Application of Meta-Modeling for Quality Assurance of Automated High Fidelity Bridge Structure Models

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We propose a quality assurance process for an automated high fidelity model (or solid element model) by using a set of consistent low fidelity models. The consistency of low fidelity models is guaranteed according to the meta-modeling theory which allocates structural mechanics as mathematical approximation of continuum mechanics. The developed automated model construction modules are capable of using two sets of digital data (Auto CAD and GIS) of a target bridge structure and of constructing a solid element model, a frame model, a consistent mass spring model or a consistent lumped mass model. We use a set of automated high fidelity multi-span bridge structures for the quality assurance process as a numerical experiment in this study. It is shown that properly chosen consistent low fidelity models can be successfully used for quality assurance of automated high fidelity model.

Key Words : *consistent modeling, automated model construction, mass spring model, bridge structure, continuum mechanics, structural mechanics*

1. INTRODUCTION

A methodology of a more reliable estimation of possible damage of a bridge structure induced by an earthquake is to improve the accuracy of seismic response analysis using a higher fidelity bridge structure model; with the progress of computers, large scale computation that is needed for the analysis of a high fidelity bridge structure model can be conducted[1), 2)]. In civil engineering, it is rare that solid element analysis is made even for a bridge structure of complicated configuration[3), 4)]. While there are several reasons for such a rare use of a solid element analysis for bridge structure, major reasons are; 1) difficulty of validation of a solid element model which includes a huge number of degrees of freedom as compared to a structure element mode, and 2) laboriousness of construction of solid CAD model.

The modeling of solid CAD model is laborious, when the number of structures and structure components analyzed is huge. The development of automated model construction, i.e., conversion of digital data available for a target structure to an analysis model which is directly input to a suitable seismic response analysis method, is thus required[8)]. In this study, a prototype of an automated model construction modules is used to construct a set of different fidelity models for a target bridge structure.

According to a typical validation approach, we need to observe input ground motion and seismic responses of a target bridge structure, but installing a monitoring system of ground motion and seismic response is expensive, specially, when the target bridge structure occupies several kilometers. As an alternative of the typical validation approach that uses observed data, we are proposing a sequence of models, from a low fidelity to a high fidelity. The quality of a low fidelity model is more easily examined than a high fidelity model, since a low fidelity model such as a mass spring model, a frame element model, ect., uses a fewer parameters which have to be examined. Using a low fidelity model as a reference, we examine the quality of a high fidelity model or solid element model which has more parameters. We emphasize the necessity for all the models in the sequence to share the same fundamental dynamic characteristics such as natural frequencies or mode shapes.

The authors are proposing meta-modeling[5), 6), 7), 8), 9), 10)] theory, which allocates structural mechanics as mathematical approximation of solving a Lagrangian problem of continuum mechanics. In other words, structure mechanics solves the same physical problem of continuum mechanics applying distinct mathematical approximations. Therefore, it is well expected to construct a set of different low fidelity models of the same fundamental dynamic characteristics as a continuum mechanics model, according to the meta-modeling theory.

This paper proposes a method of quality assurance of automated high fidelity model (solid element model) by employing a set of consistent low fidelity models those shares the same fundamental dynamic characteristics as a continuum mechanics model. The target structure is bridge structure. The contents of this paper are as follows. First, the modules of automated construction are briefly explained in Section 2. In Section 3, meta-modeling theory, which constructs a set of models for one structure, is explained; this theory is the basis of constructing a model of assured quality, and its key point is consistency of model. We carry out numerical experiment to examine behavior of response of a set of different fidelity consistent models for six different multi-span bridge structures in Section 5. Some concluding remarks are made at the end.

2. AUTOMATED CONSTRUCTION OF BRIDGE MODEL

There are different formats of digital data sources which can be employed to construct a target bridge structure. Each data source has unique data structure which is suitable for their own special purposes. Construction of a bridge structure needs global and local information of the target bridge structure. The global data include an overall 3D configuration of the target bridge structure and the local data contain a detailed information about each component of the target bridge structure. Both the local and global data of the target bridge structure need to be extracted from each of the data sources and reorganized, so that an analysis model is constructed from the reorganized data. The reorganized data is called ameliorated data; see **Fig. 1**.

The most important point of bridge structure modeling is an identification of relation between the global and local data of each data source. This relation can be developed by using pier identification numbers, pier coordinates, or other information of the target bridge structure to which the global and local data are related.

In this study, automated model construction modules are introduced to overcome laboriousness of bridge structure modeling, when the number of structures and structure components analyzed is huge. Employing ameliorated data of the target bridge structure, the developed automated model construction modules which are able to use two sets of digital data (Auto CAD and GIS) of the target bridge structure and to construct analysis models for the target bridge structure automatically. A key issue is that model could be a solid element model or a frame model. Then an eigenanalysis of the developed solid or frame element model is able to generate a consistent fundamental seismic response analysis model such as a consistent lumped mass model (CLMM) or a consistent mass spring model (CMSM) for a target bridge structure.

