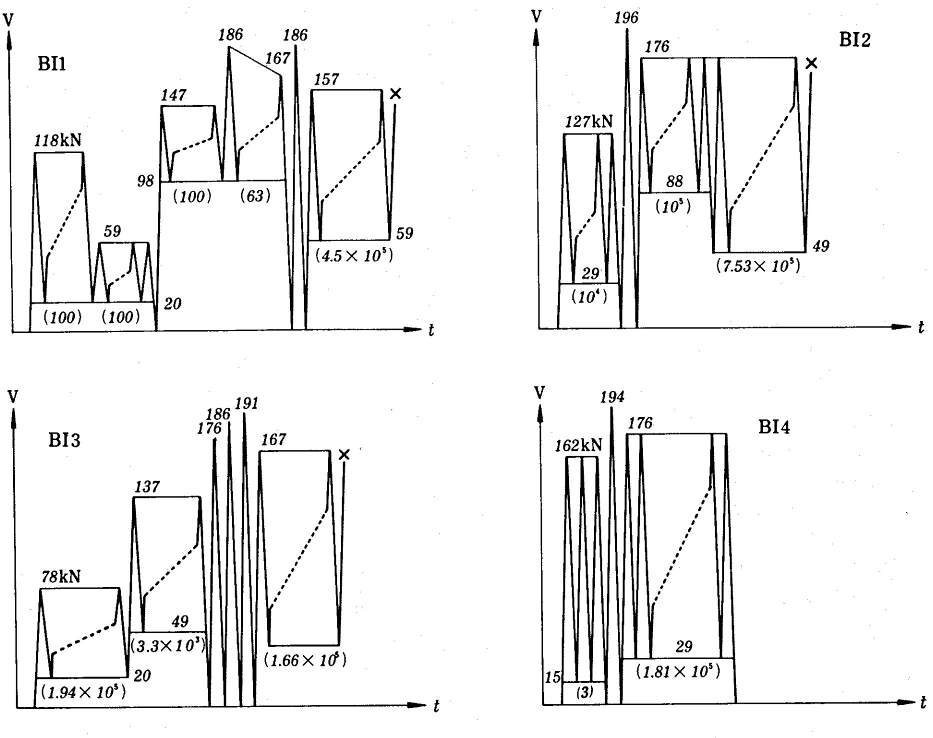


Fig. 4. Loading histories of rectangular beams with bent-up bars (numerals indicate the applied shear force, ( ) indicates loading cycles).

loaded dynamically 210 cycles per minute. A hydraulic jack was used for the cyclic and the static loading. Electrical resistance strain gauges of 5 mm in length were used for measuring the strains of shear reinforcement. The pulsator was stopped after appropriate loading cycles, and the strains of shear reinforcement were measured under static loading and the propagation of diagonal cracks was recorded. Concrete cover was removed to confirm the fatigue fracture of

Table 3. Properties and test results of beams without web reinforcement.

Specimens	a mm	d mm	b mm	p_w %	f_c' MPa	V_f kN	V_{cu} kN	V_{max} kN	V_{min} kN	$\frac{V_{min}}{V_{max}}$	$\frac{V_{max}}{V_{cu}}$	N_f 10 kilo	Notes
1a	1,540	440	200	0.68	33.4	65	69	49	5	0.1	0.72	0.05	
1b	1,540	440	200	0.68	33.4	65	69	49	30	0.6	0.72	314	
2a	1,540	440	200	0.68	45.5	66	76	46	5	0.1	0.61	1.86	
2b	1,540	440	200	0.68	45.5	66	76	46	28	0.6	0.61	447.9<	(1)
								74			0.97		(2)
3a	1,540	400	200	1.67	33.4	138	99	60	24	0.4	0.61	1,032<	(1)
								82			0.83		(3)
3b	1,540	440	200	1.67	33.4	138	99	71	36	0.5	0.72	0.07	
4a	1,540	440	200	1.67	45.5	143	110	67	7	0.1	0.61	43	
4b	1,540	440	200	1.67	45.5	143	110	79	32	0.4	0.72	0.23	
5a	770	220	400	0.68	34.2	65	85	58	6	0.1	0.69	[24.5]	(4)
5b	770	220	400	0.68	34.2	65	85	58	23	0.4	0.69	34.1	
6a	770	220	400	0.68	46.0	66	93	57	6	0.1	0.61	[31.1]	(4)
6b	770	220	400	0.68	46.0	66	93	57	23	0.4	0.61	312.1<	(1)
								62	25	0.4	0.66	23.15<	(1)
								67	34	0.5	0.72	36.9	
7a	770	220	400	1.67	34.2	139	115	98	59	0.6	0.85	0.049	
7b	770	220	400	1.67	34.2	139	115	98	10	0.1	0.85	0.024	
8a	770	220	400	1.67	46.0	143	127	99	79	0.8	0.78	70.6	
8b	770	220	400	1.67	46.0	143	127	108	97	0.9	0.85	398.4<	(1)
								108	86	0.8	0.85	387<	(1)
								117	94	0.8	0.92	123.5<	(1)
								117	70	0.6	0.92	0.267	

(1) Non failure.
(2) Flexural failure under static test.
(3) Shear failure under static test.
(4) Fatigue failure due to fracture of tension bars.

stirrups after the tests. All the bars used in the tests were deformed bars having two longitudinal ribs and parallel transverse lugs perpendicular to the bar axis. Their material constants together with the compressive strengths of concrete at the ages of testing were as shown in Table 4.

3. Fatigue Strength in Shear of Beam without Shear Reinforcement

The experimental researches on fatigue strength of beams without shear reinforcement have made clear that fatigue strength at 1 mega cycle is about 60% of the static strength in the cases of a large span depth ratio [1], [5], [8], [11], so-called S–N curves of beams with a large span depth ratio are different from those with small one [1], and that the beam, which should fail in flexure under the static loading, sometimes fails in shear under the fatigue loading [11].

However, the influences of load range and size of specimen on the fatigue strength have not yet been reported. Consequently, no S–N curve including these factors has been reported.

Table 4. Properties of materials.

(a) Concrete

	T-beam with stirrups			Rectangular beams				
				with bent-up bars				without web reinforcement
	FS1 FL2 FS3 FL4	FS5 FL6 FS7 FL8	FS9 FL10 FS11	BI1	BI2	BI3	BI4	
f_c' (MPa)	24.3	25.5	29.8	23.8	26.9	28.6	39.1	See Table 3
M.S. (mm)		20			25		15	25

(b) Reinforcement

	T-beam with sturrups		Rectangular beams				
			with bent-up bars			without web reinforcement	
ϕ (mm)	10	25	10	6	25	16	25
A_s (mm^2)	71.4	506.7	71.4	31.7	490	200	490
f_y (MPa)	383	342	383	344	370	400	370
E_s (GPa)	186	—	185	186	172	—	—

Therefore, fatigue tests of rectangular beams were carried out. The main parameters were the load range and the height of beams as shown in Table 3.

The evaluation of static strength is important when the fatigue strength is represented by the ratio of the applied maximum shear force to the static strength. The following equation, from which the most accurate static strength is obtained, is used for the calculation of the static strength. This equation can estimate it with less than 10% of the coefficient of variation [12].

$$V_{cu}=0.20 f_c'^{1/3}\,(0.75+1.40\,d/a)\,(1+\beta_p+\beta_d)b_w d \tag{4}$$

Figure 5 shows that the relationship between the ratios of the previous tested fatigue shear strengths to the values calculated by eq. 4 and the tested fatigue lives of the beams. The calculated static strengths are multiplied by 0.8 for the cases of light weight concrete. The solid line in Fig. 5 is a calculated line by the following equation, which does not include the effect of load range.

$$\log\,(V_{max}/V_{cu})=-0.035\log N_f \tag{5}$$

The average ratio of the tested maximum shear forces to the calculated ones is 1.00 and the

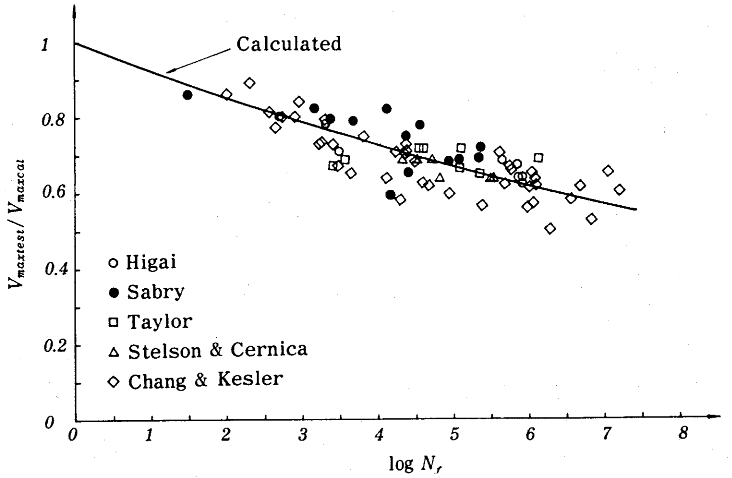


Fig. 5. Fatigue strength in shear of beam without web reinforcement normalized with static strength, V_{cu}, calculated by eq. 4 [1], [5], [8]–[11].

Table 5. Fatigue lives of beams without web reinforcement, kilo cycle.

V_{max}/V_{cu}	V_{min}/V_{max}					
	0.1	0.4	0.5	0.6	0.8	0.9
0.92				2.67	(1235)	
0.85	0.24			0.49	(3870)	(3984)
0.78					706	
0.72	0.5	2.3	0.7, 369	3140		
0.69	[245]	341				
0.66		(231.5)				
0.61	18.6, 430, [311]	(10320) (3121)		(4479)		

[ ]: Failure due to fatigue fracture of tension bars.
( ): Non-failure.

coefficient of variation is 7.4%.

Load levels and fatigue lives of our specimens are shown in Table 3. Twelve of the sixteen specimens failed in shear due to propagation of the main diagonal crack. Two of the others failed in flexure due to the fatigue fracture of the tensile bars at the maximum moment region and two failed under the static loading after the fatigue tests.

