Automated Construction of Consistent Lumped Mass Model for Road Network

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This paper presents automated construction of a lumped mass model which is consistent with a continuum mechanics model for a road network. GIS data conversion is first introduced to estimate parameters for the network, which include configuration of the road network. A set of hierarchy objects are then introduced to construct a lumped mass model. The applicability of the automatically constructed lumped mass model to seismic response analysis is studied, and it is shown that drift ratio is computable by using the model in a systematic manner. A possibility of extending the automated model construction methodology to more sophisticated models is discussed.

Key Words: *lumped mass model, continuum mechanics, structural mechanics, bridge, GIS data*

1. INTRODUCTION

For a more reliable estimation of possible damage induced by an earthquake, we need to improve the accuracy of seismic response analysis using a model of higher fidelity; with the progress of computers, large scale computation that is needed for the analysis of a high fidelity model can be conducted. It is modeling that hinges this analysis, since the number of target structures are huge. The development of automated model construction i.e., a data conversion of the available digital data of a target structure to an analysis model which can be directly inputted to a suitable seismic response analysis method, is thus required.

The quality of an analysis model that is produced by the automated model construction must be examined. In general, there is a trade-off relation between the accuracy and the complexity of the model. A more complicated model produces a more accurate estimate, but it ought to be difficult to examine the quality of all model components. In developing automated model construction, we have to pay full attention to this trade-off relation for the model quality.

The authors propose a methodology of automated construction which accounts for the model quality. The key point of the proposed methodology is to construct a set of consistent models for one structure. The consistency means to solve the same problem of the target structure response. Suitable mathematical approximations are made, so that approximate solutions of different accuracy are obtained for each of the consistent models. The model quality is more easily examined for a simpler consistent model. Comparing the analysis results with the simpler model, the quality of which is examined, we are able to examine the quality of a more complicated model. By repeating this comparison, we will realize the quality check of a most complicated model of a target structure.

Meta-modeling[1)] theory is established as a theoretical foundation of the methodology of constructing a consistent model set. This theory starts from continuum mechanics and identifies the target problem of structure response in the form of a Lagrangian. A subset of continuum mechanics function space is used to choose suitable approximate functions, and the resulting Lagrangian produces a consistent model. Meta-modeling theory actually proves that a shear beam model is consistent with a continuum mechanics model; a Bernoulli-Euler beam model at quasi-static state is consistent but one at dynamic state is not consistent[1).

This paper is aimed at developing an automated model construction for a consistent lumped mass model, which is the fundamental model for the seismic response analysis[2), 3)]. A road network, which consists of numerous bridges, is chosen as a target structure. The contents of this paper are as follows. First, the concept of meta-modeling is briefly explained in Section 2. According to meta-modeling, we clarify an approximation which is made in deriving governing equation for lumped mass system from continuum mechanics theory in Section 3. Data conversion from GIS data with practical example and an automated construction of consistent lumped mass model with proper example are discussed in Section 4. We carry out numerical experiments of applying the developed consistent lumped mass bridge model under selected ground motion in Section 5. Concluding remarks are made in Section 6.

2. META-MODELING

In meta-modeling, modeling means to pose a mathematical problem. For a common physical problem, there are many ways to pose a different mathematical problem, depending on the accuracy required. Meta-modeling specifies a set of modeling (or mathematical problem) which produces an approximate solution of the most sophisticated modeling. For structural problems, meta-model uses continuum mechanics problem as the most sophisticated modeling. Some structure mechanics modeling are specified as a consistent modeling of this continuum mechanics modeling. It should be emphasized that a consistent structure mechanics modeling produces an approximate solution of the continuum mechanics modeling.

