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STUDIES ON SPLITTING TENSILE STRENGTH TEST OF CONCRETE

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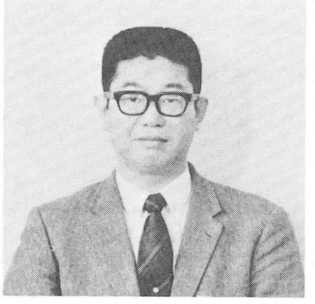

Atsuhiko MACHIDA

SYNOPSIS

The behaviors of specimens subjected splitting load were investigated in detail, based on high-speed motion pictures and others, and the effect of local crushing of loaded portions in specimens loaded using no strips on the test results, the effect of strips, the relation between splitting strength and pure tensile strength, and the applicable range of the splitting method were made clear. It was concluded from the analisis of high-speed motion pictures that the local crushing of loaded portions in a specimen loaded using no strips has no adverse effect on test results, and that the specimen collapses near the vertical plane which contains the central axis ,loosing the load-carrying capacity at the same time. On the contrary, specimens loaded through strips collapse more gradually and it can hardly concluded that the vertical plane loses the load-carrying capacity at the same time. The use of strips does not have any merits except there are severe irreguralities on the surfaces of specimens. Although there are severe irreguralities on the surface of the specimen,the use of strips makes errors due to the irregularities negligible. When concrete of ordinary quality are tested, the method using no strips gives results nearly equal to the pure tensile strength, whereas the method using strips will give a little higher results. The applicable range of the splitting method as a tensile strength test is such that the strength under the biaxial stresses is not widely different from the one of concretes of ordinary quality and the ratio of tenslie strength to compressive strength is up to in the range of 1/6-1/8.

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1. INTRODUCTION

The splitting method has many merits as compared with other tensile strength test methods. This method, therefore, has been prescribed in standards of many countries as the standard test method for tensile strength of concrete.[1],[2],[3] There are two practical ways to test by this method. One is to insert bearing strips between the specimen and bearing plates of the testing machine and apply load through the strips.[4] The other is to apply load to the specimen directly without any strips.[5] Although these splitting methods are being used widely in practice, both ways involve some problematic points to be solved. In other words, portions of a specimen loaded without strips may be damaged before the whole specimen collapses, and the effects of this phenomenon on the stress distribution of the specimen, on its failure, on test results and so on, are very important points to be clarified immediately. The problems with the method in which strips are used are essentially the same as those with the method without strips.

This paper describes the results of studies conducted to elucidate these problematic points. Fracture precesses of specimens were analized by means of observations using a high-speed camera as well as microscopic observations of cracking patterns until immediately before failure and measurements on stress distributions. Based on these tests, it was attempted to clearly reveal phenomena in fracture processes of specimens subjected to splitting loads, and the points described above were discussed. Furthermore, the limits of application of the splitting method were studied, based on results of several kinds of strength tests on materials of which ratios of tensile strengths to compressive strengths were larger than those of ordinary concretes.

This paper is a summary of the auther's doctorate thesis submitted to the University of Tokyo. The auther was fortumate to receive a Yoshida Subsidy from the Japan Society of Civil Engineers for this investigation. This paper was critically studied by Profs. M. Kokubu and Y. Higuchi. Their helpful comments are also gratefully acknowledged.

2. BEHAVIORS OF SPECIMENS LOADED WITHOUT STRIPS UNTIL IMMEDIATELY BEFORE FAILURE

2.1 Stress distributions

Horizontal strains perpendicular to the vertical planes which contain the central axes of specimens (hereafter, this plane will be called only "the vertical plane") were measured at their top and bottom surfaces and compared with values calculated usig the elastic theory.[6] These results showed that measured strains coincide rather well with calculated ones of diametrally distributed loads, of which distribution widths are 1/15 of the diameters of specimens, compared with the ones of ideal concentrated line loads up to the loading stage of 20-80% of ultimate load (see Fig.1). This is because a specimen is crushed locally at the parts near the contact lines between the surface of the specimen and bearing plates of the testing machine due to large compressive stresses of vertical direction, and loads come to be distributed ones of which distribution widths correspond to the widths of locally crushed portions. In other words, loads can not be concen-

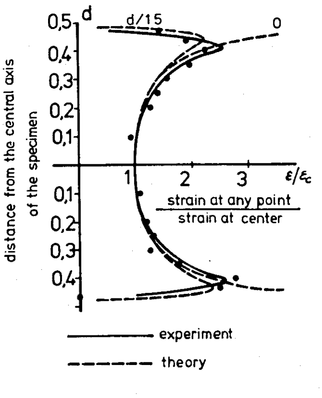


Fig.1 Comparison of experimental and theoretical strains (without strips)

trated because of the partial crushing, and strains in the specimen become as if distributed loads are applied.

According to the elastic theory, tensile stresses perpendicular to the vertical plane which contains the central axis of a specimen are not uniformly distributed along the entire plane but decrease gradually, nearer to the loaded parts, and finally become compressive stresses. If the distribution widths are not greater than d/15, however, these stresses can be regarded as uniform in the region from the central axis to points 0.35d distant and stresses of other points are not different greatly from those of concentrated loads. In the case of the testing method in which no strips are used, therefore, stresses which are nearly equal to the theoretical ones of concentrated line loads may be caused at almost all parts of a specimen.

Although the facts described above hold true for loads under 80% of the ultimate load, measured strains in the regions from the central axis to 0.4d also showed good agreement with calculated ones until a loading stage of 90-95% of the ultimate load. Central parts of a specimen, therefore, still behave elastically up to this loading stage and stresses do not differ widely from those of lower loading stages. Strains in the region from 0.4d to the loading point, however, did not show good agreement with calculated ones and the rate of increase in strains grew higher. This may be because local crushing at the loaded portions developed to the positions of strain gages.

When loads reach higher levels than described above, microcracks occur also at the region from the central axis to points 0.4d distant, as described in 2.2. When microcracks are formed, tensile stresses which had been caused at the cracked parts are redistributed to other parts and stress conditions at these local regions become quite complicated. In other words, behaviors of the central parts of a specimen become inelastic when the loads are over 90-95% of ultimate load. The average stress at a region of a specimen also may differ somewhat from the one under the elastic theory.

2.2 Cracking immediately before failure

In order to make clear behaviors of a specimen immediately before failure, microcracks occurring at the top and bottom surfaces of specimens unloaded after loading until immediately before failure were observed using a microscope. It was indicated from the observations that, although many short cracks perpendicular to the vertical plane are observed at parts near the loaded portions, these cracks are limited to those portions and it is not recognized that they advance into the central portion. At the central portion, rather long cracks perpendicular to the vertical plane are seen at parts near the vertical plane (see Fig.2).

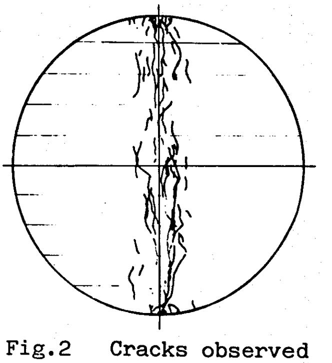


Fig.2 Cracks observed immediately before failure

The reason why cracks near the loaded portions do not advance into the central portion may be that stresses perpendicular to the vertical plane become compressive at parts near the loaded portions because of loads being distributed due to local crushing. In other words, cracks occurring near the loaded portions are prevented from advancing into the central part by these compressive stresses and it is hardly conceivable that these cracks cause the failure of a specimen. To make this more clear, the specimens were loaded again until failure and all cracks observed at failure were matched against cracks before failure, and cracks corresponding to the ones at failure were selected out of the cracks before failure. The result is as shown in Table 1 and it is recognized that short cracks at the loaded portions rarely correspond to cracks at failure but cracks at the central portion of a specimen correspond rather well to the cracks at failure. This shows clearly that specimens are broken down owing to cracks of the central portions advancing to outer portions.

Table 1 Numbers of cracks observed before and at failure

portion	number of cracks immediately before failure	number of cracks at failure
1 central axis ~0.1d	26 (22%)	14 (54%)
2 0.1~0.2d	18 (15%)	9 (50%)
3 0.2~0.3d	22 (18%)	11 (50%)
4 0.3~0.4d	22 (18%)	8 (36%)
5 0.4d~ loading point	33 (27%)	7 (21%)
Total	121 (100%)	49 (40%)

3. BEHAVIORS OF SPECIMENS LOADED WITHOUT STRIPS AT FAILURE

Although behaviors of specimens until immediately before failure have been made clear to a considerable extent, it can hardly be stated that where failure starts from, or whether given strengths depend on the starting position of failure or not, have been clarified. In order to make clear these points, motion pictures of either the top or bottom surface of a specimen being tested were taken using a high-speed camera, and behaviors at failure were studied from begining to end. An example of fracture processes taken is as shown in Photo. 1. The pictures in Photo. 1 are ones enlarged every 10 to 40 frames (about 1/400 to 4/400 second) of the films. The followings can be comprehended from Photo. 1.