3. META-MODELING THEORY

(1) Summary of the meta-modeling theory

In general, continuum and structure mechanics share the same equations for kinematics and dynamics but they have different constitutive relation of strain and stress which creates inconsistency between continuum and structure mechanics. For instance, in bar or beam theory, one dimensional stress-strain relation is used as a constitutive relation[11)], i.e., $\sigma = E\epsilon$ where σ and ϵ are normal stress and strain component in the same direction and E is Young's modulus. The material model of structure mechanics fully ignores the of Poisson's ration ν. The difference in the material property clearly appears for isotropic material state in strain energy densities, (e_{s} and e_{c}), as

$$e_{\mathrm{s}} = \frac{1}{2}E\epsilon^2 \text{ and } e_{\mathrm{c}} = \frac{1}{2}\frac{(1-\nu)E}{(1+\nu)(1-2\nu)}\epsilon^2.$$

Where e_{s} is for bar or beam theory and e_{c} is for continuum mechanics.

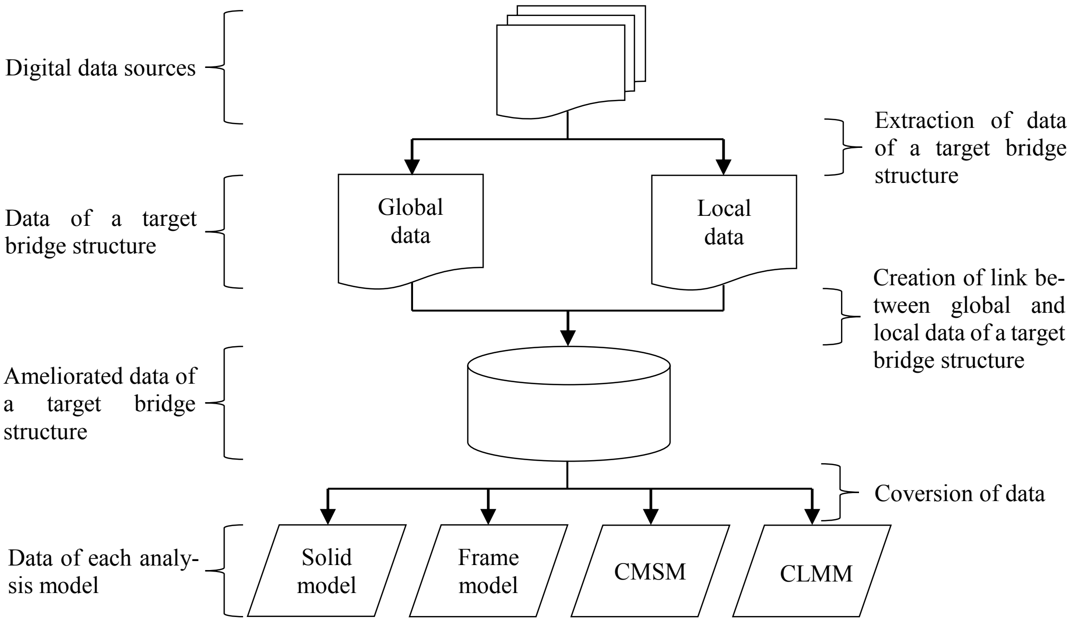


Fig. 1 Construction of a set of different fidelity analysis models for a target bridge structure.

This different treatment in material property results in a different problem of structure mechanics and continuum mechanics. This is clearly seen in terms of the following ordinary Lagrangian:

$$\mathcal{L}[\mathbf{v},\boldsymbol{\epsilon}] = \mathcal{K}[\mathbf{v}] - \mathcal{P}[\boldsymbol{\epsilon}], \tag{1}$$

where $\mathbf{v}$ and $\boldsymbol{\epsilon}$ are velocity and strain, $\mathbf{v} = \dot{\mathbf{u}}$ and $\boldsymbol{\epsilon} = \mathrm{sym}\{\boldsymbol{\nabla}\mathbf{u}\}$ with $\dot{(\cdot)}$ and $\boldsymbol{\nabla}(\cdot)$being temporal derivative and gradient, and sym standing for the symmetric part. While $\mathcal{K}[\mathbf{v}] = \int \rho\mathbf{v}\cdot\mathbf{v}\,\mathrm{d}v$ is common, $\mathcal{P}[\boldsymbol{\epsilon}]$ is different for structure mechanics and continuum mechanics, since it is the integration of e_{s} or e_{c}. It is clear that fundamental dynamic characteristics of a structure mechanics model need to be different from of a continuum mechanics model.