The continuum mechanics modeling is formulated as a Lagrangian problem. This Lagrangian is slightly different from the standard one; the Lagrangian consists of a standard kinetic energy term and a potential term which includes a complementary strain energy. For simplicity, assuming linear elasticity, we can write

$$\boldsymbol{\mathcal{L}}[\mathbf{v},\boldsymbol{\epsilon},\boldsymbol{\sigma}] = \int_V \frac{1}{2}\rho\mathbf{v}.\mathbf{v} - \left(\boldsymbol{\sigma}:\boldsymbol{\epsilon} - \frac{1}{2}\boldsymbol{\sigma}:\mathbf{c}^{-1}:\boldsymbol{\sigma}\right) dv, \tag{1}$$

where ρ and $\boldsymbol{c}$ are density and elasticity. It is readily proved that this $\boldsymbol{\mathcal{L}}$ is equivalent with the standard Lagrangian, by substituting $\mathbf{v} = \dot{\mathbf{u}}$ and $\boldsymbol{\epsilon} = sym\{\boldsymbol{\nabla}\mathbf{u}\}$, where $\dot{()}$ and $\boldsymbol{\nabla}()$ stand for temporal and spatial differentiation for a function $()$.

Without making an additional assumption, such as one-dimensional stress-strain relation4) of $\sigma = E\epsilon$ with σ and ϵ being normal stress and strain components and E being Young's modulus, we can derive a governing equation for a beam problem or a plate problem. The derivation needs the selection of a subset of the continuum mechanics function space of $\{\mathbf{u},\boldsymbol{\sigma}\}$ (not only for $\mathbf{u}$). A distinct initial boundary value problem is derived in stationarizing the Lagrangian. This mathematical problem (or modeling) is consistent with the continuum mechanics' three-dimensional initial boundary value problem (or continuum modeling).

3. CONSISTENT LUMPED MASS MODELING

According to meta-modeling explained above, a lumped mass model which is consistent with a continuum mechanics model is obtained by substituting approximate functions of displacement to the modified Lagrangian. As a simple example, we consider a pier of a road network. We regard the pier as a cantilever of span L, and choose approximate functions that corresponds to the beam at quasi-static state. Functions which correspond to the beam at dynamic state can be used as other approximate functions; see Appendix A.

According to meta-modeling, the approximate displacement functions of the following form are used:

$$\{u_1, u_2, u_3\} = U(t)\{-zw'(x), 0, w(x)\}, \tag{2}$$

where w is a solution of the beam problem,

$$\begin{cases} (EIw'')'' = 0 & 0 < x < L, \\ (w,w') = (0,0) & x = 0, \\ (w,w'') = (1,0) & x = L. \end{cases} \tag{3}$$

Note that the coordinate x is chosen in the vertical direction and, for simplicity, z is in the direction perpendicular to the bridge; E is Young's modulus and I is the second moment of inertia with respect to the z direction. This w is fully determined since Eq. (3) has a unique solution. Substituting $\mathbf{u}$ of Eq. (2) and the corresponding $\boldsymbol{\sigma}$, we can compute the modified Lagrangian as

$$\boldsymbol{\mathcal{L}} = \frac{1}{2}M\dot{U}^2(t) - \frac{1}{2}KU^2(t), \tag{4}$$

where

$$\{M,K\} = \int \{\rho(Aw^2 + I(w')^2), EI(w'')^2\}\, dx \tag{5}$$

with A being the cross-sectional area of the pier.

It should be emphasized that, while $\int \rho A w^2\, dx$ is usually used as a mass of the pier, meta-modeling yields $\int \rho I (w')^2\, dx$ as additional mass, which accounts for the effect of angular momentum. While Bernoulli-Euler beam modeling neglects it, shear beam modeling includes the angular momentum effect. Therefore, the lumped mass modeling is given by the Lagrangian of Eq. (4) is in accordance with the shear modeling that is known to be more accurate than Bernoulli-Euler beam modeling.