There are three pairs of the specimens, 1a–1b, 2a–2b and 7a–7b, which are identical except for the magnitude of the applied minimum load. The fatigue life of one specimen, whose ratio of the minimum load to the maximum was larger, was longer than that of the other. And the specimen 8b did not fail under the third repeated loading, but failed under the fourth repeated loading whose minimum load was smaller than that of the third one without change of the maximum load. The test results are rearranged in Table 5 to confirm the influence of load

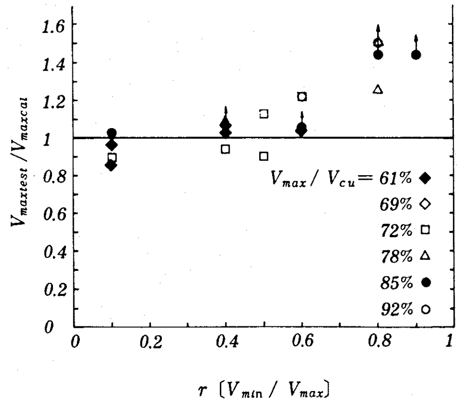
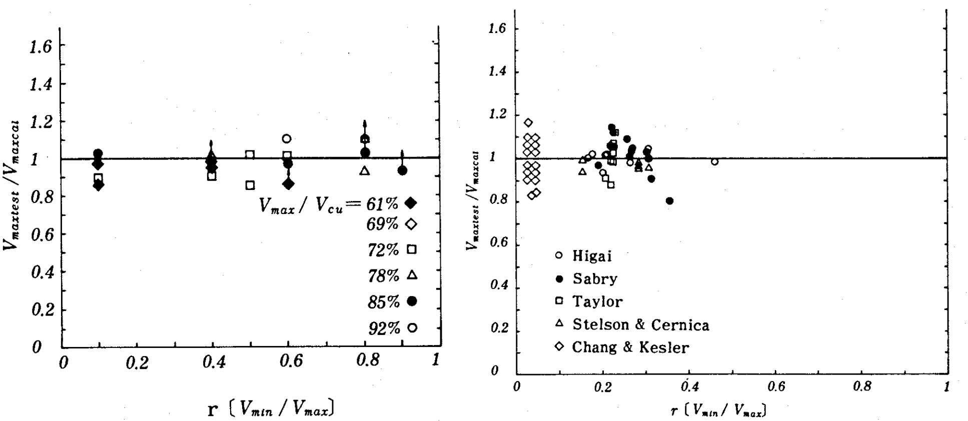


Fig. 6. Influence of ratio, r, of minimum shear force to maximum one on fatigue strength in shear of beam without web reinforcement.



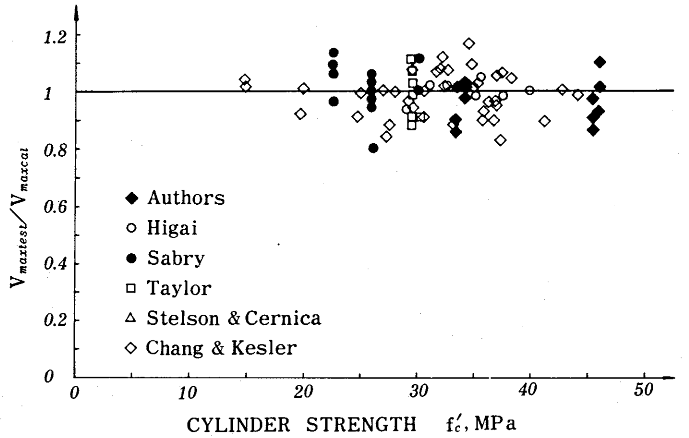
(a)

(b)

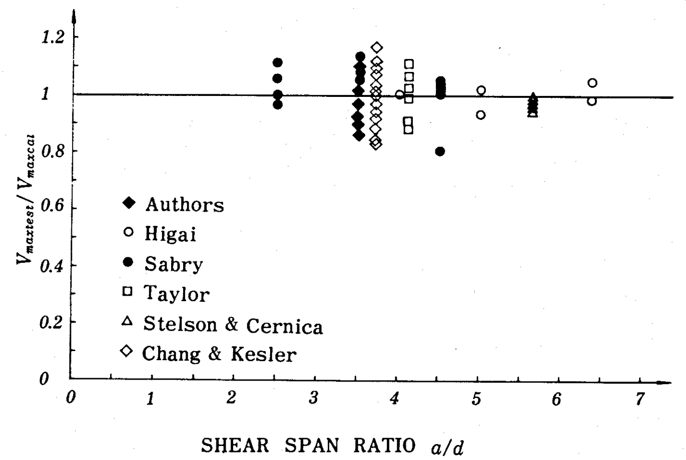
Fig. 7. Relationships between ratio of tested value of fatigue strength in shear of beam without web reinforcement to that calculated by eq. 6 and ratio, r, of minimum shear force to maximum one: (a) authors' tests; (b) reported data [1], [5], [8]–[11].

range. The tested values of fatigue lives are classified according to the value of the ratio of the applied maximum shear force to the static shear strength, V_{max}/V_{cu}. The table shows that the smaller the ratio of the applied minimum shear force to the maximum, V_{min}/V_{max} (called as 'r' hereafter), is, the shorter is the fatigue life between the specimens with the same V_{max}/V_{cu} ratio. Consequently, it can be said that the larger the load range is, the shorter is the fatigue life.

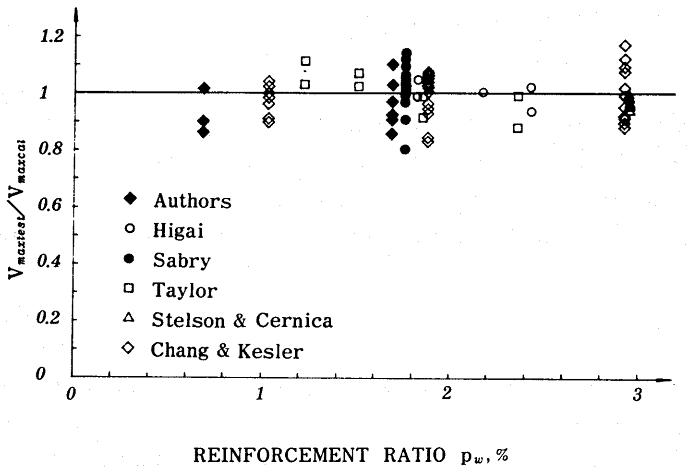
The relationship between the ratio, r, and the ratio of the tested value of fatigue strength to the value, $V_{max\ test}/V_{max\ cal}$, calculated by eq. 5 which was proposed without any consideration of the influence of load range is shown in Fig. 6. The calculated values have a tendency to become smaller than the tested ones with increase of the r values. Although the fatigue strength can be evaluated from eq. 5 in the case of r smaller than about 0.5, it cannot be evaluated in the case of r larger than 0.6. From this result it can be supposed that the influence


1.2
1
0.8
0.6
0.4
0.2
0
$V_{maxtest}/V_{maxcal}$
◆ Authors
○ Higai
● Sabry
□ Taylor
△ Stelson & Cernica
◇ Chang & Kesler
0
10
20
30
40
50
CYLINDER STRENGTH f'_c, MPa

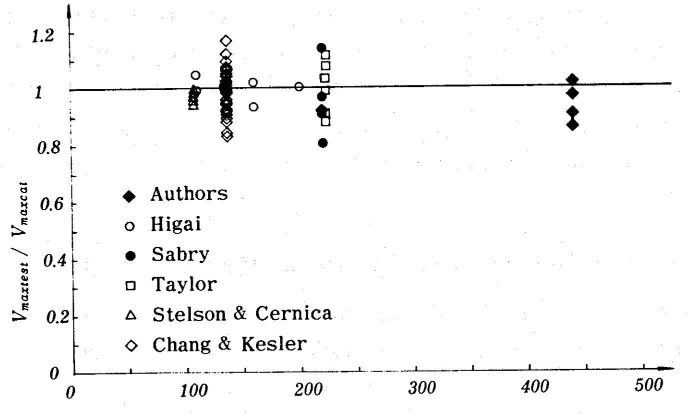
(a)


1.2
1
0.8
0.6
0.4
0.2
0
$V_{maxtest}/V_{maxcal}$
◆ Authors
○ Higai
● Sabry
□ Taylor
△ Stelson & Cernica
◇ Chang & Kesler
0
1
2
3
4
5
6
7
SHEAR SPAN RATIO a/d

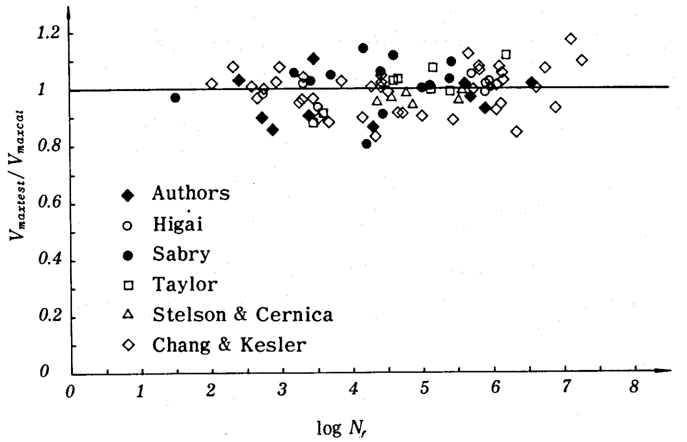
(b)


1.2
1
0.8
0.6
0.4
0.2
0
$V_{maxtest}/V_{maxcal}$
◆ Authors
○ Higai
● Sabry
□ Taylor
△ Stelson & Cernica
◇ Chang & Kesler
0
1
2
3
REINFORCEMENT RATIO p_w, %

(c)



(d)



(e)

Fig. 8. Relationships between ratio of tested value of fatigue strength in shear of beam without web reinforcement to that calculated by eq. 6 and various factors: (a) cylinder strength, f_c'; (b) shear span depth ratio, a/d; (c) reinforcement ratio, p_w; (d) effective depth, d; (e) fatigue life, N_f.

of load range appeared not so clearly from the previous tests where the most of values of r were smaller than 0.5. Finally, with consideration of the influence of load range a following equation for the prediction of the fatigue strength in shear of beam is proposed.

$$\log (V_{max}/V_{cu}) = -0.036\,(1-r^2)\log N_f \qquad (6)$$

The relationship between r and the ratio of the tested value of the fatigue strength to that calculated by eq. 6 is shown in Fig. 7(a). Figure 7(b) shows the ratio of the previously tested value of the fatigue strength [1], [5], [8]–[11] to the calculated one. The average is 0.99 and the coefficient of variation is 7.4%.