Meta-modeling theory[5] introduces another Lagrangian for velocity, strain and stress to overcome this inconsistency of material model, i.e.,

$$\mathcal{L}^*[\mathbf{v},\boldsymbol{\epsilon}] = \mathcal{K}[\mathbf{v}] - \mathcal{P}^*[\boldsymbol{\epsilon}], \tag{2}$$

where

$$\mathcal{P}^*[\boldsymbol{\epsilon},\boldsymbol{\sigma}] = \int \boldsymbol{\sigma}:\boldsymbol{\epsilon} - \frac{1}{2}\boldsymbol{\sigma}:\mathbf{c}^{-1}:\boldsymbol{\sigma}\,\mathrm{d}v, \tag{3}$$

with $\mathbf{c}^{-1}$ being the inverse of $\mathbf{c}$ and : standing for the second order contraction. This $\mathcal{L}^*[\mathbf{v},\boldsymbol{\epsilon}]$ is equivalent with $\mathcal{L}[\mathbf{v},\boldsymbol{\epsilon}]$, since $\delta\int\mathcal{L}^*\,\mathrm{d}t = 0$ yields $\boldsymbol{\sigma} = \mathbf{c}:\boldsymbol{\epsilon}$ and

$$\rho(\mathbf{x})\ddot{\mathbf{u}}(\mathbf{x},t) - \boldsymbol{\nabla}\cdot\big(\mathbf{c}(\mathbf{x}):\boldsymbol{\nabla}\mathbf{u}(\mathbf{x},t)\big) = \mathbf{0}, \tag{4}$$

for $\mathbf{u}$ which makes $\mathbf{v} = \dot{\mathbf{u}}$ and $\boldsymbol{\epsilon} = \mathrm{sym}\{\boldsymbol{\nabla}\mathbf{u}\}$. Equation (4) coincides with the wave equation of $\mathbf{u}$ which is derived from ordinary Lagrangian ($\mathcal{L}[\mathbf{v},\boldsymbol{\epsilon}]$).

It is clear that $\delta\int\mathcal{L}^*\,\mathrm{d}t = 0$ leads to the identical governing equation with that of bar or beam theory by choosing suitable subset of function space of $\{\mathbf{u},\boldsymbol{\sigma}\}$ without making any assumption such as dimensional stress-strain relation[11]. It is also clear that the resulting solution of bar or beam is an approximate solution of continuum mechanics. In this sense, we regard solution space of bar or beam theories as a subset of solution space of continuum mechanics.

(2) Construction of consistent mass spring model based on the meta-modeling theory

As a simplest case of mass spring model, we consider a mass spring system that contains two mass points. According to the meta-modeling theory, an approximate displacement function for the mass spring system can be considered as of following form:

$$\mathbf{u}(\mathbf{x},t) = \sum_{\alpha=1}^{2} U^\alpha(t)\boldsymbol{\phi}^\alpha(\mathbf{x}), \tag{5}$$

where U^α is displacement of the α-th mass point and $\boldsymbol{\phi}^\alpha$ is the corresponding displacement mode; by definition, $\boldsymbol{\phi}^\alpha(\mathbf{x}^\alpha) = 1$, with $\mathbf{x}^\alpha$ being the location of the α-th mass point, and $\boldsymbol{\phi}^\alpha(\mathbf{x}^\beta) = 0$ for $\alpha \neq \beta$, which we can call as requirement 1.

For simplicity, we substitute $\mathbf{u}$ of Eq. (5) into $\mathcal{L}[\mathbf{v},\boldsymbol{\epsilon}]$ of Eq. (1) rather than $\mathcal{L}^*[\mathbf{v},\boldsymbol{\epsilon}]$of Eq. (2) which requires setting of $\boldsymbol{\sigma}$ and obtain

$$\mathcal{L} = \sum_{\alpha,\beta=1}^{2} \frac{1}{2}m^{\alpha\beta}\dot{U}^\alpha\dot{U}^\beta - \frac{1}{2}k^{\alpha\beta}U^\alpha U^\beta, \tag{6}$$

where

$$m^{\alpha\beta} = \int_V \rho\boldsymbol{\phi}^\alpha\cdot\boldsymbol{\phi}^\beta\,\mathrm{d}v, \tag{7}$$

$$k^{\alpha\beta} = \int_V \nabla\boldsymbol{\phi}^{\alpha} : \mathbf{c} : \nabla\boldsymbol{\phi}^{\beta} \, \mathrm{d}v.$$

If we can find $\boldsymbol{\phi}^1$ and $\boldsymbol{\phi}^2$ which can vanish m^{12} and k^{22} (requirement 2), then the above $\mathcal{L}$ becomes

$$\mathcal{L} = \frac{1}{2}m^{11}(\dot{U}^1)^2 + \frac{1}{2}m^{22}(\dot{U}^2)^2 - \frac{1}{2}k^{12}(U^2 - U^1)^2 - \frac{1}{2}k^{11}(U^2)^2.$$

As is noticed, this $\mathcal{L}$ corresponds to a Lagrangian of a conventional mass spring system. However it is impossible to find $\boldsymbol{\phi}^1$ and $\boldsymbol{\phi}^2$ that vanish both m^{12} and k^{22} simultaneously. For instance, we now assume that $\boldsymbol{\phi}^1$ and $\boldsymbol{\phi}^2$ satisfy only requirement 1.