(i) The first crack which can be recognized by the high-speed motion pictures occurs at an almost constant position, which is, near the vertical plane and distant from the central axis by 0.1-0.15d. At that time when this first crack is recognized, there are no cracks at the parts near loading points. Since such a microcrack that can not be analyzed by photographs, the possibility that another microcrack not visible in photographs is a starting point of failure can not be denied completely. If there is another crack which is the origin of failure , however, this crack should appear at first in motion pictures though the period may be later than the time of its formation in a strict sense and signs of its length and width increasing accompanied by the fracture process progressing might be taken in photographs. Therefore,it could be concluded that the crack to be the origin of failure is the one which is the first to be recognized on film and appears at a region near the vertical plane and distant from the central axis by 0.1-0.15d. This result not only corresponds to the already described fact that failure of a specimen is not induced by local crushing near the loaded portions but also corresponds to the fact which can be predicted to some extent from the relation between the cracking pattern before failure and the one at failure.

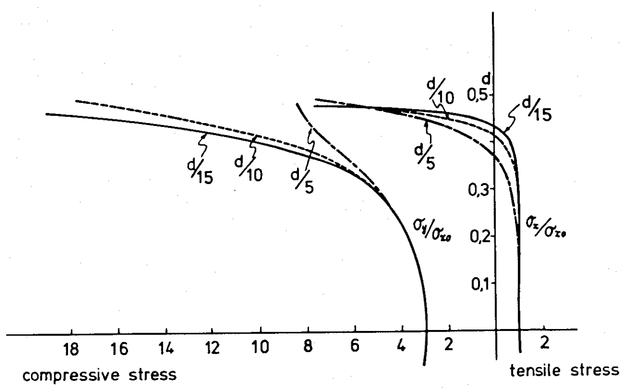
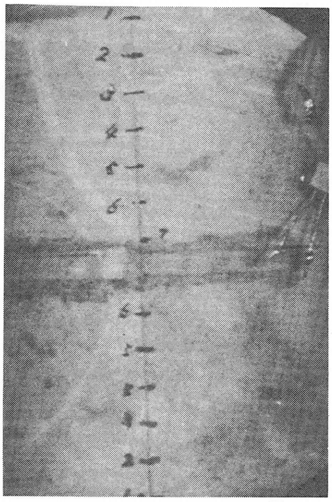
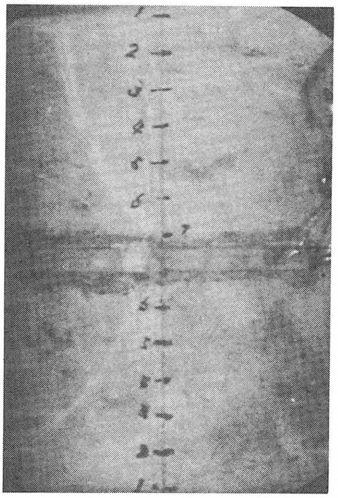


Fig.3 Stress Distributions of specimens subjected to splitting loads

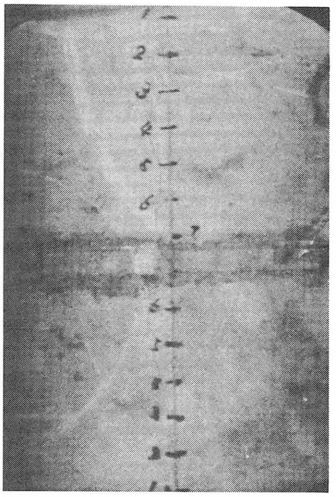
Photo 1 Example of fracture process of specimen loaded without strips.



① The first crack occurs.



② after 1/400 sec



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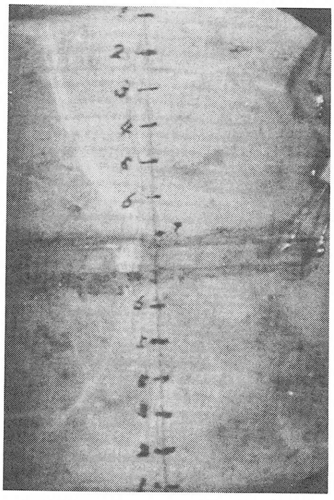

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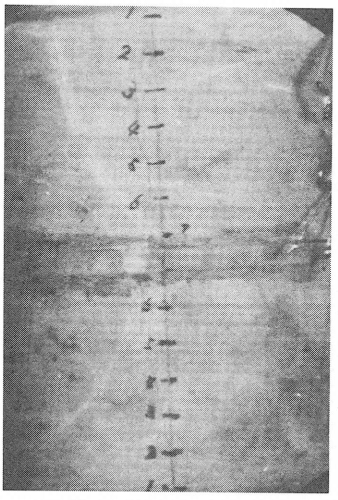

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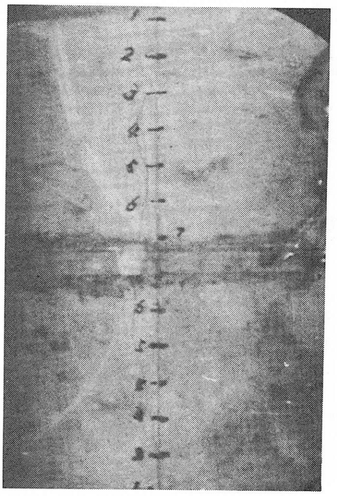

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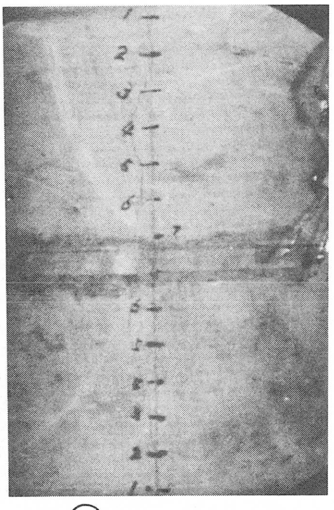
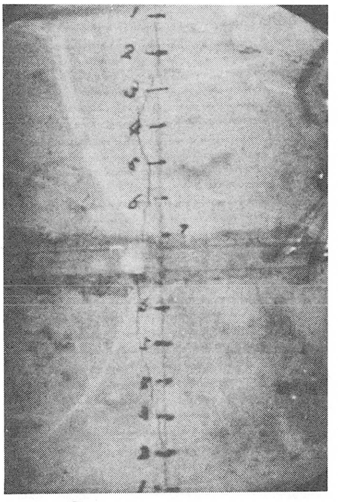
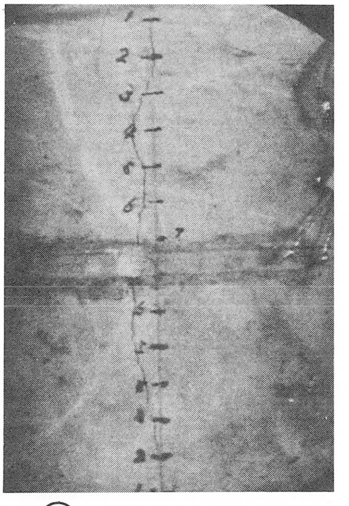
Photo 1 (cont.)



(7) after 7/400sec

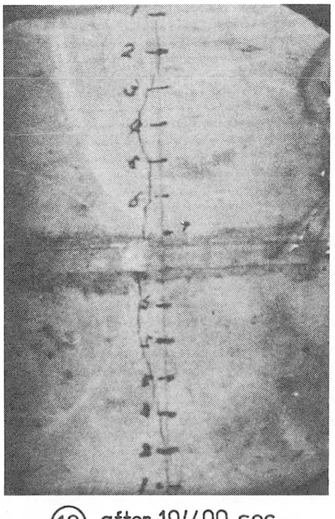


(8) after 11/400 sec

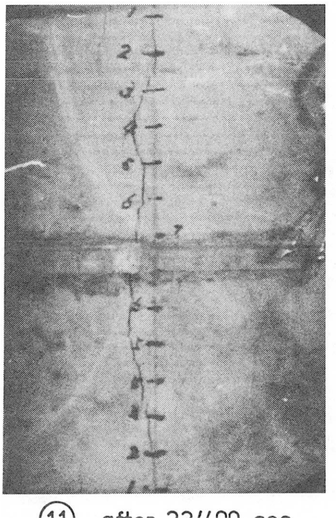


(9) after 15/400 sec

Cracks link upper and lower loaded points.