The relationship between the value of $V_{max\,test}/V_{max\,cal}$ and the cylinder strength, the ratio of shear span to effective depth, the reinforcement ratio and the effective depth which are parameters for the calculation of the static shear strength are givn in Figs. 8(a) (b) (c) (d). The relationship between the value of $V_{max\ test}/V_{max\ cal}$ and the tested value of fatigue life is given in Fig. 8(e). The values of $V_{max\ test}/V_{max\ cal}$ are not correlated to any parameter. This means that eq. 6 can be used for the evaluation of fatigue shear strength of beams without shear reinforcement.

4. Stirrup Strain under Fatigue Loading with Constant Maximum and Minimum Load

Based on the assumption that the decrease of shear force carried by concrete is essentially the same as that of the fatigue strength of beam without shear reinforcement, equations for the calculation of stirrup strain under fatigue loading with constant maximum and minimum load are derived.

4.1 Stirrup strain at the applied maximum shear force

The applied maximum shear force is carried by two components, V_s and V_c, where V_s is the shear force carried by the assumed truss with 45° diagonals and V_c is the one carried by concrete. The following equation is thus obtained, where V_{max} is the applied maximum shear force.

$$V_s = V_{max} - V_c \tag{7}$$

This equation is for the part where the influences of supports or loading points are negligibly small. For the part where these influences exist, V_s is lightened by multiplying the coefficient β_x as indicated in Fig. 2.

$$V_s = \beta_x (V_{max} - V_c) \tag{8}$$

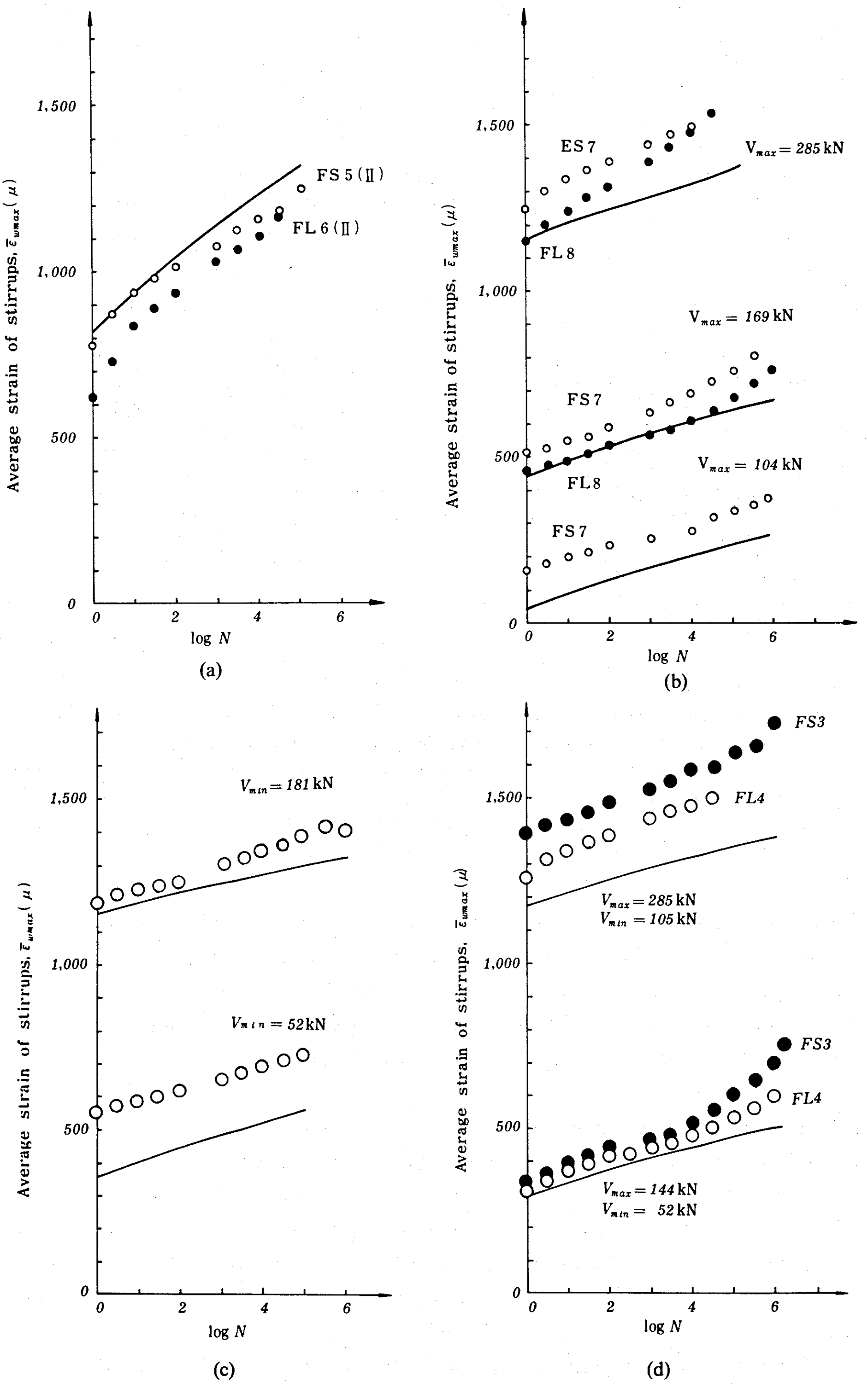
As the decrease of V_c under fatigue loading is assumed to be the same as that of the fatigue strength of beam without shear reinforcement, V_c is expressed as eq. 9, which is derived from eq. 6.

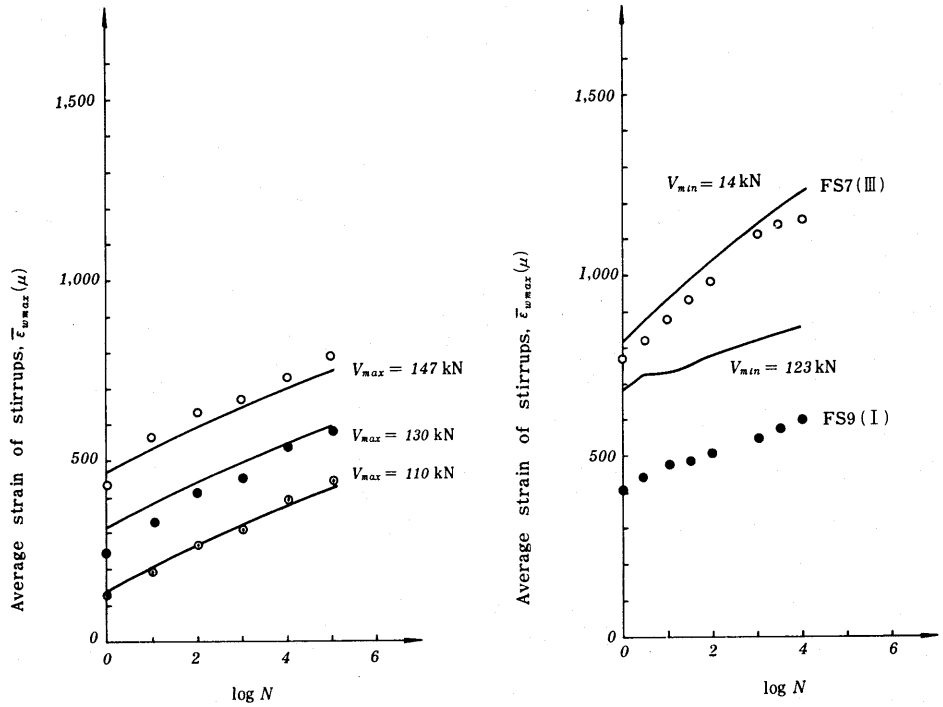
$$V_c = V_{co} 10^{-0.036(1-r^2)\log N} \tag{9}$$

When V_{max} is constant, the value of V_s increases with the decrease of V_c as indicated in eq. 8. Finally, eq. 10 is obtained for the calculation of the average strain of vertical stirrups at the applied maximum shear force.

$$\varepsilon_{w\ max} = \beta_x \{V_{max} - V_{co} 10^{-0.036(1-r^2)\log N}\}/(A_w E_w z/s) \tag{10}$$

The average strains calculated by eq. 10 are compared with the tested ones as shown in Fig. 9. Equation 10 is confirmed to be applicable to our T-beam tests with different minimum load as wells as the Sabry's rectangular beam tests with constant minimum load [1]. Figure 9(b) shows the cases where the maximum load is different with constant minimum load. Figure 9(c) shows the case where the minimum load is different with constant load range.


Average strain of stirrups, $\bar{\varepsilon}_{wmax}$ (μ)
1,500
1,000
500
0
FS 5 (II)
FL 6 (II)
log N
(a)
ES 7
FL 8
V_{max} = 285 kN
V_{max} = 169 kN
FS 7
FL 8
V_{max} = 104 kN
FS 7
(b)
V_{min} = 181 kN
V_{min} = 52kN
(c)
FS3
FL4
V_{max} = 285 kN
V_{min} = 105 kN
FS3
FL4
V_{max} = 144 kN
V_{min} = 52 kN
(d)



(e) (f)

Fig. 9. Average strain of stirrups at applied maximum shear force: (a) $V_{max}=150$ kN, $V_{min}=27$ kN, $a/d=4$; (b) maximum load was changed with constant minimum load (=26 kN), $a/d=2$; (c) minimum load was changed with constant load range (=110 kN), specimen FS11; (d) load range, maximum load and minimum load were changed; (e) maximum loads were different with constant minimum load (=20 kN), $a/d=2.5$ [1]; (f) minimum load was changed with constant maximum load (=150 kN).

Figure 9(d) shows the case where the load range is different as well as the maximum and minimum loads. The solid lines in these figures derived from eq. 10 agree with the tested values. The ranges of V_{max}/V_{min} (1.23 to 11.1) and V_{max}/V_{co} (1.42 to 2.94) cover all the practical cases. Equation 10 is confirmed to be applicable also to the cases where the influence of support or loading point is either small ($a/d=4.0$) or large ($a/d=2.0$).