We need to utilize dynamic modes of continuum mechanics in constructing a mass spring model, so that it shares the same dynamic fundamental characteristics with a continuum model. We suppose that two dynamic modes $\{\boldsymbol{\psi}^{\alpha}, \omega^{\alpha}\}$ ($\alpha = 1$ or 2), are given; $\boldsymbol{\psi}^{\alpha}$ is a mode shape and ω^{α} is a natural frequency. Recall that the dynamic mode satisfies

$$\rho(\omega^{\alpha})^2\boldsymbol{\psi}^{\alpha} + \nabla\cdot(\mathbf{c}:\nabla\boldsymbol{\psi}^{\alpha}) = 0, \qquad (8)$$

and

$$\int_V \rho\boldsymbol{\psi}^{\alpha}\boldsymbol{\psi}^{\beta}\,\mathrm{d}v = 0, \quad \int_V \nabla\boldsymbol{\psi}^{\alpha}:\mathbf{c}:\nabla\boldsymbol{\psi}^{\beta}\,\mathrm{d}v = 0, \qquad (9)$$

for $\alpha \neq \beta$.

It is clear that $\boldsymbol{\psi}^1$ and $\boldsymbol{\psi}^2$ cannot satisfy requirement 1. Hence, we try to find suitable linear combinations of $\{\boldsymbol{\psi}^{\alpha}\}$ that satisfy the requirement 1. To this end, we consider the following combination:

$$\boldsymbol{\phi}^{\alpha} = \sum t^{\alpha\beta}\,\boldsymbol{\psi}^{\alpha}, \qquad (10)$$

where $t^{\alpha\beta}$ is a component of two-by-two matrix. It is readily seen that this matrix can be determined when $\boldsymbol{\psi}^1$ and $\boldsymbol{\psi}^2$ is in the same direction and are parallel to each other.

4. QUALITY ASSURANCE OF AUTOMATED HIGH FIDELITY BRIDGE STRUCTURE MODEL BY EMPLOYING CONSISTENT LOW FIDELITY MODELS

(1) Problem setting

Three straight (SC) and three curved (CC) multi-span bridge structures with different types of pier arrangement along the longitudinal direction of each bridge structure are studied in this numerical experiment for the quality assurance of an automated high fidelity bridge structure model; see **Fig. 2** and Appendix A for the geometric arrangement of these bridge structures. The continuous deck structures of target bridges are only allowed to move in the longitudinal direction of the bridge, the piers are fixed to the

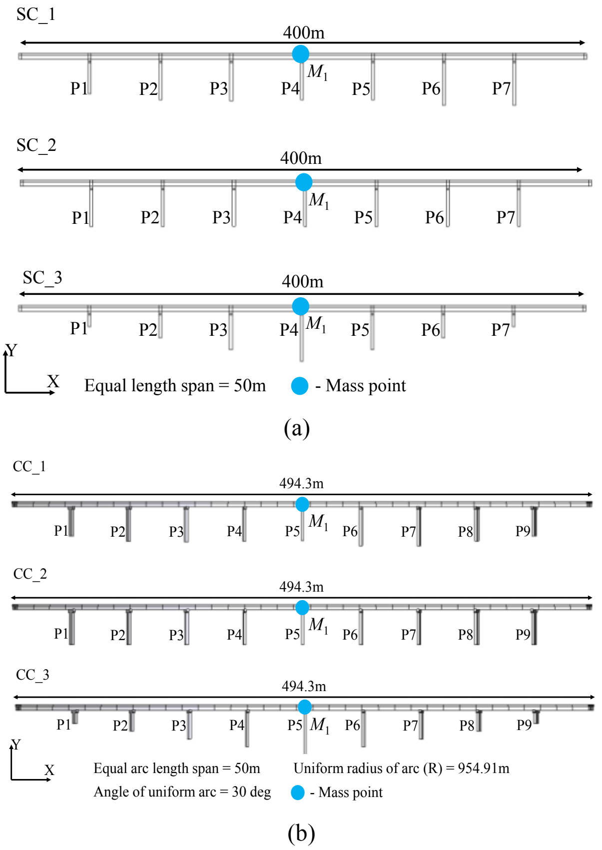


Fig. 2 Geometric and mass points' information about multi-span bridge structures: (a) straight continuous (SC); and (b) curved continuous (CC).

ground at the pier base, and tie connection is used for the connection between the pier and the deck. Tie connection is the simplest, and more sophisticated connection could be used if more detailed information is available for the connection.