(10) after 19/400 sec



(11) after 23/400 sec

Figures on photos indicate distances from upper or lower loaded point in cm.

Assuming stresses equal to theoretical ones are caused in a specimen, the position where the fracture is most probable can be predicted from the results of studies on fracture strengths of concretes under biaxial stress. Calculated horizontal tensile stresses and vertical compressive stresses along and perpendicular to the vertical plane are as shown in Fig.3 and every point on the vertical plane of a specimen is assumed to be under these biaxial stress conditions. Many reports have been published on strengths of concrete under biaxial stress but the report published by H. Kupper and others [7] is considered to be most reliable (see Fig.4). Using their result on the concrete of compressive strength of 315 kg/cm^2 (30.9 N/mm^2) ratios of fracture strengths of every point on the vertical plane to the one at the central axis were calculated with the results shown in Fig. 5. It can be predicted from this result that a region from 0.35d to 0.40d has the highest possibility of failure because the tendancy that the fracture strength is less than the tensile stress caused is most marked at this region. This prediction, however, does not correspond to the result of the high-speed motion pictures. The reason for this probably exists in the fact that the results of Fig.5 are ones which take into account only the stress condition at the vertical plane without any consideration of the stress conditions at other parts somewhat distant from the vertical plane. According to the elastic theory, decreases of tensile and compressive stresses at parts distant from the vertical plane become larger the farther from the central axis (see Fig.6). It can be stated from this tendency that a part nearer to the central axis, where stresses are more uni-

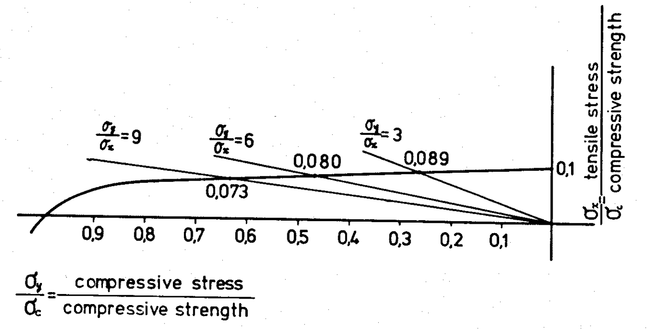


Fig.4 Fracture strength of concrete subjected to biaxial stress (H. Kupper and others)

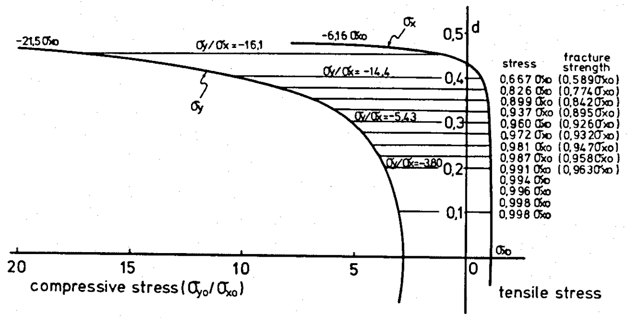


Fig.5 Relation between stress distributions and fracture strengths of specimen loaded without strips

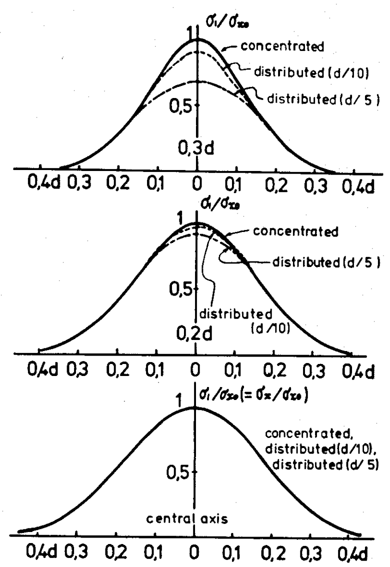


Fig.6 Principal tensile stresses of specimens subjected to splitting loads

form than at the region from 0.35d to 0.40d, may have higher possibility of fracture.

(ii) As time goes by, the crack recognized at first develops to the upper and lower loaded portions while its width increases. About 1/400-2/400 second after the first crack appears, other cracks which are not connected to the first one can be recognized elsewhere. Since the development states of the second cracks are more indistinct than that of the first one, it may be considered that the first one has the most dominant influence on the failure of the specimen. As time goes further by, other cracks are recognized elsewhere and all of those cracks including the first one develop towards the upper and lower loaded portions. Finally, all cracks are connectes with each other and link the loaded portions, resulting in failure of the specimen. The time until cracks linking the upper and lower portions after they first appear is very short, and it was about 7/400 to 16/400 second in this experiment though it differed slightly depending on specimens.

Considering together the development of cracks and the fact that microcracks had already been formed, it may be said that specimens in the splitting test are broken much more gradually than assumed. The fact that the time from the appearance of the first crack until the other cracks were recognized elsewhere was very short, however, shows that at the loading stage when such a crack that can be recognized by high-speed motion pictures occurs, parts near the loading plane of the specimen are very close to their failures and are at a point immediately before losing their load-carrying capacities. When loads are increased without controlling the strain rate in the testing method not using distribution strips, it is not at great error to consider from a macroscopic view point that the whole specimen will be broken at the same time.

(iii) The fact that no cracks are recognized near the loaded portions at the loading stage when the first crack is recognized at the central portion of a specimen does not change even after the fracture process progresses considerably, and cracks still can not be recognized near the loaded portions until cracks of the central portion are connected with each other. After this stage, however, cracks which appear from points on the circumference distant from the loading point by about 1 cm develop towards the central portion. The regions near the loading points of a specimen subjected to splitting loads show very complicated fracture characteristics resulting in the misunderstanding that the loaded portions are the origins of failure.[8],[9] The main reason for this misunderstanding may be due to cracks of this kind appearing. As described above, however, the period when cracks of this kind occur corresponds to a considerably later stage of the fracture process, which is, the period immediately before the whole specimen loses its load-carrying capacity and is split open. In other words, cracks of this kind are caused as secondary effects of failure and do not act as an essential part of the fracture process.

4. EFFECT OF USE OF STRIPS ON BEHAVIORS OF SPECIMENS

4.1 Effects of strips on stress distributions

Splitting tests were carried out using strips made of plywood, with thickness of 3 mm and 10 mm and widths of d/10 and d/5, and horizontal strains of parts along the vertical plane were measured at the top and bottom surfaces of specimens. An example of these results is as shown in Fig. 7 and it is indicated that measured strains corresponed well with ones calculated using the elastic theory, in which uniformly distributed loads of distribution width equal to the widths of strips are assumed, until about 80% of the ultimate load. This shows specimens which are loaded through strips also behave as if they are elastic bodies when load are not near ultimate ones, and stresses in the specimens are considered to be nearly equal to ones calculated using the elastic theory. The results of calculations of stresses at various parts of specimens subjected to distributed loads of various distribution widths are as shown in Figs. 3 and 6. The following can be said from these figures.

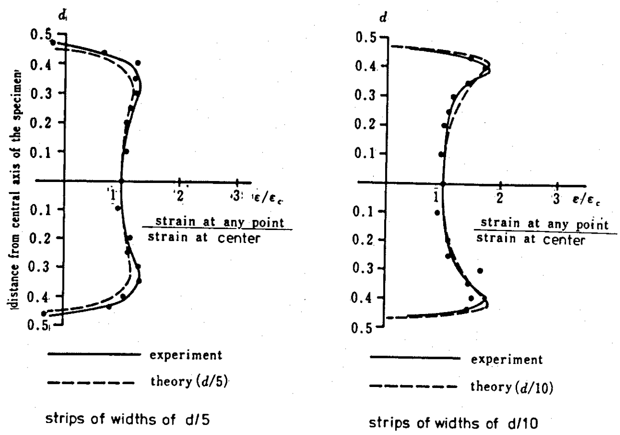


Fig.7 Comparisons of experimental and theoretical strains (with strips)

(i) Horizontal tensile stresses perpendicular to the vertical plane at the central axis do not show any change even though the distribution widths of loads are increased. The range in which uniform tensile stresses equal to these stresses are caused, however, decreases, as the distriubution widths of loads increase and the region where the stresses change from tensile to compressive is expanded nearer toward the central axis. This is an important difference between the methods with and without strips.

(ii) The vertical compressive stresses at the central axis, which are perpendicular to the stresses described above, are also constant not depending on distribution widths of loads, but the degree to which vertical compressive stresses increase the nearer to the loaded portions decreases as the distribution widths of loads are enlarged.