Beside for translation, we can readily include rotation in the consistent lumped mass modeling. We use other approximate displacement functions,

$$\{u_1, u_2, u_3\} = \Theta(t)\{-zh'(x), 0, h(x)\}, \tag{6}$$

where h is a solution of the other beam problem,

$$\begin{cases} (EIh'')'' = 0 & 0 < x < L, \\ (h, h') = (0,0) & x = 0, \\ (h', h''') = (1,0) & x = L. \end{cases} \tag{7}$$

The modified Lagrangian is computed in the same form as Eq. (4) if $\{M, K\}$ of Eq. (5) are computed by using h instead of w.

The modified Lagrangian for coupling of translation and rotation is readily obtained by substituting the sum of $\mathbf{u}$ given by Eqs. (2) and (6). That is,

$$\boldsymbol{\mathcal{L}} = \frac{1}{2}M_U\dot{U}^2(t) - \frac{1}{2}K_U U^2(t) + \frac{1}{2}M_\Theta\dot{\Theta}^2(t) - \frac{1}{2}K_\Theta\Theta^2(t) + M_{U\Theta}\dot{U}(t)\dot{\Theta}(t) - K_{U\Theta}U(t)\Theta(t), \tag{8}$$

where $\{M_U, K_U\}$ and $\{M_\Theta, K_\Theta\}$ are given by Eq. (5) using w and h, respectively, and

$$\{M_{U\Theta}, K_{U\Theta}\} = \int \{\rho(Awh + Iw'h'), EIw''h''\}\, dx \tag{9}$$

As it is seen, the coupling between translation and rotation naturally appears via $\{M_{U\Theta}, K_{U\Theta}\}$ of Eq. (9).

In substituting the sum of $\mathbf{u}$ given by Eqs. (2) and (6), we have to pay attention to coupling of translation and rotation. For instance, at $x = L$, we have

$$u_3 = U + \Theta h(L), \quad \Theta\left(\frac{\partial u_3}{\partial x_1}\right) = Uw'(L) + \Theta;$$

recall, $w(L) = 1$ and $h(L) = 1$. $\{u_3, \theta\}$ that can be measured at site. Therefore, the measured data should be compared with $\{U + \Theta h(L)\}$, $\{Uw'(L) + \Theta\}$ rather than $\{U, \Theta\}$.

4. DATA CONVERSION

Automated modeling is required for a large network of highway that runs a few kilometers in length and includes many bridge structures, when digital data of such a road network is available. Accuracy is of primary importance in constructing a model of high fidelity by automated modeling. Meta-modeling enables us to construct a sequence of modeling that lead to higher fidelity. The initial modeling is to construct a lumped mass model, which is the simplest as it consists of only two parameters as explained in the preceding section.

(1) Decoding of GIS data

The following two types of GIS data are used for automated modeling: 1) 2D GIS data that include 2D polygon data about a road network; and 2) 3D GIS data of ground surface elevation. Information about how polygons are connected to form a structure is not included in the 2D GIS data. Thus, the 3D GIS data and the attribute tables in the 2D GIS data are used to guess the connection of neighboring polygons, so that the configuration of a structure is identified.

Decoding programs are developed for these GIS data. They create a separate file which includes a set of connected polygons for a particular segment of a road network. The segment is specified according to class information of the road network; there are four main classes as shown in Table 1. Fig.1 presents a typical example of the segment that is created by the decoding classes. As it is seen, there are no flaws in the segment. Detailed manual inspection is made to examine this segment by comparing it with photos provided by Google Earth.

The decoded GIS data need to be interpreted in order to construct an analysis model. As an example, the interpretation of the configuration and elevation of the center line of the road network is presented. First, the plane configuration of the line is interpreted by using the 2D GIS data, as follows:

1. Convert vector data of polygons to raster data.
2. Apply thinning to identify the center line[5].
3. Prune line segment shorter than 10m in each junction.
4. Separate line segments at each junction, removing complexity at junctions.

Next, the elevation of the center line is estimated by using the difference between the profile and the terrain data that are stored in the 3D GIS data. Noises of estimated elevation data are removed.