Four consistent models are developed for each bridge structure. They are (1) a CLMM, (2) a CMSM, (3) a frame model, and (4) a solid element model. First, the automated construction module is used to develop a solid element model and a frame model separately; see Sec. 2 for the automated construction process. Then, a CLMM is constructed from the frame model; see reference[8] for the construction of the CLMM. Next, a CMSM is constructed from the solid element model; see Sec. 3.2 and reference[6), 10)] for construction of CMSM. The CMSM for the longitudinal direction uses only first mode in this numerical experiment. This is because in the longitudinal direction, the first mode has much lower natural frequency than other modes. The location of mass point of each bridge structure for the CMSM is shown in **Fig. 2**.

Table 1 shows the material properties of the pier and the deck structures. Linearly isotropic elasticity is assumed. The configuration of the pier is displayed

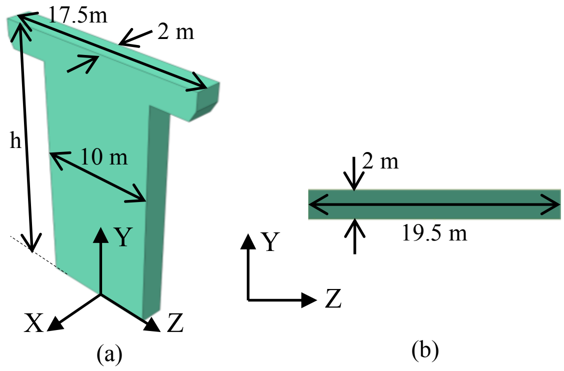


Fig. 3 Multi-span bridge models (SC & CC): (a) pier geometry; and (b) deck cross-section geometry.

Table 1 Material data of multi-span bridge models.

Item	E / GPa	ρ / Kgm^{-3}	v
Pier	24	2400	0.2
Deck	200	2000	0.3

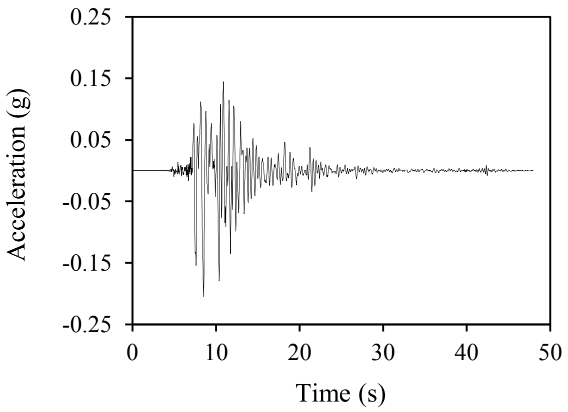


Fig. 4 Input ground motion for bridge structure

in **Fig. 3(a)**, and the cross-section of the deck is shown in **Fig. 3(b)**. Frequency and time domain analyses are conducted for the target bridge structures, in order to check the consistency of the developed models and the applicability of the consistent low fidelity model for quality assurance process of the automated solid element model. The ground motion displayed in **Fig. 4** is employed.

(2) Results and discussion

Natural frequencies of the CLMMs in the longitudinal direction are presented in **Table 2**; the natural frequencies of the first mode of the frame models are presented, too. As is seen, the natural frequencies of the CLMMs do not have a good agreement with those of the frame models, except for the cases of SC_1 and SC_2. This is due to the contribution of stiffness from the deck structure to the first mode in the longitudinal direction; in the current CLMM, the deck structure is assumed to be a rigid body[8]. **Figure 5** shows the axial strain distribution in the first mode in the longitudinal direction for SC_2 and CC_3. These two models are, respectively, the best and the worst, in comparison of the frequency with that of the frame models. It is clear that the deck structure of CC_3 generates more axial strain than that of SC_2, which induces stiffer responses for the first mode in the longitudinal direction. When the target structure becomes complicated such as case CC_3, the current CLMM cannot perform well; we need to improve it in the future by considering a deformable deck structure.

Natural frequencies of the CMSMs in the longitudinal directions is presented in **Table 3**; the natural frequencies of the original solid element models are presented, too. As is seen, the natural frequencies of the CMSMs coincide with those of the solid element models, as expected. Next, first three natural frequencies of the frame model and the solid element model in the longitudinal directions is also presented in **Table 4**. It is seen that there is good agreement of the natural frequencies between solid element and frame element model in the longitudinal direction too.