(iii) Principal tensile stresses at parts distant from the vertical plane become more uniform as distribution width is increased. That is to say, principal tensile stresses decrease with more distance from the vertical plane and the degree of decrease becomes extreme as the distribution width increases.

When loads exceeded 80% of ultimate loads, measured strains became different from theoretical ones at regions distant vertically from the central axis by more than 0.4d, and the increase of stresses due to increase of load became larger than at lower loading levels. The degree of this increase, however, was less marked than with the methods of testing without strips inferring that the behaviors of various parts of specimens loaded through strips are more elastic than those of specimens loaded without strips. This difference concerning the behaviors of the loaded portions may be due to the difference in the compessive strsses in the vertical direction. This difference, however, may not be looked upon as important, because , in the testing method without strips, inelastic behaviors of the loaded portions do not have an adverse effect on the test results, as described previously.

4.2 Effects of strips on cracking patterns immediately before failure

It was indicated in the results that the number of short cracks at the loaded portions, which are of the same kind as the ones observed in specimens loaded without strips, decreases considerably (see Table 2). This is because, if strips are used, compressive stresses in the direction of loads become smaller at the loaded portions, besides which the range in which stresses perpendicular to these compressive stresses also become compressive is enlarged. It can be concluded from this fact that the use of strips is effective in preventing local crushing. However, it must be said to be meaningless for strips to be used only to prevent local crushing, because the local crushing does not have an adverse effect on the test results and cracks at loaded portions can not be prevented completely even though strips as wide as 1/5 of the diameter of a specimen are used. The meaning of the use if strips, as described later, exists in the point that they can minimize errors in tests results when there is excessive irregularity at the surface of a specimen.

Table 2 Numbers of cracks observed immediately before failure with testing method using strips

d \ strip	none	d/10, 3mm	d/5, 3mm	d/5, 10mm
	%	%	%	%
0 ~ 0.1	26 (22)	14 (14)	20 (17)	29 (25)
0.1 ~ 0.2	18 (15)	24 (24)	29 (24)	23 (19)
0.2 ~ 0.3	22 (18)	26 (26)	25 (21)	25 (21)
0.3 ~ 0.4	22 (18)	18 (18)	22 (19)	24 (20)
0.4 ~ 0.5	33 (27)	18 (18)	23 (19)	17 (15)
Total	121 (100)	100 (100)	119 (100)	118 (100)

In the testing method in which strips are used, the same as in the method without strips, cracks independent of the ones at loaded portions occur at the central portion and there is a high possibility that they correspond to failure cracks. Therefore, it may be considered that failure of a specimen starts from the central portion also when strips are used.

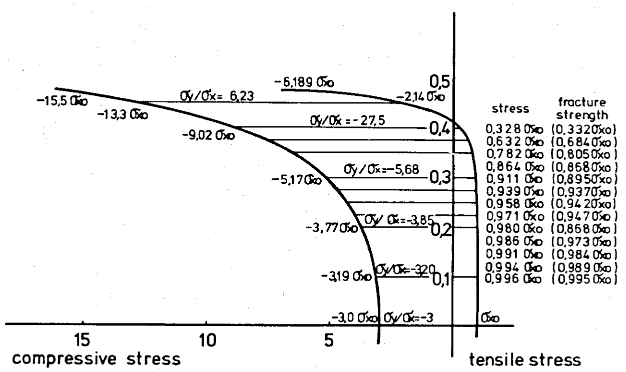


Fig.8 Relation between acting stresses and fracture strength of specimen loaded through strips 10/d wide

4.3 Effects of strips on fracture process

In order to make clear the fracture process of a specimen loaded through strips, high-speed motion pictures were taken in the same way as described in 3. Photographs which were enlarged from the films are as shown in Photo. 2 and the following can be recognized from these photographs.

(i) The first crack which is recognized in the photographs occurs at a part distant from the central axis by 0.1-0.2d. This crack is not necessarily the one caused first, as described in 3.2, but this crack can be regarded as the crack which acts as the precursor of failure. As described above, the location where the crack

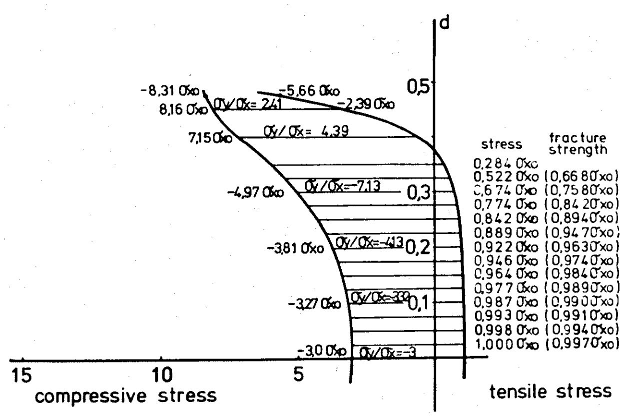
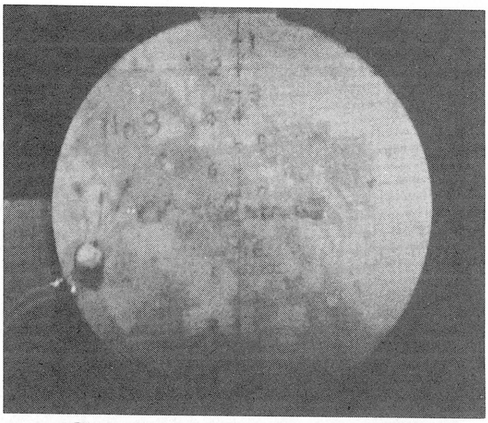
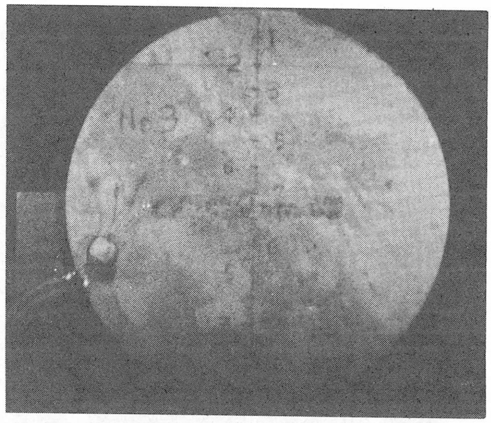


Fig.9 Relation between acting stresses and fracture strengths of specimen loaded through strips 5/d wide

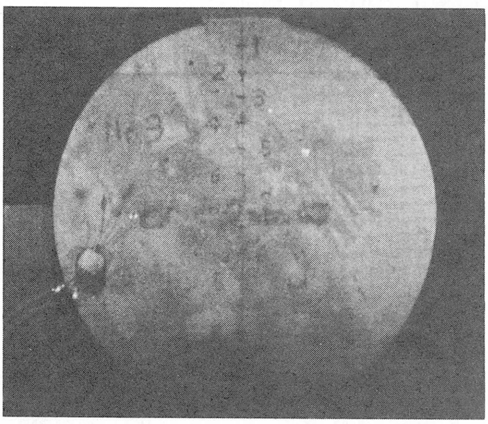
Photo 2 Example of fracture process of specimen loaded using strips.



① The first crack occurs



② after 2/400 sec



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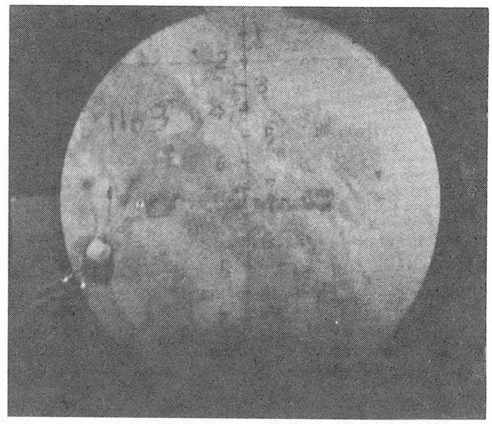

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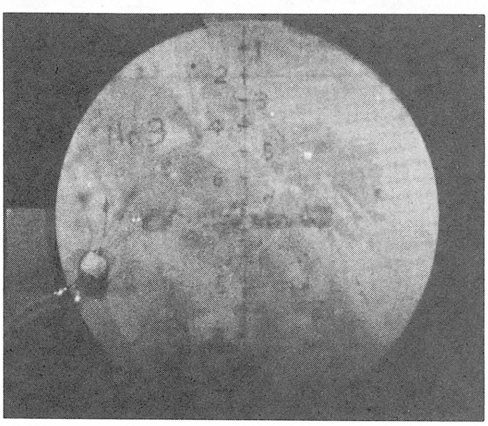