4.2 Influence of load range on stirrup strain

It is shown by eq. 10 that the larger the ratio of r is, the smaller is the increase of stirrup strain, and that the increase does not exist, when the r value is equal to one. Actually the stirrup strain increases even under the sustained loading ($r=1$), because the duration of loading causes the stirrup strain to increase. However, eq. 10 can be applied to the cases, where the influence of fatigue loading is dominant, since the increase due to the duration of loading is much smaller than the one due to the loading cycles in those cases.

The increase of average stirrup strains in FS9(I), whose value of V_{min} is very close to that of V_{max}, is smaller than those in other cases as shown in Fig. 9(f). The calculated line represents the small increase nicely. It may be said that Fig. 9(f) shows the influence of load range on the decrease of V_c clearly.

4.3 Stirrup strain at the applied maximum shear force less than the shear capacity of concrete

Sabry reported that the stirrup strains hardly increased in the early stage of loading and stirrup strains began to increase noticeably after some cycles when the applied maximum shear force was less than the shear capacity of concrete [1]. In the authors' tests it was observed that the average of strains in stirrups increased very little until two or three diagonal cracks appeared in the same shear span and one of them became so long to cross some of the stirrups.

The idea on which eq. 10 is based can be extended to estimate not only the number of cycles, N_c, when stirrup strains begin to increase but also the increase of the strains thereafter. The shear capacity of concrete is assumed to be same as the shear force carried by concrete at the initial loading and to decrease like the fatigue strength of the identical beam without web reinforcement until N_c cycles. After N_c cycles the applied maximum shear force is carried by two components, V_s and V_c, and the decrease of V_c can be evaluated from eq. 9 so that eq. 11 is obtained.

$$V_{max} = V_{co}\, 10^{-0.036(1-r^2)\log N_c} \tag{11}$$

The stirrup strain at the applied maximum shear force can be calculated by eq. 10, substituting

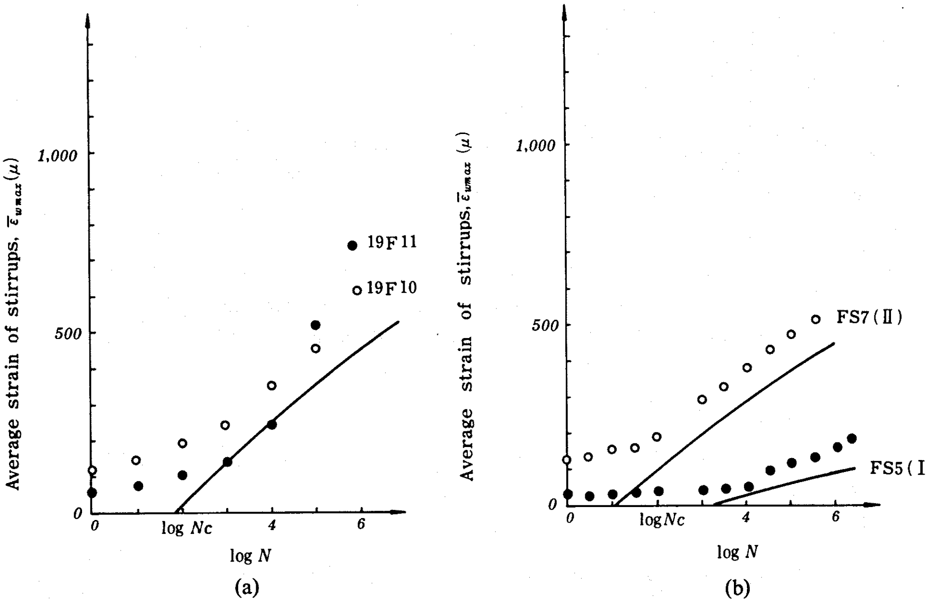


Fig. 10. Average strain of stirrups at applied maximum shear force less than the shear capacity of concrete ($V_{max} < V_{co}$). (a) Sabry's tests, rectangular beams, $V_{max}=69$ kN, $V_{min}=20$ kN, $V_{co}=79$ kN, $a/d=2.5$. (b) authors' tests, T-beams, $V_{max}=89$ kN, $V_{min}=14$ kN, $V_{co}=97$ kN, $a/d=4$ for FS7, $V_{max}=77$ kN, $V_{min}=26$ kN, $V_{co}=97$ kN, $a/d=2$ for FS5.

the total loading cycles from the start of fatigue loading for N. The calculated values are compared with the experimental values in Figs. 10(a) (b). These figures show that this procedure can be used for the calculation of stirrup strains when the applied maximum shear force is less than the shear capacity. of concrete.

4.4 Stirrup strain at the minimum shear force

In order to know the fatigue fracture of stirrups in a beam, the stress range under fatigue loading need be determined. The equation for the calculation of the stress range was temporarily proposed as a result of observation which showed that the stirrup stress changed linearly with the change of load [1]. But the equation was incomplete because of the neglected residual strain. In the authors' study the stirrup strain at the applied minimum load was examined in detail, and an improved equation was derived.

Figure 11(a) shows the typical relationship between applied shear force, V, and average strain, $\bar{\varepsilon}_w$. Although V–$\bar{\varepsilon}_w$ curve was a straight line at unloading of the first cycle, the one was a folded line at reloading of the second cycle. The both curves at unloading and reloading of the 10 kilo cycle approached each other and made folded lines. The shear force at the folded point became larger with loading cycles. In another specimen, however, no folded point was observed, when the applied minimum shear force was larger than the shear force at the folded point (Fig. 11(b)). The applied minimum shear force is usually larger than the shear force at the

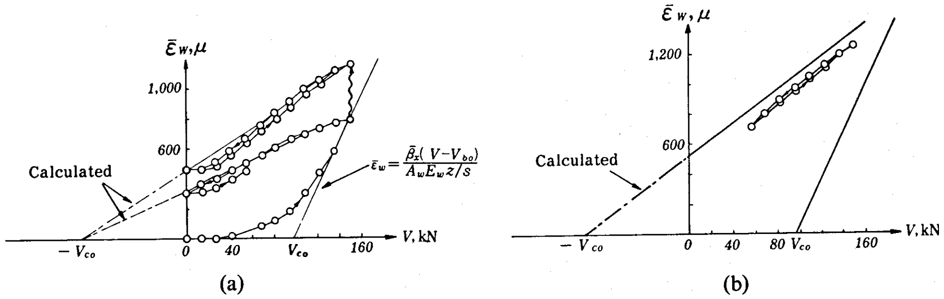


Fig. 11. Observed and assumed relationships between applied shear force and average strain of stirrups: (a) specimen FL6, $a/d=4$, $N=1$ and $N=10^4$; (b) specimen FS3, $a/d=4$, $N=10^6$.

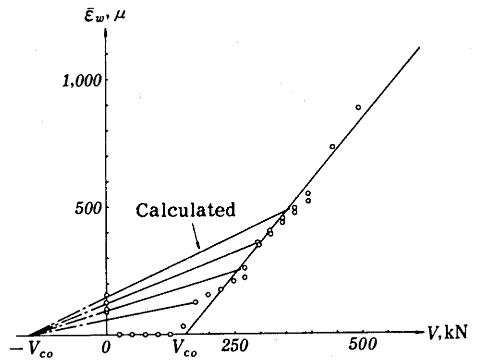


Fig. 12. Observed and assumed relationships between applied shear force and average strain of stirrups in the reported test specimen No. 3 in [14].

folded point so that the V–$\bar{\varepsilon}_w$ curve can be roughly said to be a straight line.

The inclination of the straight line became larger with increase in number of cycles of fatigue loading so that the strain range corresponding to the same range of shear force became larger. And a significant increase of the residual strain was observed during the fatigue loading.

The observed V–$\bar{\varepsilon}_w$ relationship shown in Fig. 11 can be explained by assuming that V–$\bar{\varepsilon}_w$ curve is always on the straight line between the point representing the strain at the applied maximum shear force and the fixed point on the shear axis. At present this point is regarded as the point where the shear force is equal to minus V_{co}. The strain range and the residual strain are assumed to increase during the fatigue loading, because the maximum strain increases almost proportional to the logarithm of loading cycles according to eq. 10. The assumed lines agree with the actual V–$\bar{\varepsilon}_w$ curve not only in the authors' tests (Fig. 11) but also in a previous test [14] (Fig. 12).

From the assumption, the following equations are derived.

$$\bar{\varepsilon}_{wr} = \bar{\varepsilon}_{w\,max}(V_{max} - V_{min})/(V_{max} + V_{co}) \tag{12}$$

$$\bar{\varepsilon}_{wo} = \bar{\varepsilon}_{w\,max}(V_{co})/(V_{max} + V_{co}) \tag{13}$$

The calculated values are compared with the tested values in Figs. 13(a), 13(b) and 14. The authors' tested values are shown in Figs. 13(a) and 14, and Sabry's ones [1] are in Fig. 13(b). These figures clearly show that these equations can express the average behavior of stirrup in the authors' tests. While the calculated values are generally smaller than the measured values in Sabry's tests. It was observed that the calculated strain ranges tended to be smaller than the

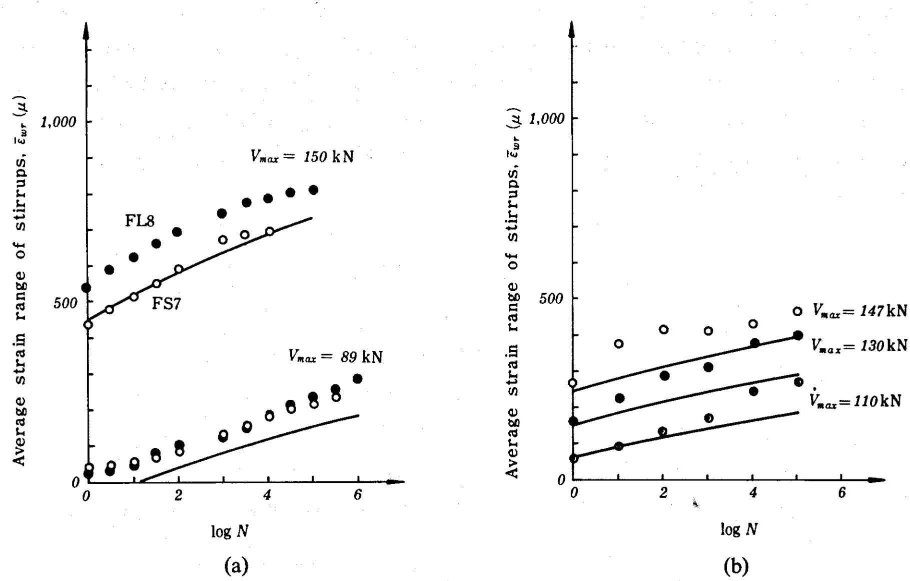


Fig. 13. Average strain range of stirrups: (a) authors' tests, $V_{min} = 14$ kN, $a/d = 4$; (b) Sabry's tests [1], $V_{min} = 20$ kN, $a/d = 2.5$.