After the frequency analysis, we conduct time history analyses for the longitudinal direction of each bridge structure. Responses of the CLMMs, CMSMs and frame models are compared with those of the solid element model; see **Figs. 6, 7** and **8** for the case of SC_2 and CC_3, respectively. It is seen that the responses of the CMSM and frame model matches well with those of the solid element model, but that the response of CLMMs does not match well except the case of SC_2. **Figures 9(a)** and **(b)** show input ground motion in the frequency domain and the natural frequency of each model of the case of SC_2 and CC_3 are designated. The natural frequency of the CLMM of CC_3 shifts to the peak amplitude range of the input ground motion, which causes a larger difference in displacement response; see **Figs. 8(b)** and **9(b)**. The natural frequencies of all the modes of the case of SC_2, which is the simplest bridge structure, coincide with each other; see **Fig. 9(a)**. Relative errors of the maximum displacement in the longitudinal direction of the each model are presented in **Table 5**. As is seen, the maximum error is 16.049% for CLMMs of the case of CC_3.

We need to choose a suitable consistent model of low fidelity for the quality assurance of a model of higher fidelity (solid element model) that is constructed in an automated manner. The model of low

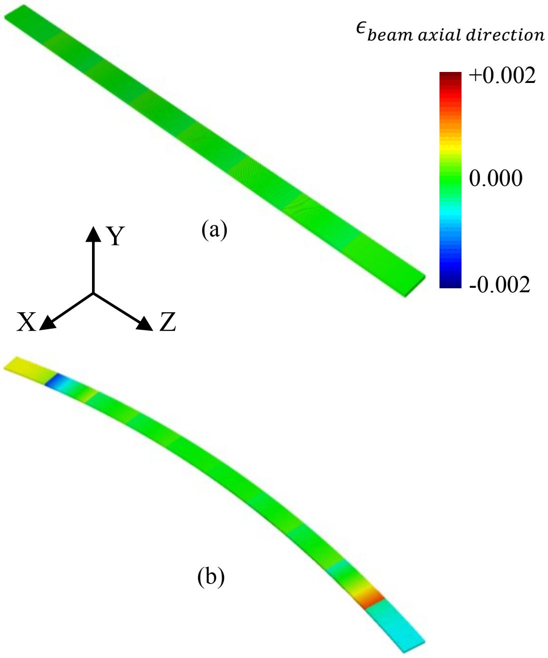


Fig. 5 Axial strain contours plot of deck structure (frame element model) for first mode along longitudinal direction of bridge: (a) SC_2; and (b) CC_3.

Table 2 Natural frequency of multi-span bridge structure (CLMM and frame element models) along longitudinal direction.

Case ID	Frequency / (Hz)		Diff. / (%)
	CLMM	Frame	
SC_1	0.614	0.623	1.444
SC_2	0.665	0.673	1.189
SC_3	1.504	1.620	7.160
CC_1	0.628	0.690	8.985
CC_2	0.656	0.711	7.735
CC_3	1.398	1.610	13.167

Table 3 Natural frequency of multi-span bridge structure (CMSM and solid element models) along longitudinal direction

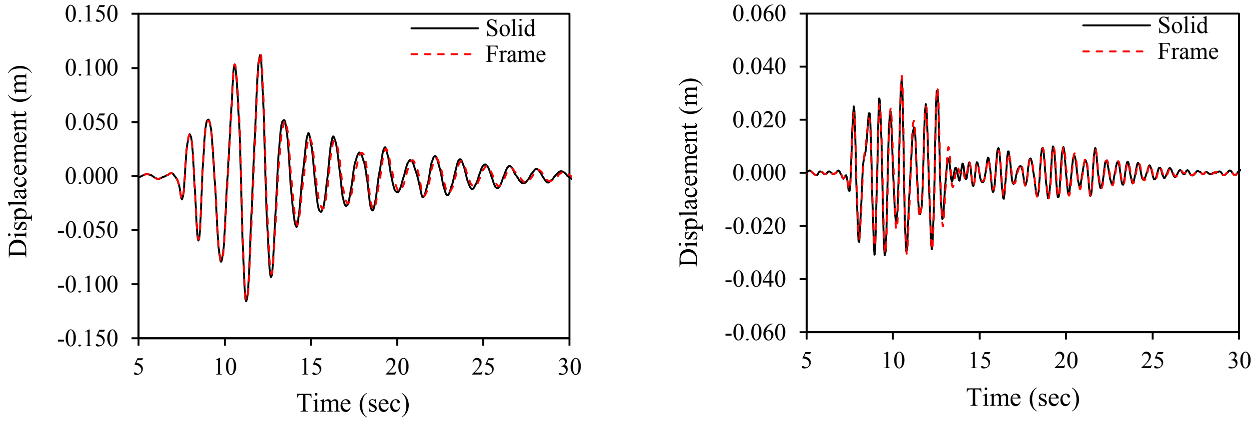
Case ID	Frequency / (Hz)		Diff. / (%)
	CMSM	Solid	
SC_1	0.630	0.630	0.000
SC_2	0.680	0.680	0.000
SC_3	1.640	1.640	0.000
CC_1	0.713	0.712	0.140
CC_2	0.730	0.730	0.000
CC_3	1.624	1.623	0.062

Table 4 Natural frequency of multi-span bridge structure (frame and solid element models) along longitudinal direction.