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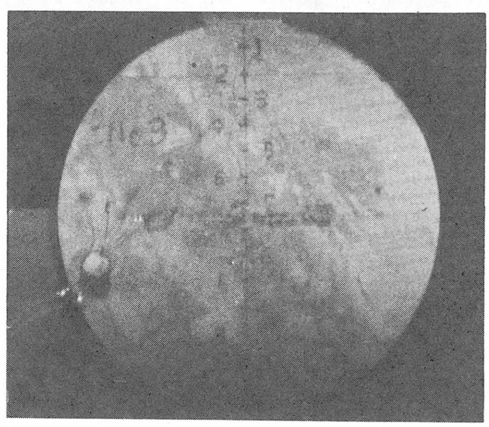

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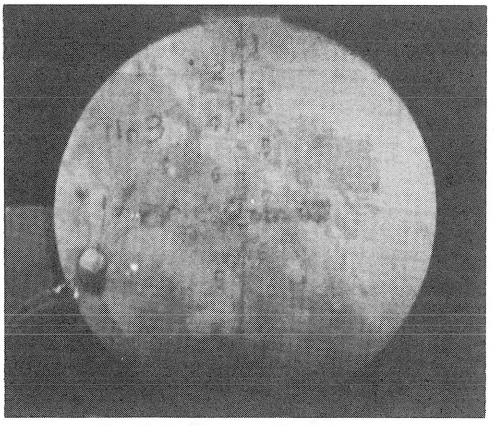
Photo 2 (cont)



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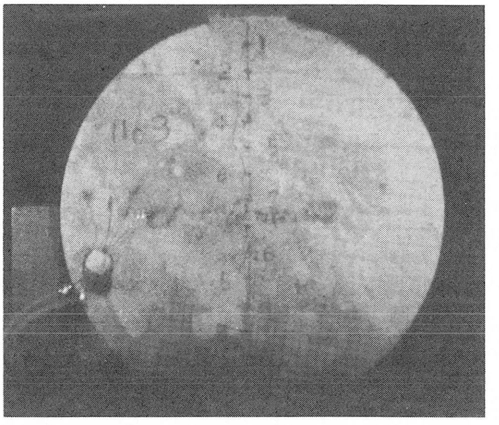
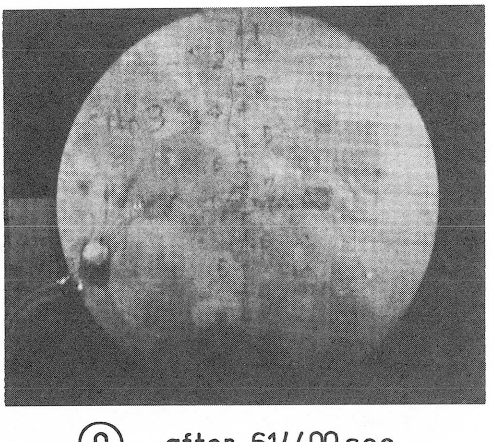
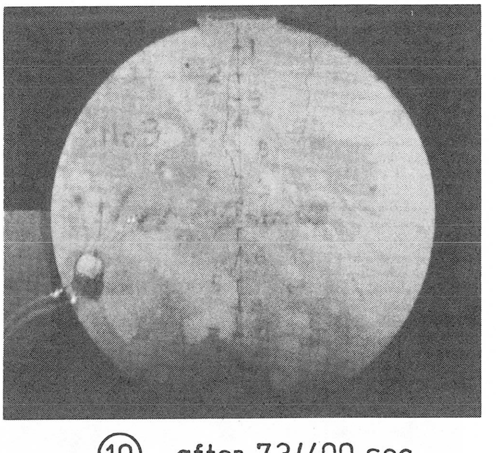


⑧ after 49/400sec

Cracks link upper and lower loaded points.



⑨ after 61/400 sec



⑩ after 73/400 sec

Figures on photos indicate distances from upper or lower loaded point in cm.

acting as the precursor of failure occurs is not greatly different from that in the method of testing without strips. The location where fracture is most probable can be predicted also on the testing method using strips by the same procedure as described in 3, by the elastic theory and fracture strength of concrete subjected to biaxial stress. The results of this prediction are as shown in Figs. 8 and 9, and the location where fracture is most probable is in a region from 0.2-0.25d in the case of strips of d/10 width, and from the central axis to 0.1d in the case of strips of d/5 width. Since the strips used were ones 3 mm thick, the distribution widths of loads are considered to be a little smaller than d/5, and so, the actual location where fracture is most probable is probably nearer to the loading points than in the predictions described above. That is to say, it can be stated that the location of the crack which is recognized in the high-speed motion pictures at first and is the precursor of failure corresponds to the location which is predicted using the elastic theory and fracture strength of concrete under biaxial stress.

The reason why the location of the precursor of fracture corresponds to the predicted one in the case of the testing method using strips though it does not in the case of the method without strips can be found in the difference in degrees of decreases of principal tensile stresses at regions distant from the vertical plane. That is to say, the decrease of principal tensile stresses in case of using strips is smaller than in case of no strip, as shown in Fig. 6, and therefore, when the first crack occurs at a location which can be predicted by the elastic theory and fracture strength, a region distant somewhat from this location has a lower capacity to carry the redistributed stress caused by the occurrence of the crack, and it is supposed that the crack to be the precursor of fracture is caused finally at a part near this location.

(ii) As time elapses, the first crack recognized develops towards the loading points roughly parallel to the vertical plane. Other cracks independent of the first one can be recognized after about 2/400-4/400 second. The crack which has a dominant effect on the progress of fracture is the first one since the latter ones develop more slowly than the first one. As time elapses further, all of the cracks are connected with each other and reach the loaded portions bringing the specimen to breakage. The fracture process described above is not greatly different qualitatively from that of the testing method without strips. When strips are used, however, not only does the first crack develop more slowly than the one of a specimen loaded without strips, but also the increase of its width with time is less marked. Especially, the time when all cracks link the upper and lower loading points after the first one has been recognized is about 25/400-49/400 second, and almost twice the time for a specimen loaded without strips, in which it is about 7/400-16/400 second. The reason for the difference in fracture processes between the two testing methods described above may be attributed to the difference in stress distributions along vertical planes. In other words, it is recognized from Figs. 8 and 9 that the range where the fracture strength of concrete under biaxial stress is smaller than working stress is narrower in the case of using strips than in the case of no strip, and this is one of the reasons why developing velocities of the cracks are slower when using strips.

(iii) When strips are not used, crack widths are markedly increased immediately after all cracks have linked the upper and lower loading points. On the otherhand, when strips are used, the increase of crack width is much slower and even after 12/400 second, a great increase in crack width can not be recognized. The main reason for this is considered to be the friction between strips and the specimen which prevents the specimen from being divided into two. The existence of the friction can be recognized by the behavior of the specimen after time elapses furather, as described in (iv). Even though the effect of the friction between strips and specimen on the test results is not clear, it is reasonable to consider that it is one of the reasons why slightly higher results are indicated because obviously the friction exists before such a crack that can be recognized by high-speed motion picture occurs. This friction may also be another reason why the development of cracks is gradual.

(iv) Another crack which occurs from a point on the circumference distant from the loading point by about 2 cm (d/7.5) and develops towards the central portion roughly parallel to the direction of loading can be recognized after all other cracks have become connected with each other between the loading points. The fact that a crack of this kind is recognized is the same as in testing without strips but the growth characteristics after its appearance are considerably different. In other words, the increase in the width of a crack of this kind is more marked and its width becomes wider than those of cracks along the vertical plane, whereas, when loaded directly without any strip, it is less marked than those of cracks along the vertical plane. Furthermore,a crack of this kind turns towards the vertical plane afterwards and finally reaches the vertical plane although, unfortunately, this phenomenon is not indicated in the photographs. Immediately after this phenomenon the spsecimen is broken and fragment fly to left and right. The fracture process described above is considered to be strong proof that friction exist between a specimen and strips and the friction prevents the specimen being divided into two. When strips are used, it is often recognized that specimens are separated to left and right more abruptly than those loaded without strips. This phenomenon, as described above, occurs after the cracks of the central portion have become connected with each other between the upper and lower loading points and the specimen has lost almost all of its load-carrying capacity, and does not indicate that the fracture process of a specimen loaded through strops is more rapid than that of a specimen loaded without strips. The fracture process until cracks link the upper and lower loading points after microcracks occur is more rapid in the case of no strips than in the case of using strips, as described in (ii) and (iii).