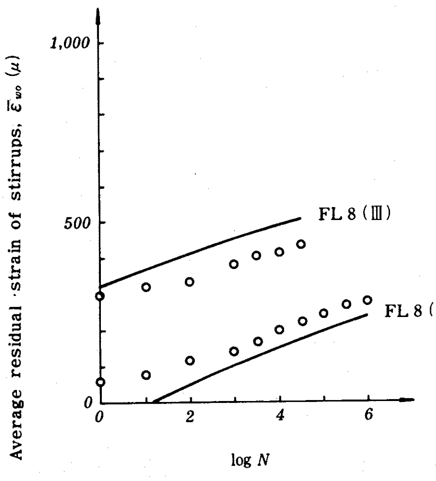


Fig. 14. Average residual strain of stirrups, a/d=4.

tested values in the cases where the number of shear reinforcement was relatively small. This tendency causes the calculated values to be smaller in Sabry's tests. However, for simplicity eq. 12 is considered to be used for practical cases.

4.5 Scatter of measured strain in each stirrup from calculated one

Measured strain in each stirrup lies scatteringly around the calculated one in each stirrup. The observed scatter is influenced by propagation of diagonal crack. The fuller the propagation is, the smaller is the scatter. The scatter of stirrup strain range at the last measurement before the first finding of the damaged gauge is shown in Fig. 15. The average ratios of tested value to calculated one are 1.09 (a/d=4.0) and 1.00 (a/d=2.0), and the coefficients of variation are 38% (a/d=4.0) and 28% (a/d=2.0). The ratios tend to be smaller than 1.0 in the cases of stirrups near to loading points and larger than 1.0 in the cases of stirrups near to supports.

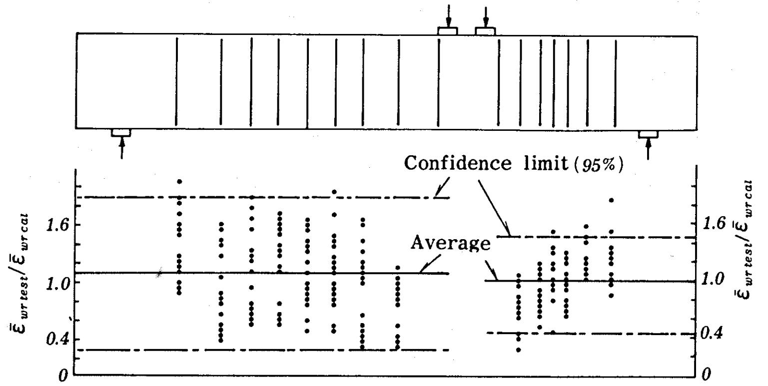


Fig. 15. Scatter of measured strain of each stirrup around calculated one.

5. Stirrup Strain under General Variable Loading

5.1 Equivalent number of loading cycles

It is necessary to make clear the behavior of web reinforcement under fatigue loading with varied load range, since the actual structures are not subjected to fatigue loading with the constant maximum and minimum load.

When general fatigue loading is divided into some sets of repeated loading with constant maximum and minimum shear force, each set is named the first repeated loading, the second repeated loading and so on according to the sequence of the loading. Although the stirrup strain after subjected to the first repeated loading shall be calculated as mentioned in Section 4, the strain during the second repeated loading can not be calculated. But this strain can be estimated if it is assumed that the loading history of the first repeated loading is equivalent to a certain number (N_{eq}) of cycles of repeated loading whose maximum and minimum shear force are equal to those of the second repeated loading. By the similar way the stirrup stain under the subsequent repeated loading can be estimated.

To obtain the equivalent loading cycles a new idea is developed. When the stirrup strains produced by a certain applied shear force are equal in the identical beams under different loading histories, the strains during subsequent loading are essentially the same in spite of the difference of the previous loading histories. In other words, the behavior of a stirrup after subjected to a certain loading, static or fatigue or sustained loading, is only dependent on the strain corresponding to the shear force applied. Consequently any loading history can be substituted by an equivalent fatigue loading with the constant maximum and minimum load.

5.2 Stirrup strain under fatigue loading with varied load range

The line (b) in Fig. 16 is drawn between the fixed point ($-V_{co}$, 0) and the point representing the strain of stirrup after subjected to N_1 cycles of the first repeated loading whose maximum and minimum shear force are $V_{max\,1}$ and $V_{min\,1}$. When the maximum shear force of

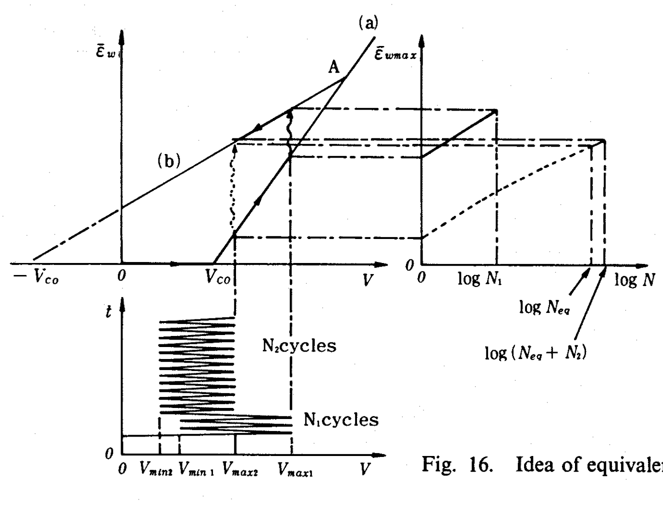


Fig. 16. Idea of equivalent number of loading cycles.

the second repeated loading is below the point A in Fig. 16, the points representing the strains at the beginning of the second repeated loading are on the line (b), as mentioned in Section 4.4. The strain, ε_w, at $V_{max\,2}$ is calculated by the following equation, where $V_{max\,2}$ is the maximum shear force of the second repeated loading and $\varepsilon_{w\,max\,1}$ is the maximum stirrup strain calculated by eq. 10 with $V_{max}=V_{max\,1}$, $V_{min}=V_{min\,1}$ and $N=N_1$.

$$\varepsilon_w=\varepsilon_{w\,max\,1}\,(V_{max\,2}+V_{co})/(V_{max\,1}+V_{co}) \tag{14}$$

Based on the newly developed idea, it is assumed that the state of strain after subjected to N_1 cycles of the first repeated loading is equivalent to the state of strain after subjected to the equivalent number of cycles of loading whose maximum and minimum shear force are equal to that of the second repeated loading.

The assumption is expressed by eq. 15.

$$\bar{\beta}_x\{V_{max\,2}-V_{co}10^{-0.036(1-r_2^2)\log N_{eq}}\}$$
$$=\bar{\beta}_x\{V_{max\,1}-V_{co}10^{-0.036(1-r_1^2)\log N_1}\}(V_{max\,2}+V_{co})/(V_{max\,1}+V_{co}) \tag{15}$$

The equivalent number of loading cycles is obtained by transposing eq. 15.

$$\log N_{eq}=\frac{1}{0.036\,(1-r_2^2)}\log\left\{\frac{(V_{max\,2}-V_{max\,1})+(V_{max\,2}+V_{co})^{-0.036(1-r_1^2)\log N_1}}{V_{max\,1}+V_{co}}\right\} \tag{16}$$

where $r_1=V_{min\,1}/V_{max\,1}$; $r_2=V_{min\,2}/V_{max\,2}$.
After subjected to N_2 cycles of the second repeated loading, stirrup strain, $\varepsilon_{w\,max\,2}$, at $V_{max\,2}$ can be calculated by eq. 10, substituting $N_{eq}+N_2$ for N. This means that the rate of increase in $\varepsilon_{w\,max\,2}$ or $\varepsilon_{wr\,2}$ is very small if the order of N_2 is smaller than that of N_{eq} (see Fig. 17). Solid and

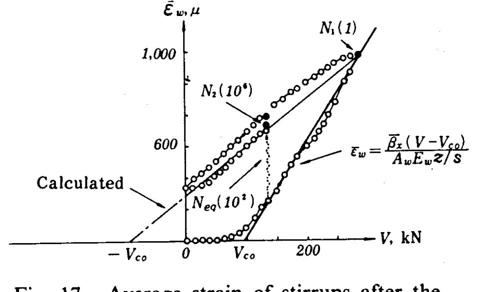


Fig. 17. Average strain of stirrups after the first repeated loading in a case of a large influence of previous fatigue loading, specimen FL2, $a/d=2$.

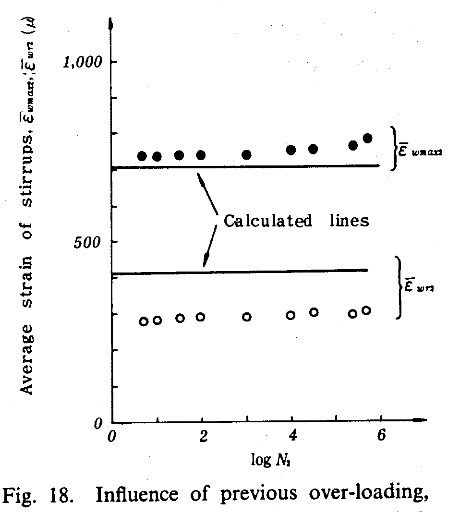


Fig. 18. Influence of previous over-loading, specimen FL2, $a/d=2$, $V_{max\,1}=283$ kN, $V_{min\,1}=0$, $N_1=1$, $V_{max\,2}=136$ kN, $V_{min\,2}=49$ kN.

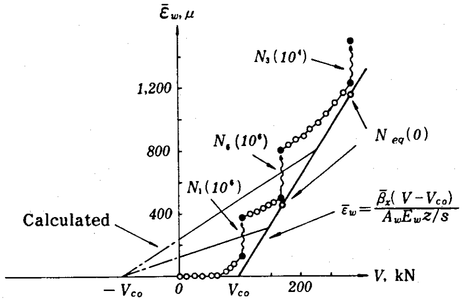


Fig. 19. Average strain of stirrups after the first repeated loading in a case of no influence of previous fatigue loading, specimen FS7, $a/d=2$.