In Fig.2, we present an example of the decoding procedures explained above. The target is a segment of a highway road network, the most part of which is bridges, though the tunnel sub-class is excluded; see Fig.2(a). The ramp part of the segment, the configuration of which changes in a short distance, is removed due to the poor resolution of the profile elevation data which are available for this area; see Fig.2(b).

Table 1 Main and sub classes of road network.

Main Class	Sub Class
Highway	Tunnel
National road	Intersection
General road	Surface
Main local road	

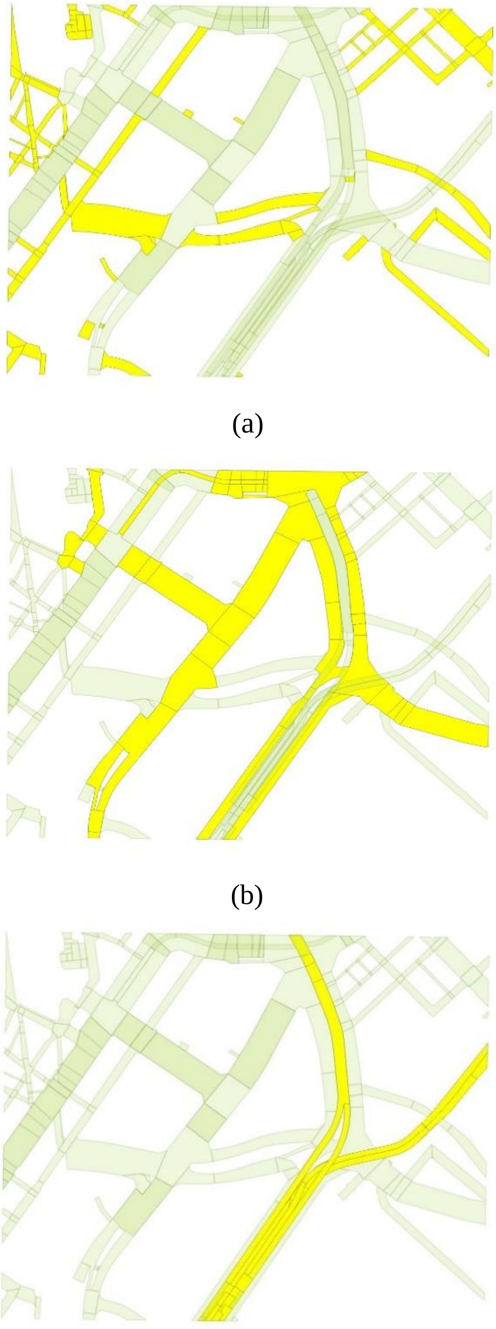

Fig.1 Example of decoding of GIS data to identify network configuration: (a) decoded configuration; (b) extraction of main local road; and (c) extraction of highway.