5. CONSIDERATIONS ON RELATIONS BETWEEN SPLITTING STRENGTH AND TENSILE STRENGTH OF CONCRETE

It has been recognized that the results given by the splitting test are nearly equal to those by the pure tension test when concretes of ordinary quality are tested. 10 The reason for this can be made clear to some extent by the facts already described in considered comprehensively.

According to the results of high-speed motion pictures, specimens subjected to splitting loads, whether or not strips are used, are broken from a point of the vertical plane and distant from the central axis by about 0.1-0.2d. At this region, a horizontal tensile stress and a vertical compressive stress which is about 3.2-3.8 times larger than the tensile stress are caused (see Fig.3). Comparing the fracture strength of concrete under this stress condition with the one under a pure tensile stress condition using Fig. 4, it is evident that the former is lower than the latter by 15%. It could be concluded,therefore, that the results given by the splitting test will be lower than that by the pure tensile strength test by about 15% if every part of a specimen is under such a stress condition that a uniform tensile stress and 3.2-3.8 times larger compressive stress perpendicular to the tensile stress are acting and the equation $\sigma = 2P/\pi DL$ is used. Since the stresses in a specimen subjected to splitting loads are not uniform ones as above, however, the effect of nonuniformity of stresses on fracture strength of concrete under biaxial stress must be made clear.

According to the results of the high-speed motion pictures, each point along the vertical plane loses its load carrying capacity almost at the same time when loaded without strips. In this case, therefore, it will suffice for only the horizontally distributed stresses to be taken into account without any consideration of the vertically distributed stresses. Considering together the fact that, in flexural strength tests, the center-point loading method gives larger strengths than the third-point loading method by 20%-25% and the fact that the decrease of stresses at parts distant from the vertical plane in the splitting method is less marked than that of the center-point loading flexure test, it is surmised that the fracture strength at a point which becomes the precursor of failure is nearly equal to pure tensile strength, the decrease of stresses in the horizontal direction setting off the effect of compressive stresses in the vertical direction. In the testing method using no strips, therefore, results nearly equal to pure tensile strength will be given when concrete of ordinary quality is tested, since each part along the vertical plane of a specimen loaded without strips loses its load carrying capacity at the same time, as described above.

In contrast to this, in the testing method using strips, the part from where failure originates is under the same stress condition as the corresponding part in testing without strips, but as shown in Figs. 8 and 9, stresses along the vertical plane tend to become lower than the fracture strength as the loading points are approached. Furthermore, it is recognized from high-speed motion pictures that the fracture process with the method using strips is more gradual than with the method not using strips and that friction hinders a specimen from being divided into two. All of these facts infer that strengths indicated will be larger than those for the testing method not using strips or the pure tensile strength test method. Since the tendency for stresses along the vertical plane to be lower than the fracture strength becomes stronger the wider the widths of strips, the results indicated will also be increased.

In order to confirm the above discussion, splitting tensile strength tests on concretes of the same compressive strengths were carried out using no strip, d/10 wide strips and d/5 wide strips. The results are as shown in Table 3 and it was recognized that the testing method in which strips are used gives higher strengths than the testing method without strips by 3-14% in the case of d/10 wide strips and 14-25% in the case of d/5 width strips. As described above, the strength given by splitting tests differ from each other depending on the testing method, and when concrete of ordinary quality is tested, results nearly equal to pure tensile strength will be obtained if the testing method not using strips is employed, whereas considerably higher strengths will be obtained if the testing method in which wider strips are used is adopted. It can be seen in Table 3 that differences between strengths indicated which vary depending on the testing method differ if strengths of concretes are varied. This indicates that not only the distribution width of loads but also compressive strength of concrete has an important effect on the results obtained by the splitting test method.

Table 3 Effect of use of various strips on test results

W/C compressive strength	strips	(1) average tensile strength (kg/cm^2)	coefficient of variation (%)	strength ratio in (1)	(2) modified tensile strength (kg/cm^2)	strength ratio in (2)	number of specimens
40% 519 kg/cm^2	none	34.4	5.8	1	27.4	1	10
	d/10, 3mm	35.4	7.1	1.03	25.6	0.93	7
	d/10,10mm	35.1	3.2	1.02	25.4	0.93	3
	d/5, 3mm	39.2	7.4	1.14	(22.0)	(0.80)	6
	d/5,10mm	39.1	3.5	1.14	(22.0)	(0.80)	6
55% 360 kg/cm^2	none	28.6	5.3	1	22.8	1	5
	d/10, 3mm	30.1	3.8	1.08	21.8	0.96	5
	d/5, 3mm	33.2	4.4	1.16	(18.7)	(0.82)	3
	d/5,10mm	33.2	6.4	1.16	(18.7)	(0.82)	3
70% 215 kg/cm^2	none	21.5	4.3	1	17.1	1	5
	d/10, 3mm	24.5	3.4	1.14	(17.7)	(1.04)	5
	d/5, 3mm	26.9	2.3	1.25	(15.1)	(0.88)	5

6. SCATTERING OF THE TEST RESULTS IN SPLITTING TESTS

It can be said that the first cause of scattering of test results is the existence of irregularities on the surface of specimens leaving openings between specimens and bearing plates of the testing machine or strips. In order to study this matter, splitting tests were carried out using specimens on surfaces of which six kinds irregularities were made. The results are as shown in Table 4. It can be recognized from Table 4 that scattering due to irregularities on the surface is less for the testing method using strips than for the method not using strips. The smallest irregularity of the specimens tested was a depth of 0.8 mm and length of 1/6 of specimen length, an extreme condition which ordinarily would be inconceivable, so that errors due to such irregularities as would occur in ordinary specimens can be reduced to a negligible degree for practical purposes if strips of proper quality are used. This is a prominent advantage of the testing method using strips. It can be stated from these studies that the use of strips is meaningful in reducing errors of test results though it is less meaningful in preventing local crushing at loaded portions.

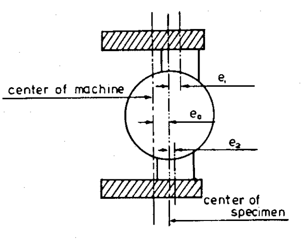
According to Table 4, errors due to irregularities can not be neglected when the testing method not using strips is adopted. It is recognized from this result that scattering of results due to irregular surfaces can not be ruled out if tests are performed without any consideration of irregularities and without any strips. In the testing method not using strips, however, it is quite easy to inspect whether or not there are irregularities before testing. In other words, even small irregularities can be found by inspection from one side after a specimen is laid down on the lower bearing plate and the upper bearing plate is made to lightly touch the specimen. According to this method, it is very easy to find irregularities, and even irregularities slighter than the smallest ones which did not cause any strength reduction in these experiments can be recognized easily. If testing were to be performed carefully without neglecting the abovementioned simple operation, it is thought that irregularities on the side surfaces of specimens can be kept to a negligible degree.

Table 4 Effect of irregularities on test results

strip \ irregularity	none	1/6 of one side 0.53% smaller	1/3 of one side 0.53% smaller	1/3 of one side 1.1% smaller	1/3 of bothsides 0.53% smaller	1/6 of one side 0.53% larger	1/3 of one side 0.53% larger
no	kg/cm² 29.7 (100)	kg/cm² 29.9 (101)	kg/cm² 27.7 (93)	—	kg/cm² 24.0 (81)	kg/cm² 24.4 (82)	—
d/10 3 mm	31.1 (100)	31.1 (100)	30.0 (96)	kg/cm² 28.5 (92)	30.3 (97)	29.5 (95)	kg/cm² 26.7 (86)
d/5 10 mm	33.6 (100)	33.4 (99)	33.7 (100)	—	34.3 (102)	34.0 (101)	—

Table 5 Effect on test results of eccentrical setting of specimen or strips

eccentricity	specimen	$e_0=0$	cooresponds to center of testing machine $e_0=0$				$e_0=2.5$mm	$e_0=5$ mm
	strip	$e_1=0$ $e_2=0$	$e_1=2.5$ mm $e_2=0$	$e_1=5$mm $e_2=0$	$e_1=0\sim5$ mm $e_2=0$	$e_1=2.5$ mm $e_2=2.5$	corresponds to center of specimen $e_1=e_2=0$	
strength kg/cm² (ratio)		29,8 (1,00)	29,8 (1,00)	29,2 (0,98)	29,8 (1,00)	29,1 (0,98)	29,6 (0,99)	28,9 (0,97)
remarks		specimen: Φ15×15~20cm, strip: d/10 thickness: 3 mm concrete: W/C=50% slump = 8 cm age = 7days						