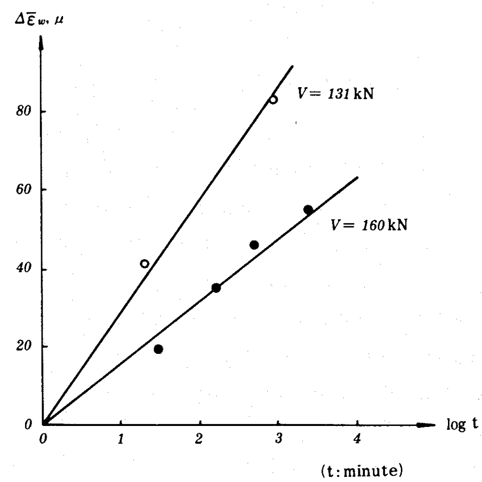


Fig. 20. Increment of average strain of stirrup under sustained loading, $\Delta\varepsilon_w$, specimen FS11, $a/d=2$.

open circles in Fig. 18 indicate the observed $\varepsilon_{w\,max\,2}$ and $\varepsilon_{wr\,2}$ respectively in the case of beam, in which N_1 is one, and the solid lines in the figures are calculated by eqs. 10 and 12, substituting $N_{eq}+N_2$ for N. In the figures the small increase is recognized in the early stage of the second repeated loading, and the calculated lines express nicely the small increase.

When the maximum shear force of the second repeated loading is above the point A in Fig. 16, the point representing the stirrup strain at $V_{max\,2}$ is on the line (a). The line (a) represents the relationship between applied shear force and stirrup strain under static loading. Therefore, there is no influence of the previous fatigue loading on the stirrup strains under the second repeated loading, and eq. 10 can be used without any modification for calculating $\varepsilon_{w\,max\,2}$ (see Fig. 19). Measured and calculated strains in this case are shown in Figs. 9(a)–(d).

Stirrup strains increase clearly, even if sustained load is applied. It was observed that the rate of increase was approximately in proportion to logarithm of duration of the loading as shown in Fig. 20. When a stirrup, whose strain had increased due to sustained loading, was subjected to fatigue loading, it was observed that the increase of strain was small in the early stage of the fatigue loading. When sustained load was applied after some cycles of fatigue loading, the increase of stirrup strain was hardly recognized. Therefore, the increase of strain due to sustained loading is considered essentially the same as the one due to fatigue loading. Considering this fact, loading speed may be one of the important factors, but the effect of sustained loading during a usual fatigue test is generally negligible.

This method of calculation for stirrup strains can be used for design of concrete structures subjected to any variable loads. However, simple conservative procedures can be used for such structures as railway bridges in which the design variable loads are determined so as to hardly exceed the actual loads. One of the procedures is to set the value of V_c constant such as 0.5 V_{co}

or 0.6 V_{co} which corresponds to the case of log $N=8$ or 6 with $r=0$.

6. Strains of Bent-up Bar under Fatigue Loading

6.1 Different characteristics of bent-up bar under static loading

The bent-up bar plays the anchorage part of the tensile bar against bending moment as well as the shear reinforcement. Because of this double role, the strain characteristics under fatigue loading may be different from those of stirrups.

Two beams whose one shear span had only bent-up bars as shear reinforcement and the other had only inclined stirrups were tested. Both the shear spans were reinforced essentially the same against shear force (see Fig. 1(b)). The different point between the two shear spans is that the tensile reinforcement ratio is constant for the shear span with inclined stirrups and not constant for the other because the bent-up bars do not play a role as tensile bar in the vicinity of the support. Strains of the inclined stirrups under static loading are to be expressed by eq. 10, substituting one for N and considering the angle, α, between shear reinforcement and the axis of the member (see Fig. 21(a)).

$$\bar{\varepsilon}_{w\,max}=\bar{\beta}_x(V_{max}-V_{co})/\{A_w\,E_w(z/s)(\cos\alpha+\sin\alpha)\} \tag{17}$$

The strains of bent-up bar could be expressed by eq. 17 with consideration of the little decrease of V_{co} due to the decrease of reinforcement ratio, if the bent-up bar plays only a role of shear reinforcement. The tested values were actually larger than the calculated ones (see Fig. 21(b)), because of role of the anchorage of tensile reinforcement. The tensile stress in the bent-up portion is equal to the one as the anchorage, when the tensile stress as anchorage is larger than that as shear reinforcement. In the reversed case the tensile stress in the bent-up portion is equal to the tensile stress as shear reinforcement. On the other hand, the portion just before bent portion plays a role of anchorage of shear reinforcement as well as a role of tensile reinforcement. Therefore, the stress at this portion is swayed by the stress as tensile reinforcement or the stress as anchorage of shear reinforcement, whichever is larger. For example, Fig. 22(a) indicates the strains in the vicinity of bent portion of the bent-up bar closest to the loading point in the specimen BI3. The bent-up bar was located in the region where bending moment was superior to shear force. The strain at the portion just before bent porton was almost the same as that of tensile reinforcement against bending moment, and the strain at the bent-up portion was much larger than that as shear reinforcement. The case in which shear force was superior to bending moment is shown in Fig. 22(b). The bent-up bar was the second one from the loading point in the specimen BI3. The strain at the bent-up portion agreed approximately with that as shear reinforcement when the applied shear force was larger than 140 kN, and the strain at the portion just before the bent was much larger than that as tensile reinforcement.

Consequently the maximum strain in the vicinity of bent portion of bent-up bar is the strain as shear reinforcement calculated by eq. 17, substituting β_x for $\bar{\beta}_x$, or the strain as tensile reinforcement in bending calculated by the conventional method, whichever is larger. The

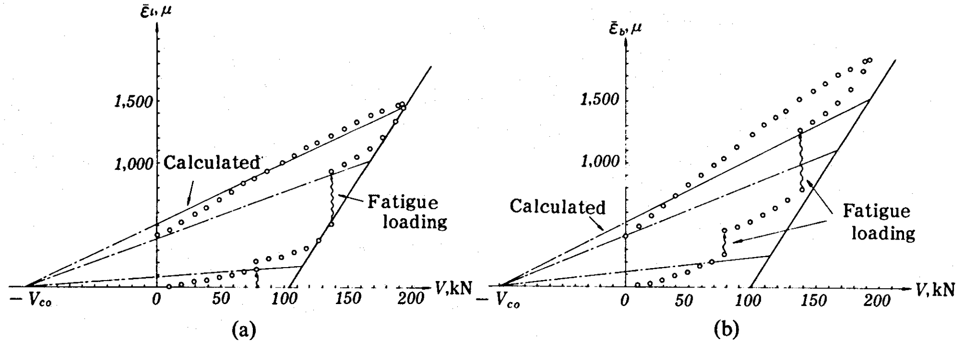


Fig. 21. Average strain of shear reinforcement in specimen BI3: (a) inclined stirrups; (b) bent-up bars.

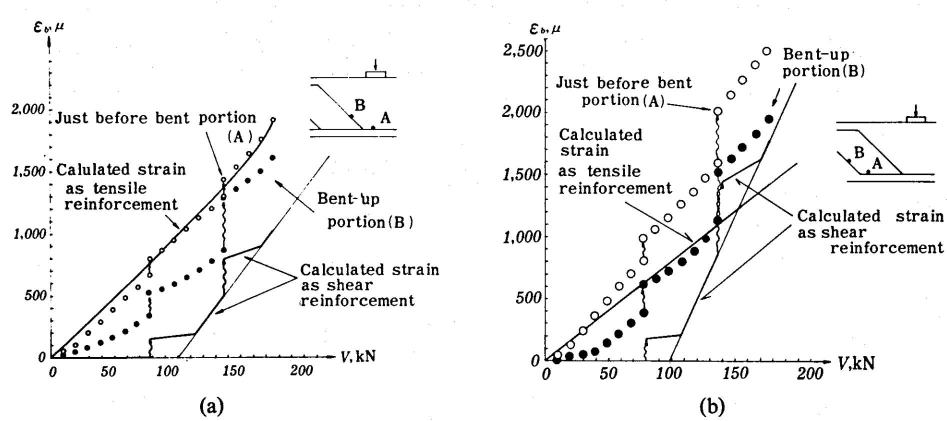


Fig. 22. Strain of bent-up bar in the vicinity of bent portion, specimen BI3.

things mentioned in this section are applicable to the average stress in the cross-section of bent-up bar, but not applicable to the local stress caused by bending operation.

6.2 Strains of shear reinforcement in case of using bent-up bars together with vertical stirrups

Bent-up bars are generally used together with vertical stirrups, and the strain characteristics should be different from the case where only one type of shear reinforcement is used.

The average strains of inclined stirrups in the right span of the beam were almost twice as large as those of vertical stirrups (Fig. 23(a)). This was observed also in the left span of the same beam, where the bent-up bars and vertical stirrups are used together (Fig. 23(b)). This observed phenomenon is explained by the following assumption. It is assumed that a pure shear strain state occurs in the shear span. The pure shear strain state indicates that there are uniform tensile strain, ε_t, field at an angle of 45° between the member's axis and uniform compressive strain, ε_c, field at an angle of 90° between the tensile strain (Fig. 24). The strains of bent-up bars (or inclined stirrups), ε_b (or ε_i), and vertical stirrups, ε_v, are expressed by these tensile and compressive strains, ε_t and ε_c.