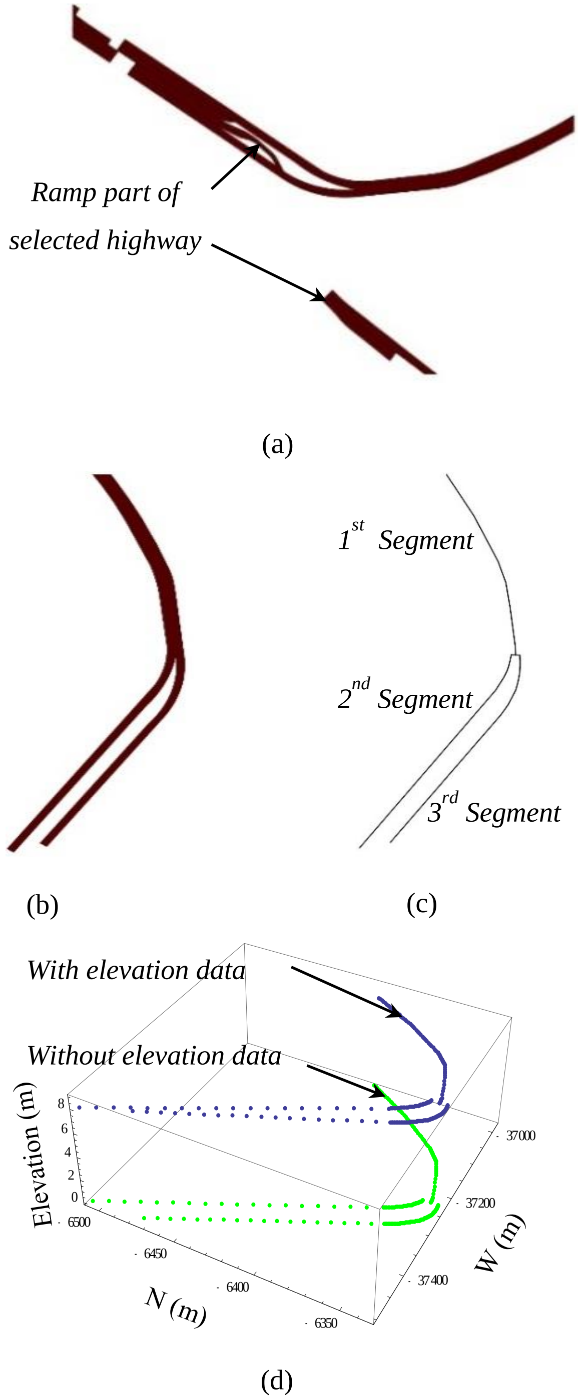

Fig.2 Example of applying decoding procedures to GIS data: (a) exclusion of highway main-class tunnel sub-class; (b) exclusion of highway main-class tunnel and ramp part; (c) extraction of 2D centre line arrangement; and (d) arrangement of 3D centre line.

(2) Automated construction of consistent lumped mass model

As explained in the preceding section, a lumped mass model requires two parameters, namely, equivalent mass and stiffness coefficient2),3). According to meta-modeling, lumped mass modeling is to use an approximate displacement function of the form of

Eq. (2); recall that U is an unknown function. The modified Lagrangian yields an ordinary differential equation for U, and the mass and stiffness are computed by using w.

As for the bridge structure, we use quasi-static beam theory, assuming that the pier provides stiffness and the deck moves like a rigid body, to determine w. That is, setting the x_1- and x_2- axes parallel and normal to the bridge direction, respectively, and x_3- axis as the vertical direction, an approximate function is

$$\{u_1, u_2, u_3\} = \begin{cases} U(t)\{-xw'(z), 0, w(z)\} & 0 < x < H, \\ U(t)\{-xw'(H), 0, w(H)\} & H < z < H+T \end{cases}$$

where $x = x_1$ and $z = x_3$; $0 < x < H$ is for a pier and $H < z < H+T$ is for a deck. Note that the displacement function of the above ensures that the deck moves as a rigid body. Posing suitable boundary conditions, we can determine w. For instance, we choose

$$w(0) = 0, w'(0) = 0, w(H) = 1, w'(H) = 0.$$

Here, setting $w(H) = 1$ means that U is the displacement of the pier top as well as the movement of the whole deck.

The approximate function w, which is selected according to quasi-static beam theory, is fully determined by solving the boundary value problem. When it is given, the modified Lagrangian becomes

$$\mathcal{L} = \frac{1}{2}M\dot{U}^2(t) - \frac{1}{2}KU^2(t), \qquad (10)$$

where

$$M = M_d + \int_0^H \rho(Aw^2 + I(w')^2), \qquad (11)$$

$$K = \int_0^H EI(w'')^2 dz, \qquad (12)$$

with A and I being the cross section area and the second moment of inertia of pier, respectively. M_d is the mass lumped of the deck.