The eccentrical set up of a specimen on a testing machine can be mentioned as the second reason for errors on results in splitting tests. Especially, with the method using strips, large errors may be brought in because not only the specimen, but also the strips must set without eccentricity. Splitting tests using bearing strips were conducted with various kinds of eccentricity, and scattering of results due to the eccentricities was invented. Table 5 shows the results of this investigation together with the eccentricities provided.It is recognized from Table 5 that errors due to eccentricities of a specimen or strips are surprisingly small. That is to say, the maximum eccentricities provided were the one in which strips were shifted by 5 mm from the center of the specimen set at the center of the testing machine, and the one in which the specimen was shifted by 5mm from the center of the testing machine with the strips remaining at the center of the specimen. Although such large eccentricities were provided, they caused errors of only about 3%. The reason for this is considered to be that the effect of eccentricities is decreased owing to the strips being deformed, so that the greater part of the loads worked at the mid-points of the loaded portions. Also, slight rotation of bearing plates during loading may cause a decrease of eccentricities and this is considered to be another reason. In order to confirm this, the effects of eccentricities were studied further using a testing machine whose bearing plates do not rotate at all. The specimens and strips used in this experiment were equal to the ones in Table 5. The experiments were carried out for no eccentricity and for such eccentricities that the specimen as well as strips were set apart from the center of the testing machine by 5 mm. The results of these tests showed slightly greater errors than those of Table 5. That is to say, the decrease in strength was 4% when the bearing plate could not rotate, whereas it was 3% when the bearing plates could rotate. It can be stated from these results that in testing using strips, errors due to eccentricities will be larger when the testing machine has unsuitable bearing plates. The results of later tests, however, show that test errors are larger for the testing method not using strips than for testing method using though the difference between them is small. When tests are conducted under the same conditions, therefore, it may be said that errors with the testing method using strips cannot be larger than those with the testing method not using strips. This, however, does not negate the fact that setting up specimens and strips without eccentricity is difficult in testing method using strips. In other words, the trouble to set both specimen and strips without eccentricity will be greater when strips are used even though the accuracy of set-up is not required so much. There is also the risk that the errors due to eccentricities will become large when strips or the testing machine are not suitable. Therefore, the difficulties in set-up in the testing method using strips can be considered as one of the drawbacks of this method.

It can be said that local crushing of loaded portions of a specimen is a reason for errors in test results in the testing method not using strips. In other words, it is considered for this testing method that distribution widths of loads differ depending on the widths of the locally crushed portions and the differences cause errors in test results. In order to clarify this matter, tests were carried out, and the widths of locally crushed portions were measured using a steel tape after load removal immediately before failure. The results are as shown in Table 6. Table shows the widths of locally crushed portions may be considered to be almost constant when strength of concrete and the diameters of specimens are not changed. For the sake of certainty, the relations between

Table 6 Widths of locally crushed parts at loaded portions

specimen	comp. st. of concrete (kg/cm^2)	① average width of local crushing (mm)	standard deviation of ① (mm)	①/diameter	number of measurements
diameter 30cm length 10~14cm	519	17,1	2,0	$\frac{d}{17,5}$	8
	360	19,4	2,1	$\frac{d}{15,4}$	8
	215	20,1	2,0	$\frac{d}{14,9}$	6
diameter 15cm length 15cm	519	8,6	1,7	$\frac{d}{17,4}$	12
	360	10,1	2,1	$\frac{d}{14,9}$	12
	215	10,1	1,5	$\frac{d}{15,2}$	10
diameter 10cm length 9~11cm	519	7,6	1,9	$\frac{d}{13,2}$	8
	360	7,9	0,8	$\frac{d}{12,7}$	8
	215	9,0	0,6	$\frac{d}{11,1}$	6

Slump was 8~10cm.
Maximum size of coarse aggregate was 25mm.

maximum loads up to load removal and the widths of locally crushed portions were investigated, but no correlation could be found between the two. As shown by this investigation, it can be stated that there is no fear of the widths of locally crushed portions causing errors in test results. When compressive strengths of concretes were changed, the widths of locally crushed portions also changed and the widths became larger as strengths became lower. However, even when the compressive strength of concrete was greatly decreased from 515 kg/cm^2 (50.5 N/mm^2) to 215 kg/cm^2 (20.1 N/mm^2), the changes in widths were extremely small and they only increased from d/17 to d/15 when specimens of diameter of 15 cm were used. This shows that the distribution widths of loads in the testing method not using strips are almost constant, not depending on compressive strength of concrete if the diameter of specimens and maximum size of coarse aggregate used are constant. When concrete with coarse aggregate of maximum size of 25 mm is tested using specimens of diameter of 15 cm, the distribution widths of loads will be in the range of d/15–d/17.

Another cause of errors in test results in the testing method using strips is the difference in quality of strips. There have been many studies made on the effects on test results of quality of strips, but a definite theory has not yet been established. In order to study this matter, splitting tests were conducted using as strips plywood of 3 mm and 10 mm thicknesses and the tensile strengths obtained were compared with each other. According to the results, tensile strengths obtained do not depend on the thickness of strips. Since the strips used in the tests were different in thickness by more than 3 times, their deformation characteristics should have been considerably different. The fact that the results described above were obtained in spite of such a condition shows that some amount of difference in the qualities of strips does not affect test results and accordingly errors are not caused. When strips of width of d/5 were used, however, strains along the vertical plane which contained the central axis of the specimen tested using strips of 3 mm thickness were somewhat different from those tested using 10 mm thick strips. This suggests that, when the ratio of width of strip to diameter of specimen is large, loading conditions for specimens differ depending on the quality of strips, and that the test results will be affected if comparisons are made using strips of which qualities are changed more widely than in this study. That is to say, the reason why the qualities of strips affect the test results is considered to be that the differences in deformation characteristics make loading conditions change and, conversely, the test results will be affected if the characteristics of strips are changed so greatly that the loading conditions are changed. As for qualities of strips, therefore, materials and thickness should be prescribed to obtain reliable results even though scattering of qualities does not have a marked effect such as to produce errors in the test results.

In summary, there are many factors which cause errors in test results with both testing methods, but if the tests are carried out carefully enough, scattering of test results with both testing methods will not be greatly different. This is shown in Table 3 and the coefficients of variations of tests results in this investigation were under 10% and around 5% whichever testing method was used.

7. LIMITS OF APPLICATION OF SPLITTING METHOD

In order to investigate limits of application of the splitting method to various materials as a tensile strength test, splitting,flexual,tensile (briquette) and compressive strength tests were conducted on mortars to which an emulsion was added.

The test results are as shown in Table 7. Table 7 shows that increases in the amount of the emulsion cause compressive strengths of the mortars to be lower and flexual and tensile strengths somewhat higher. Since flexual strengths and briquette strengths may be determined in accordance with tensile characteristics of materials, especially tensile strengths, it may be concluded that addition of emulsion makes tensile strength of mortar to be higher relative to compressive strength. In contrast, splitting strengths were decreased as the amounts of emulsion were increased. All specimens for splitting tests, regardless of mix proportions, failed showing sudden decreases in loads after reaching maximum followed by cracks along the vertical planes linking the upper and lower loading points. The fracture processes were considerably gradual and such a phenomenon as a specimen being shattered into parts and fragments flying to left and right was not seen at all, and almost all specimens retained their original shapes. Within the limits of observation of the fracture characteristics other than this phenomenon, it might be concluded that tensile strengths can be tested by the splitting method. The results of strength tests, however, do not show a positive correlation between splitting strength and tensile strength.