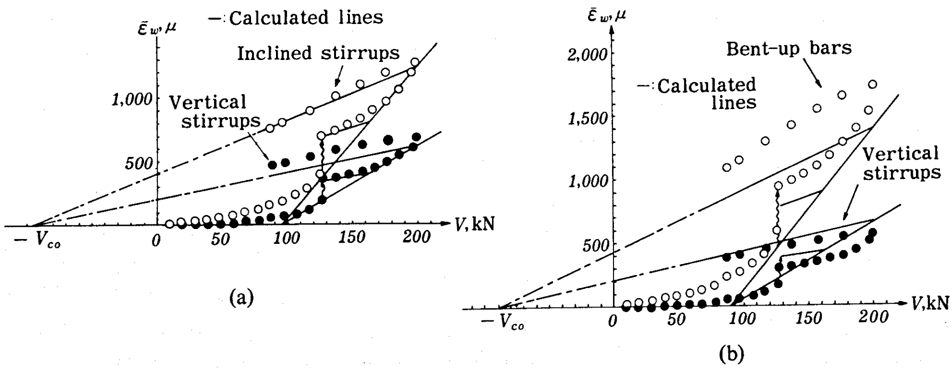
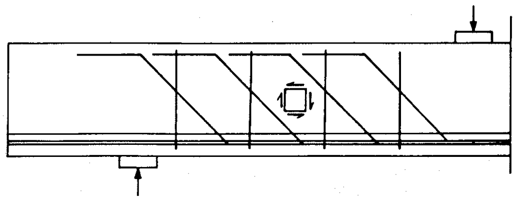


Fig. 23. Average strain of shear reinforcement in beam with bent-up bars (or inclined stirrups) and vertical stirrups, specimen BI2: (a) inclined stirrups; (b) bent-up bars.



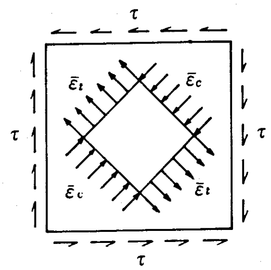


Fig. 24. Pure shear strain state.

$$\varepsilon_b \text{ (or } \varepsilon_i) = \varepsilon_t \cos^2(\alpha - 45°) + \varepsilon_c \sin^2(\alpha - 45°) \tag{18}$$

$$\varepsilon_v = \varepsilon_c \cos^2(90° - 45°) + \varepsilon_c \sin^2(90° - 45°) \tag{19}$$

Both the equations are deduced by taking $\alpha = 45°$ and $\varepsilon_t \gg \varepsilon_c$.

$$\varepsilon_b \text{ (or } \varepsilon_i) = \varepsilon_t \tag{20}$$

$$\varepsilon_v = 0.5\ \varepsilon_t \tag{21}$$

Therefore the following relationship can be obtained.

$$\varepsilon_b \text{(or } \varepsilon_i) : \varepsilon_v = 2 : 1 \tag{22}$$

The sum of shear force carried by bent-up bars, V_b (or inclined stirrups, V_i), that carried by vertical stirrups, V_v and that carried by concrete, V_c is equal to the applied maximum shear force V_{max}. The following equation is thus obtained.

$$V_b(\text{or } V_i)+V_v=V_{max}-V_c \tag{23}$$

When the influences of loading points and supports are considered,

$$V_b(\text{or } V_i)+V_v=\beta_x\,(V_{max}-V_c) \tag{24}$$

where

$$V_b=A_b(\cos\alpha+\sin\alpha)E_w\varepsilon_b z/s_b \tag{25}$$

$$V_i=A_i(\cos\alpha+\sin\alpha)E_w\varepsilon_i z/s_i \tag{26}$$

$$V_v=A_vE_w\varepsilon_v z/s_v \tag{27}$$

Consequently each strain can be calculated by using eqs. 22 and 24 and the calculated lines are shown in Figs. 23(a) (b).

6.3 *Behavior of bent-up bars under fatigue loading*

The characreristics of strain in vertical stirrups under fatigue loading were mentioned in Sections 4 and 5. It was experimentally made clear that these characteristics were also recognized in the bent-up bar and inclined stirrup.

In the case where only bent-up bars or inclined stirrups are used, the strain of bent-up bar or inclined stirrup at the applied maximum shear force under fatigue loading with the constant maximum and minimum shear force are essentially the same as that of the vertical stirrups and can be expressed by eq. 28 (see Fig. 25(a)).

$$\bar{\varepsilon}_{w\,max}=\frac{\beta_x\{V_{max}-V_{co}10^{-0.036(1-r^2)\log N}\}}{A_wE_w\ z/s\ (\cos\alpha+\sin\alpha)} \tag{28}$$

However, this equation is not applicable to the bent-up bar in the region where bending moment is superior to shear force, as mentioned in Section 6.2. Furthermore, it was indicated experimentally that the tensile strains as anchorage of tensile reinforcement hardly increase under repeated loading.

In the case where both the bent-up bars (or the inclined stirrups) and the verticl stirrups are used together, the strains of both shear reinforcement under fatigue loading can be calculated, considering that the strain of bent-up bar (or inclined stirrup) is almost twice as large as that of vertical stirrup. The calculated values are compared with the tested values in Figs. 25(b) (c).

The relationships between shear force and strain under unloading and reloading are line aiming at the certain fixed point, which can be assumed the point $(-V_{co}, 0)$ practically (Figs. 21 and 23).

The influence of over-loading, which is the previous loading whose maximum load is larger than that of subsequent repeated loading, can be explained by the idea of its equivalent repeated loading, as mentioned in Section 5.1. Figure 25(d) shows the case of $N_{eq}=263$.

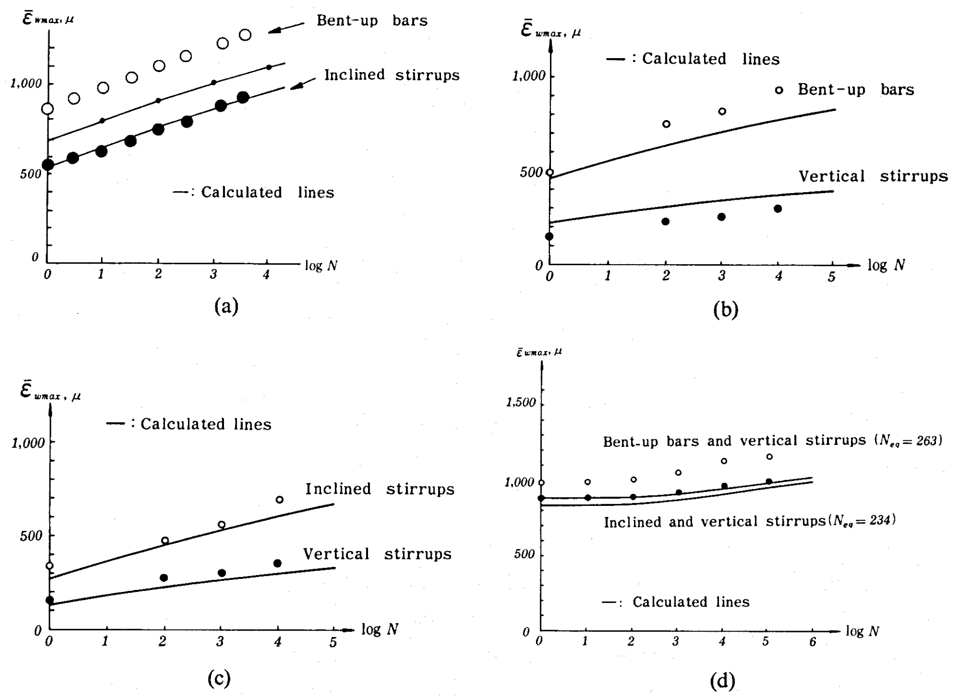


Fig. 25. Average strains of bent-up bars and inclined stirrups at applied maximum shear force: (a) specimen BI3, $V_{max}=137$ kN, $V_{min}=49$ kN; (b) shear span with bent-up bars of BI2, $V_{max}=127$ kN, $V_{min}=29$ kN; (c) shear span with inclined stirrups of BI2, $V_{max}=127$ kN, $V_{min}=29$ kN; (d) influence of previous over-loading, specimen BI2, $V_{max}=176$ kN, $V_{min}=88$ kN.

7. Beam Failure Due to Fatigue Fracture of Shear Reinforcement

Beam failure due to fatigue fracture of shear reinforcement was observed in both the eleven T-beam tests and the four rectangular beam tests with bent-up bars. In this section the fatigue strength of shear reinforcement and beam are estimated from the calculated strain of shear reinforcement.

7.1 Fatigue fracture of shear reinforcement

Nine of the eleven T-beams failed in shear due to fatigue fracture of stirrups. The applied maximum shear forces were 62% of the calculated static flexure strength. Two beams failed under static loading after fatigue tests. The specimen FS1 failed in flexure and the ultimate strength was 103% of the calculated ultimate strength. The specimen FS9 failed in shear and the ultimate strength was 97% of the calculated ultimate strength.

Two of the four rectangular beams failed in shear due to fatigue fracture of bent-up bars and one beam failed in shear due to fatigue fracture of both bent-up bars and vertical stirrups and one beam failed in shear due to fatigue fracture of main bars at the crossing point of diagonal crack with that of bent-up bars and vertical stirrups.

Fatigue fractured stirrups in the T-beams were always found in the shear span where a/d was 2.0. Fatigue fractured stirrups were also found in the specimen FS9 which did not fail under fatigue loading. Many fractured shear reinforcement in the rectangular beams were found in the shear span with bent-up bars except for the specimen BI4 in which the fracture was found in both the spans. After the first fatigue fracture of shear reinforcement, each specimen resisted one hundred thousand to three million cycles. Following the first fracture, the T-beams failed due to the fracture of four to ten more legs of stirrups and the rectangular beams failed due to the fracture of five or six more bent-up bars and several vertical stirrups. Although the total number of fractured shear reinforcement in each specimen did not seem to relate to the magnitude of fatigue loading or the fatigue lives of the beams, it was recognized that the shorter the fatigue life of beam was, the fewer was the loading cycles after the first fracture.

In the T-beams the fatigue fracture occurred not only at lower bent portion where stirrup was bent around longitudinal bars but also at middle straight portion and upper hook portion. The portion of fatigue fracture was generally along the main diagonal crack which caused the failure of beam (see Fig. 26). Many fractured legs at the middle straight portion of stirrup were found in the center of shear span, and those at the upper hook are found in the vicinity of the loading point. In the rectangular beams the fatigue fracture of bent-up bar occurred also along the main diagonal crack. But all the fracture of vertical stirrups was found at the lower bent portion. The following points can be considered as the reasons for the difference of fractured positions in vertical stirrup.