The task needed for the automated construction of lumped mass model is the automated computation of M and K by using Eqs. (11) and (12) to the decoded and interpreted data of the target structure; for instance, M_d is computed from the volume calculation of the solid CAD model of the deck.

In the same manner, we can construct a lumped mass model in the transverse direction. An approximate functions are

$$\{u_1, u_2, u_3\} = \begin{cases} U(t)\{-yw'(z), 0, w(z)\} & 0 < x < H, \\ U(t)\{-yw'(H), 0, w(H)\} & H < z < H+T. \end{cases}$$

5. NUMERICAL EXPERIMENTS

A consistent lumped mass model is automatically constructed for the first segment of the road network; see Fig.2(c). That is, the value of M and K are computed. As an example for selected pier in Fig.4(b),

$$M = 314577\ kg, \qquad K = 1.458 \times 10^8\ Nm,$$

for both the longitudinal and transverse directions; no coupling is considered in the present model.

The details in computing these value are itemized as follows:

1. The interval of piers is fixed as 20m along the center line and; see Fig.3; see also Fig. 4 for the frame and solid CAD models of this segment.
2. A steel girder bridge deck is used to estimate the mass of deck for this experiment.
3. Pin connection is used for the connection between the deck and the pier, to satisfy the posed boundary conditions.

These details are schematically presented in Fig. 5, and the material properties used are summarized in Table 2. The total mass of the deck in each span is lumped to the pin connected frame in both the longitudinal and transverse directions, so that the segment is analyzed individually or fully ignoring coupling of neighboring segments.

Seismic response is computed for each frame in the both longitudinal and transverse directions. The input surface ground motion is shown in Fig.6; this 1D motion is applied to transverse direction of bridge (Fig.4(a)). The maximum drift ratio of the pier is used as an index of the seismic response[6),7)], and they are plotted in Figs 7 and 8 for the longitudinal and transverse directions, respectively.

The quality of the constructed model ought is to be examined. Manual computation of the two parameters, M and K, coincide with those automatically computed. The natural frequency for selected pier (Fig.4(b)), denoted by f, is easily computed, as

$$f = 3.426\ Hz.$$

We can check the accuracy of the natural frequency by comparing the observed value of the natural frequency, if some data are available.

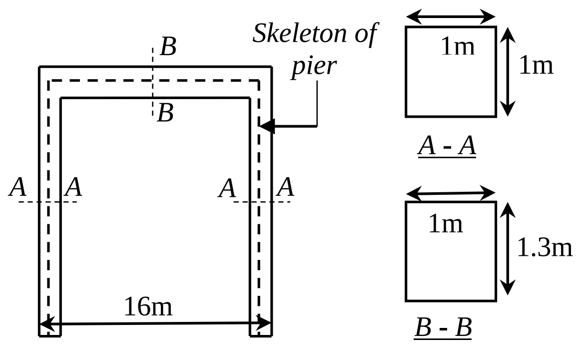

Fig.3 Assumed geometry of piers.