Table 7 Strengths of mortars with emulsion additions

	comp. st. (broken portions)	bending st. (4x4x16 cm)	tensile st. (briquette)	tensile st. (splitting)	comp. st. (φ5x10cm)
W/C=50% C:S=1:2 mortar	581 kg/cm^2	83.1 kg/cm^2	27.7 kg/cm^2	37.7 kg/cm^2	540 kg/cm^2
	1	1/6.99 1	1/21.0 1/3.00 1	1/15.4 1/2.20 1.36	0.93
W/C=50% C:S=1:2 emulsion =C×0.5	170 kg/cm^2	100.9 kg/cm^2	39.4 kg/cm^2	33.1 kg/cm^2	164 kg/cm^2
	1	1/1.69 1	1/4.32 1/2.56 1	1/5.01 1/3.05 0.84	0.96
W/C=50% C:S=1:2 emulsion = C×0.4	250 kg/cm^2	100 kg/cm^2	36.6 kg/cm^2	35.9 kg/cm^2	232 kg/cm^2
	1	1/2.57 1	6.83 1/2.75 1	6.96 2.81 0.98	0.93
W/C=60% C:S=1:2 emulsion =C×0.3	259 kg/cm^2	90.7 kg/cm^2	43.0 kg/cm^2	43.2 kg/cm^2	253 kg/cm^2
	1	2.86 1	1/6.02 1/2.11 1	1/6.00 1/2.11 1.00	0.99
W/C = 50% C:S=1:2 emulsion =C×0.2	432 kg/cm^2	89.4 kg/cm^2	40.5 kg/cm^2	53.2 kg/cm^2	426 kg/cm^2
	1	1/4.83 1	1/10.7 1/2.21 1	1/8.12 1/1.68 1.31	0.99

As described in 5, the reasons why the splitting test gives strengths not greatly different from pure tensile strengths exist mainly in the fact that the fracture strength of concrete subjected to biaxial stress is as shown in Fig. 4 and the fact that fracture of a specimen is initiated from a part where the stresses cause fracture strength not very different from pure tensile strength. The splitting method, therefore, can not be applied as a tensile strength test to materials whose fracture strengths are different from Fig.4. As described above, since the mortars used in these experiments are the ones of which tensile strengths are increased relative to compressive strengths, their fracture strengths under biaxial stress are considered to be greatly different from those of ordinary concretes, and so, results as described above were obtained.

From Table 7, of mortars to which emulsion was added, in case of that of flexual strength/compressive strength = 1/4.83, splitting strength/flexual strength = 1/1.68 and splitting strength/tensile strength = 1.31 result, which are nearly equal to the corresponding values of mortar with no emulsion, in which they are 1/2.20 and 1.36 respectively, but the values of mortars for which flexual strength/compressive strength is over 1/1.86 differ widely from those of mortars with no emulsion. Splitting strengths become nearly equal to tensile strengths unless the ratios of splitting strengths to briquette strengths or flexual strengths are greatly different from each other. The results of Table 7, therefore, show that the splitting method can be applied as a tensile strength test for materials of which flexual strength/compressive strength is equal to or less than 1/4.83, but not for materials of which ratio of bending strength to compressive strength is equal to or over 1/2.86. As a result of repetitions of the same tests as for Table 7, it was shown that the splitting method can not be applied as a tensile strength test when the ratio of flexual strengths obtained by center-point loading flexure tests on mortar specimens of 4x4x14 cm to compressive strengths obtained using portions of specimens broken in flexure becomes 1/4.4-1/4.8, and when the ratio of briquette strengths to compressive strengths becomes 1/11-1/15.

Based on these results, the rate of increase of tensile strength of the mortar, which is just at the limit of application, to that of mortar without emulsion was calculated, assuming flexual strengths and briquette strengths are proportional to tensile strengths, and then, the limit value of tensile strength/compressive strength at which the splitting method could not be applied was calculated further assuming that the tensile strength of mortar without emulsion is 1/11 of its compressive strength. It was recognized from this result that it is the application limit when the tensile strength is 1/6-1/8 of the compressive strength (see Table 8). This estimation gives only a rough value. However, the fact that the splitting method can not be applied when the tensile strength is at most 1/6 of the compressive strength should be noticed.

Table 8 Application limit of splitting method

	1 ordinary mortar	2 mortar at application limit	2/1 increase rate of tensile st.	tensile st. / comp. st. at application limit
bending st. / compressive st. basis	1/7	1/4,4	1,6	1/6,9
	1/6,7	1/4,8	1,4	1/7,9
briquette st. / compressive st. basis	1/21	1/14,5	1,5	1/7,3
	1/21	1/10,7	1,9	1/5,8

8. CONCLUSIONS

Behaviors of cylindrical specimens carrying splitting loads were investigated in detail, especially at their failures, in order to make clear the influence on test results of local crushing at loaded portions, this having been an object of criticism in case of the method not using strips, the influence of use of strips on the test results, the relations between splitting tensile strength and pure tensile strength, and the limits of application of the splitting method. As for concretes of which qualities are in the range of wide practical use, the following can be concluded within the limits of the experiments.

(1) In the testing method in which specimens are loaded without strips, the loads do not work as ideal concentrated line loads, as has been pointed out for a long time, but as distributed loads of which distribution widths are about 1/15 of the diameter of the specimen owing to local crushing at the loaded portions. The greater part of a specimen except near the loaded portions, however, behaves quite elastically until a rather high loading stage, and its stresses are near the calculated ones based on the elastic theory assuming diametrally distributed loads of which distribution widths correspond to the width of local crushing. According to the elastic theory, stresses of an elastic disk due to distributed loads for d/15 width are not greatly different from those of concentrated loads. It may be concluded, therefore, that stresses in the greater part of a specimen are nearly equal to those theoretical ones of concentrated loads. The regions of local crushing increase gradually when the load approaches the ultimate stage. Cracks at loaded portions, however, do not develop into the inner region and the specimen collapses owing to cracks, at the interior and independent of those at loaded portions, which develop to the upper and lower parts. As described above, partial crushing at loaded portions of a specimen loaded without any strip not only does not have any adverse effect on the stresses of the specimen but also does not cause failure of the entire specimen at too early a stage.

(2) Observations of the fracture process of a specimens loaded without strips using a high-speed camera showed that the first dominant crack occurs at a region which is distant from the central axis of the specimen by 0.1-0.15d and is near the vertical plane which contains the central axis and that a specimen

collapses owing to this crack linking the upper and lower loaded points within such a very short time as 7/400-16/400 second. The pictures further shows that cracks like wedges at a loaded portion which can be observed after failure occur as a secondary effect at the end of the fracture process. The region where the first crack occurs roughly corresponds to the region where the biaxial strength of concrete and stresses near this region show that fracture is most probable. The fracture process of a specimen loaded without strips is a gradual one where a crack at one part develops to other parts, as described above. However, another crack independent of the first one occurs at another part after such a short time as 1/400-2/400 second, and the cracks including the first one develop extremely rapidly. It may be concluded from these facts without great error that a specimen collapses due to all of the parts along the vertical plane losing their load-carrying capacities at the same time.

(3) In the splitting method in which no strips are used, the ultimate strength of the part where the first dominant crack occurs is considered to be nearly equal to pure tensile strength when judged according to its stress condition, and the ultimate strength of each part along the vertical axis is not greatly different from that of the first crack part. This is the reason why results obtained by the splitting method are regarded as pure tensile strengths although specimens are under complex biaxial stress conditions. Within the range of concretes of ordinary quality, therefore, the splitting strength indicates strength nearly equal to pure tensile strength because the shapes of envelopes which indicate ultimate strengths under biaxial stresses are not much different from each other. It has been claimed for a long time that the testing method in which strips are used should be adopted, being guided by many countries' examples. When tests are carried out using molded specimens made of concretes of ordinary quality, however, the testing method using strips should not be adopted because the adverse effects of no strips being used on test results are negligible, as described in (1), and the testing procedure is more simple with the method of no strips, as will be discussed in (4).

(4) Fracture processes of specimens loaded through strips are recognized as being much more gradual than those loaded with no strips. The reasons for this can be attributed to the fact that in case of using strips the degree stresses along the vertical plane become smaller than failure strength of concrete subjected to biaxial stress is greater the farther from the central axis, and to the fact that friction between the specimen and bearing strips slows down failure of the specimen. It can be concluded from these facts that slightly higher strength than pure tensile strength will be indicated if the method using bearing strips is adopted. Once this method has been prescribed as the standard method, the fact that higher strength will be indicated is not an important matter. If strips are used, however, not only will the testing procedure become more complicated, but also there will be a risk that scatter in test results will be caused, if the properties of strips as well as their widths are not standardized. Use of strips is considerably effective in preventing local crushing at the loaded portions but, as described in (1), there is little significance in preventing local crushing. Generally speaking, therefore, the testing method in which bearing strips are used can not be recognized as being a superior method. The only merit in using strips is that the error will be very small when excessive irregularity in the specimen surface can not be avoided.

(5) As described in (3), it may be readily presumed that the splitting method can not be applied as a tensile strength test to materials of which ultimate strengths under biaxial stress differ widely from those of concretes of ordinary quality. Various kinds of strength tests were conducted on mortars of which tensile strengths were increased relative to their compressive strength by adding an emulsion. These results showed that the limit of ratio of tensile

strength to compressive strength that the splitting method can be used as a tensile strength test is roughly 1/6-1/8, and that, for a material with tensile strength higher than this limit, the results obtained by the splitting method can not be regarded as the tensile strength. As described above, although the splitting method is excellent as a testing method per se, the applicable range of the splitting method is surprisingly narrow.

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