Case ID	Mode ID	Frequency / (Hz)		Diff. / (%)
		Frame	Solid	
SC_1	1	0.623	0.630	1.111
	2	3.691	3.750	1.573
	3	3.810	3.880	1.804
SC_2	1	0.673	0.680	1.029
	2	3.717	3.785	1.796
	3	3.828	3.902	1.896
SC_3	1	1.620	1.640	1.220
	2	3.789	3.884	2.446
	3	3.985	4.088	2.520
CC_1	1	0.690	0.712	3.090
	2	3.720	3.699	0.568
	3	3.826	3.896	1.797
CC_2	1	0.711	0.730	2.602
	2	3.736	3.734	0.054
	3	3.837	3.907	1.792
CC_3	1	1.610	1.623	0.801
	2	3.783	3.752	0.826
	3	3.950	4.056	2.613

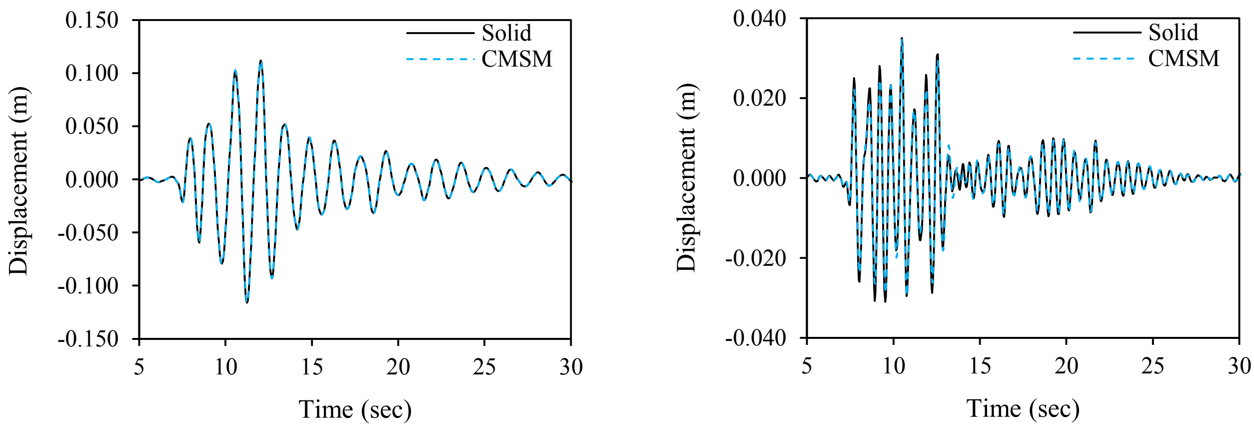
Diff - Difference

fidelity is used as a reference, in examining the response of the target structure of complex configuration. In this study, a pair of a CMSM and a frame model are for such quality assurance of the automated solid element model for all the six bridge structures; the responses in the longitudinal direction are used.



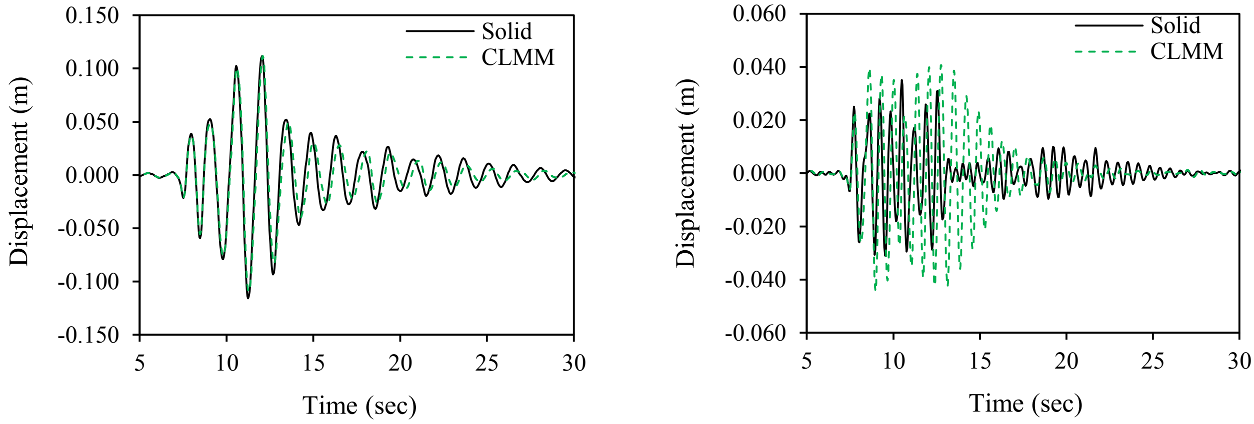
(a) (b)