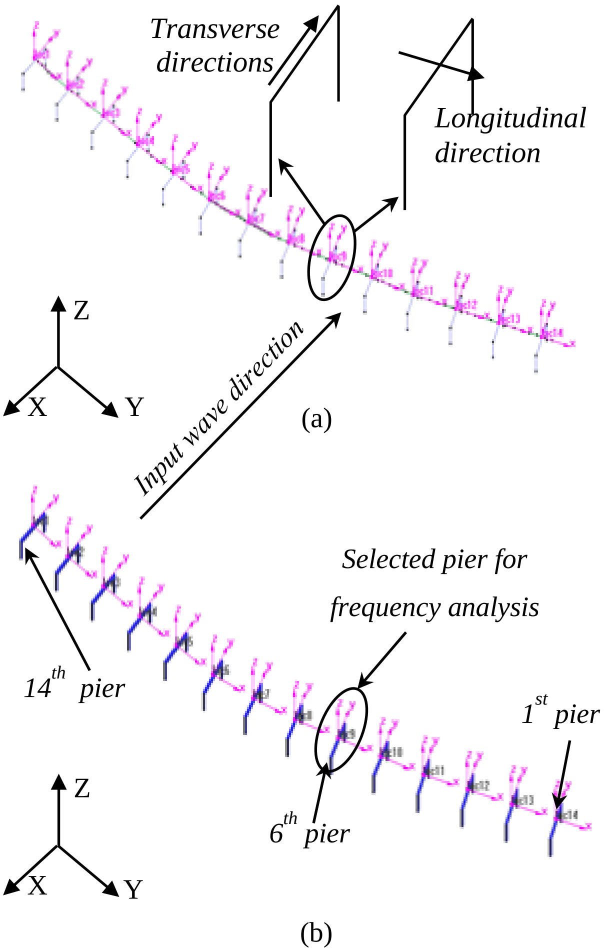

Fig.4 CAD model for selected highway segment: (a) frame CAD model; and (b) solid CAD model

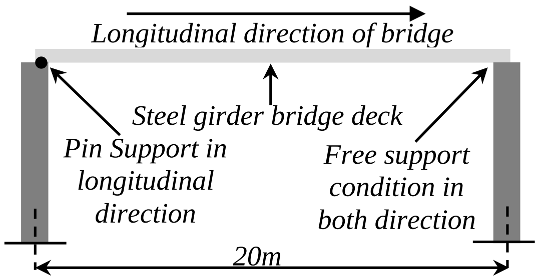

Fig.3 End condition of selected span of target highway

The automated model construction can be extended to a larger part of the road network. Actually, it would be a straightforward task, since the programs of decoding and interpreting the GIS data and constructing a lumped mass model are already developed; the decoding and interpretation programs need to be improved, in order to be applied to segments of a more complicated configuration, such as a ramp. The methodology of the automated model construction can be extended from the simplest lumped mass model to a more sophisticated model; eventual goal is a full solid element model for non-linear finite element analysis.

Table 2 Martial properties and lumped mass information.

Density of pier (Concrete)	$2400 kg/m^3$
Density of deck (Steel)	$7800 kg/m^3$
Young's modulus (Concrete)	$25 GPa$
Young's modulus (Steel)	$200 GPa$
Damping ratio (Concrete structures)	5%

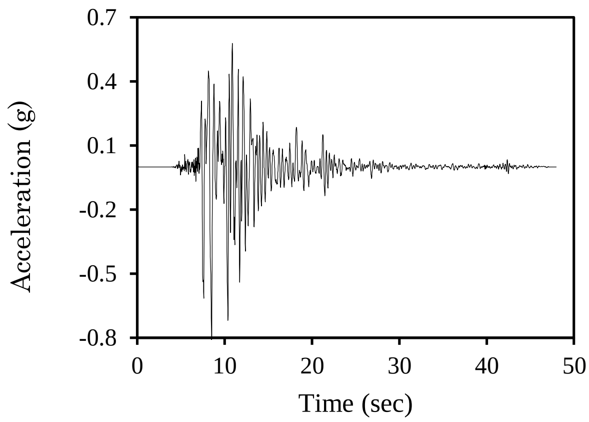

Fig.6 Applied ground motion to the transverse direction.

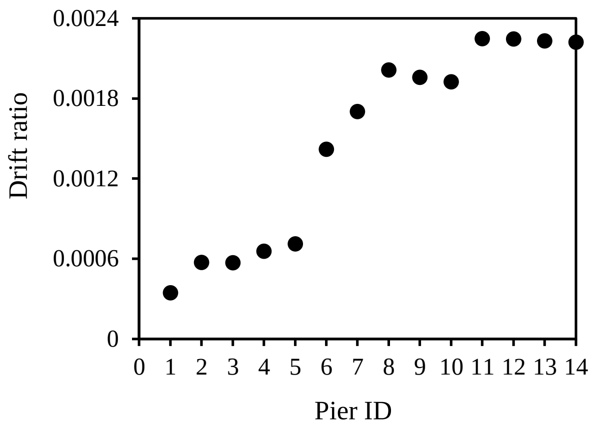

Fig.7 Drift ratio in each pier along local X-axis of pier.