(1) The effective depeth of the specimen in the T-beam tests was about two times as high as that in the rectangular beam tests. A higher effective depth prevents the deterioration of bond from extending all over the stirrup. Therefore, the stress condition at the lower bend might be considerably lightened in the T-beam tests, if the point of the diagonal crack crossing

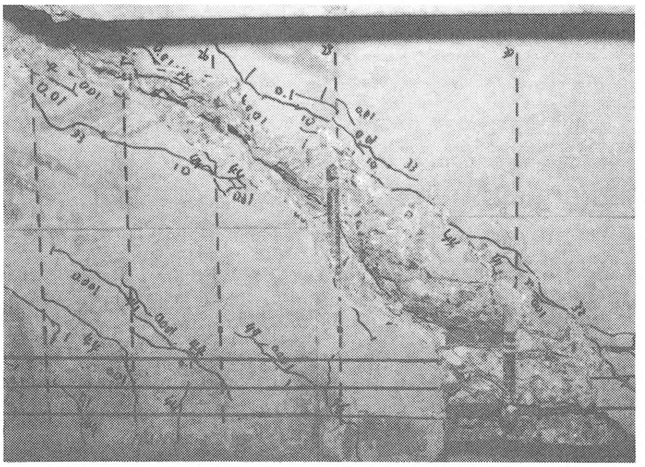


Fig. 26. Fatigue fractured stirrups.

was far from the lower bend of stirrup.

(2) A main diagonal crack in the specimen BI2 happened to propagate in the vicinity of the point in which a pair of bent-up bars crossed that of vertical stirrups. Thus the local stress of the vertical stirrup due to the crossing of diagonal crack might be lightened.

7.2 *Fatigue strength of beam*

It is considered that the fatigue strength of beam failing in shear due to stirrup fracture is related to the fatigue strength of stirrup. However, the measured strains in the T-beams show no more information than that the fatigue strength of stirrup lies between the fatigue strength of the straight bar (100%) and that of the bent bar (50%, which is reported in some previous papers [2] [15]) as shown in Fig. 27. Because it is difficult not only to recognize the fracture from the measured strains, but also to clear the relationship between the strain at the measured point and the strain at the fractured point.

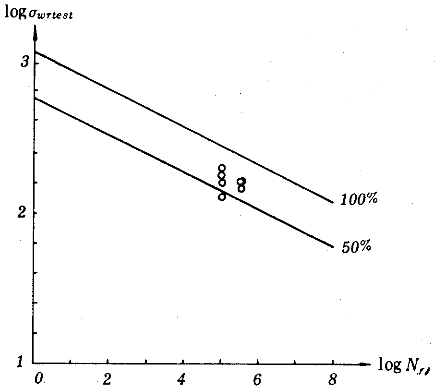


Fig. 27. Relationship between tested stress range of the first fractured stirrup and number of loading cycle at the first fracture of stirrup in T-beam.

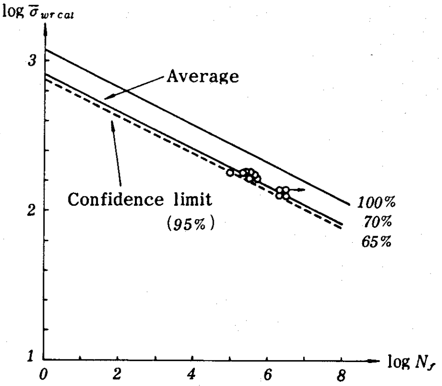


Fig. 28. Relationship between calculated average stress range of stirrups and tested fatigue life of T-beam.

On the other hand the closed relationship between calculated value of average stress range of stirrups at the failure of beam and tested value of fatigue life of the T-beam is found as shown in Fig. 28. The average of the stress ranges is 70% of the fatigue strength of the straight bar. The coefficient of variation is 4.8%. This figure clearly indicates that the fatigue strength of the T-beam can be evaluated from the average of stress range in stirrups calculated by eq. 12.

7.3 *Design recommendation*

Although the beam failure due to fatigue fracture of shear reinforcement can be evaluated from the average stress range calculated by the proposed equation, it is difficult to evaluate accurately the S–N curve for the beam failure. It is necessary to clarify the shear resisting mechanism after the first fracture of shear reinforcement occurs, in order to evaluate more correctly the fatigue life of a general beam. On the other hand the applied stress of shear reinforcement can be calculated by the authors' proposed equations before the first fracture of shear reinforcement occurs. Considering these facts, it is proper that the first fracture of shear reinforcement, which does not mean the beam failure but influences it greatly, is regarded as the fatigue limit state in design.

In design procedure the applied stress range calculated as in Section 5.2 is checked by comparing with the fatigue strength of shear reinforcement. There is not so many information on the fatigue strength, but the first fracture occurs at bent portion in most of the cases. Therefore, the fatigue strength is considered to be 50% of that of the straight bar according to the fatigue strength of bent bar. In future it is necessary to estimate the fatigue strength of shear reinforcement more correctly, considering the scatter of the applied stress (see Section 4.5).

The strains of shear reinforcement increase under fatigue loading. However, most of the increase occurs in the early stage of repeated loading, since the increase is approximately proportional to logarithm of loading cycles. The applied stress can be assumed to be constant and calculated by the proposed equations, substituting the design value of loading cycles, which is one mega cycle for example.

8. Conclusions

(1) An equation to calculate the fatigue strength in shear of beams without shear reinforcement is proposed. This equation includes the influence of load range which becomes significant when the ratio of applied minimum shear force to the maximum exceeds the value of 0.6. This equation is valid for the relatively large span depth ratio and for all the previous data collected by the authors.

(2) Based on the above equation, the equation for the calculation of the average stirrup strain at the applied maximum shear force under fatigue loading is modified. This modified equation is valid for T and rectangular beams, and for the wide range of span depth ratio. And the equation is applicable not only to the cases of the constant minimum load but also to those of the constant maximum load or the constant load range. This equation clarifies the influence of load range on the increase of stirrup strain, which is obtained from the experimental results.

(3) When the applied maximum shear force is smaller than the shear capacity of concrete, the stirrup strain does not increase at the early stage of fatigue loading but begins to increase after the specific cycles of fatigue loading. This phenomenon can be explained by extending the assumption on which the equation in (2) is based.

(4) The equation for the calculation of strain range in stirrup is modified to increase the accuracy. This equation is derived from the observation that the applied shear force–stirrup strain relationship can be considered linear at unloading and reloading and the line is assumed to cross the shear axis at the fixed point.

(5) A procedure to calculate the average strain in stirrups under general variable loading is proposed. This procedure is based on a newly developed idea that the strains during the

subsequent loading are essentially the same in spite of the difference of the previous loading history, if the stirrup strains produced by the same applied shear forces are same. Consequently any loading history can be substituted by equivalent fatigue loading with the constant maximum and minimum load.

(6) The strain of bent-up bar is equal to the larger value between the strain as shear reinforcement and the one as tensile reinforcement for bending. The strain of 45° bent-up bar as shear reinforcement is two times as large as that of vertical stirrup used together. The strain of bent-up bar under the fatigue shear loading has the same characteristics as that of vertical stirrup and can be calculated from the proposed equations in consideration of the above.

(7) In the T-beam tests nine of the eleven specimens fail in shear under fatigue loading whose maximum shear force is about 60% of the static strength, although these specimens under static loading will have failed in flexure at the load when stirrups yield. In all the cases shear failure occurs due to fracture of several legs of stirrups. Most of the fatigue fracture occurs along the main diagonal crack. The fatigue strength of beam can be evaluated from the calculated average of stress ranges in stirrups.

(8) A design method for stirrup under fatigue loading is presented, assuming that the fatigue limit state is the first fracture of shear reinforcement.

References

[1] Sabry A. Farghaly: Shear Design of Reinforced Concrete Beams for Static and Repeated Loads, Dissertation for the Degree of Doctor of Engineering, submitted to the University of Tokyo, March 1979.

[2] Hawkins, N. M.: Fatigue Characteristics in Bond and Shear of Reinforced Concrete Beams, ACI Publication **SP-41**, pp. 203–236, 1974.

[3] Ruhnau, J.: Influence of Repeated Loading on the Stirrup Stress of Reinforced Concrete Beams, ACI Publication **SP-42 V. 1**, pp. 169–181, 1974.

[4] Kaar, P. H. and Mattock, A. H.: High Strength Bars as Concrete Reinforcement—Part 4 Control of Cracking, PCA Development Department, Bulletin **D59**, January 1963.

[5] Higai, T.: Fundamental Study on Shear Failure of Reinforced Concrete Beams, Proceedings of the Japan Society of Civil Engineers (JSCE), **No. 279**, pp. 113–126, November 1978.

[6] Price, K. M. and Edwards, A. D.: Fatigue Strength in Shear of Prestressed Concrete I-Beams, ACI Journal, **Vol. 68**, pp. 282–292, April 1971.

[7] Hanson, J. M., Hulsbos, C. L. and VanHorn, D. A.: Fatigue Tests of Prestressed Concrete I-Beams, Proceedings of ASCE, **Vol. 96, No. ST11**, pp. 2443–2464, November 1970.

[8] Chang, T. S. and Kesler, C. E.: Static and Fatigue Strength in Shear of Beams with Tensile Reinforcement, Journal of ACI, **Vol. 54**, pp. 1033–1057, June 1958.

[9] Taylor, R.: Discussion of a Paper by Chang, T. S. and Kesler, C. E.: Fatigue Behavior of Reinforced Concrete Beams, Journal of ACI, **Vol. 55**, pp. 1011–1015, March 1959.

[10] Stelson, T. E. and Cernica, J. N.: Fatigue Properties of Concrete Beams, Journal of ACI, **Vol. 55**, pp. 255–259, August 1958.

[11] Chang, T. S. and Kesler, C. E.: Fatigue Behavior of Reinforced Concrete Beams, Journal of ACI, **Vol. 55**, pp. 245–254, August 1958.

[12] Okamura, H. and Higai, T.: Proposed Design Equation for Shear Strength of Reinforced Concrete Beams without Web Reinforcement, Proceedings of JSCE, **No. 300**, pp. 131–141, August 1980.

[13] Okamura, H. and Sabry A. Farghaly: Shear Design of Reinforced Concrete Beams for Static and

Moving Loads, Proceedings of JSCE, **No. 287**, pp. 127–136, July 1979.

[14] Matsumoto, Y., Nakamura, S., Kohno, K., Nakayama, N. and Okamura, H.: Flexural Behavior of Reinforced Concrete Beams with High Strength Deformed Bars, Proceedings of JSCE, **No. 122**, pp. 1–28, October 1965.

[15] Pfister, J. F. and Hognestad, E.: High Strength Bars as Concrete Reinforcement, Part 6. Fatigue Tests, PCA Development Department, Bulletin **D74**, January 1964.