Fig. 6 Displacement results of deck structure (solid and frame element models): (a) SC_1; and (b) CC_3



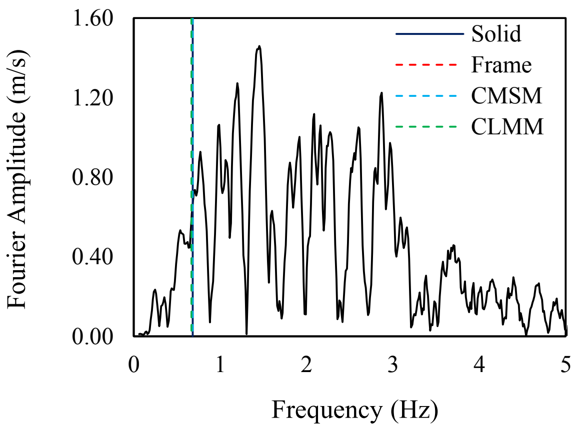
(a) (b)

Fig. 7 Displacement results of deck structure (solid element model and CMSM): (a) SC_1; and (b) CC_3

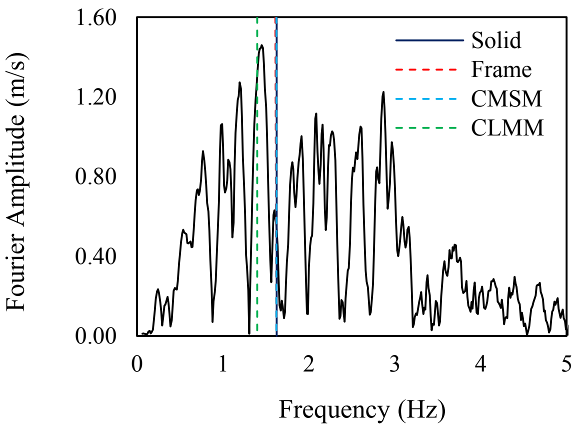


(a) (b)

Fig. 8 Displacement results of deck structure (solid element model and CLMM): (a) SC_1; and (b) CC_3



(a)



(b)

Fig. 9 Input ground motion in frequency domain with first natural frequency of each model along longitudinal direction: (a) SC_2; and (b) CC_3.

Table 5 Relative difference for maximum displacement between solid and other models along longitudinal direction.

Case ID	Difference / (%)		
	CLMM	CMSM	Frame
SC_1	5.945	1.027	1.708
SC_2	0.204	1.145	1.792
SC_3	14.797	1.403	4.234
CC_1	5.059	2.230	4.365
CC_2	4.457	1.313	2.770
CC_3	16.049	1.953	3.875

5. CONCLUDING REMARKS

This paper presents a quality assurance of an automated high fidelity model (or solid element model) by employing consistent low fidelity models. The consistency is assured according to the meta-modeling theory. The developed modules of the automated model construction are able to use two sets of digital data (Auto CAD and GIS) of a target bridge structure and to generate a solid element model, a frame model, a CMSM or a CLMM. The quality assurance process of automated solid element model is tested with a set of multi-span bridge structures successfully in this study. The quality of automated solid element model is assured by comparing the fundamental properties and the synthesized response of low fidelity models of a target bridge structure.

The current CLMM needs to be improved by introducing a deformable deck structure in its formulation, which will give complete consistency with the first mode of the frame model of a bridge structure for any arbitrary configuration. There is a possibility to improve the automated model construction modules by introducing other structure element models such as plate and shell. Also, there is a possibility of extending meta-modeling to non-linear analysis modeling. At least, it is straightforward to apply the meta-modeling theory to incremental response of a non-linear elasto-plastic structure.

APPENDIX A PIER HEIGHT INFORMATION OF CONTINUOUS BRIDGE MODELS IN NUMRICAL EXPRIMENT REFERENCES

Table a.1 Pier height data of straight bridge models (SC).

Pier ID	Height / m		
	SC_1	SC_2	SC_3
1	22.0	25.0	8.0
2	26.3	25.0	16.0
3	27.1	25.0	24.0
4	26.3	25.0	32.0
5	26.1	25.0	24.0
6	30.2	25.0	16.0
7	30.1	25.0	8.0

Table a.2 Pier height data of curved bridge models (CC).

Pier ID	Height / m		
	CC_1	CC_2	CC_3
1	22	25.0	8.0
2	26.3	25.0	14.0
3	27.1	25.0	20.0
4	26.3	25.0	26.0
5	26.1	25.0	32.0
6	30.2	25.0	26.0
7	30.1	25.0	20.0
8	26.3	25.0	14.0
9	22.0	25.0	8.0

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