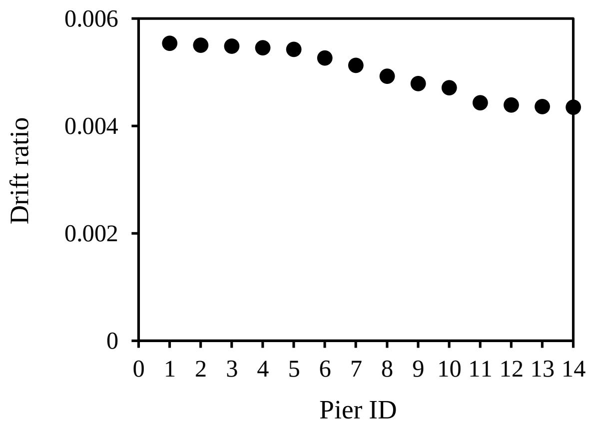

Fig.8 Drift ratio in each pier along local Y-axis of pier.

6. CONCLUDING REMARKS

This paper presents the automated construction of a consistent lumped mass model for a road network. It is shown that meta-modeling could be used to construct a lumped mass model in a systematic manner. It is also shown that the choice of approximated functions is a key issue to make an accurate model.

The GIS data conversion and interpretation are employed to estimate the global parameters of a road network, namely, center line and elevation data of the road network. An example of the data conversion and interpretation demonstrates the usefulness of the developed programs. The interpreted data are used to automatically construct a set of lumped mass models for the example. It is shown that these models are used to conduct seismic response analysis.

This automated model construction will be extended to a model of higher fidelity. An accurate modeling methodology of connection between deck and pier and a robust method to estimate each geometric detail of the bridge components are needed. When a large road network is analyzed, we have to consider input ground motion since each pier may have distinct input due to the local topographical effects.

APPENDIX A USE OF MODAL ANALYSIS FOR LUMPED MASS MODELING

Approximate displacement functions that are used to construct consistent lumped mass modeling are found by using modal analysis. Modal analysis means solving the following eigen-value problem of w:

$$\begin{cases} \omega^2 Aw + (EIw'')'' = 0 & 0 < x < L, \\ (w, w') = (0,0) & x = 0, \\ (w, w'') = (1,0) & x = L. \end{cases} \tag{a.1}$$

Here, ω is the natural frequency of the cantilever subjected to translation boundary conditions. Another modal analysis is made for the rotation boundary conditions, i.e.,

$$\begin{cases} \omega^2 Ah + (EIwh'')'' = 0 & 0 < x < L, \\ (h, h') = (0,0) & x = 0, \\ (h, h''') = (1,0) & x = L. \end{cases} \tag{a.2}$$

The same symbols, w and h, are used as those at quasi-static state; see Eqs. (3) and (7).

For each eigen-value of ω, the corresponding eigen-function for w or h is determined. The approximate displacement function is expressed in terms of such eigen-functions as

$$\begin{aligned} \mathrm{u}_3(x,t) &= \sum_\alpha U^\alpha(t) w^\alpha(x) \\ &\quad + \sum_\beta \Theta^\beta(t) h^\beta(x), \end{aligned} \tag{a.3}$$

where w^α and h^β are the α^{th} and β^{th} eigen-functions for w and h.

Substitution of u_3 with corresponding $\mathrm{u}_1 = -zu_3'$ into the Lagrangian leads to the modified Lagrangian. For simplicity, replacing the symbol $\{\Theta^\beta, h^\alpha\}$ with $\{U^{N+\beta}, w^{N+\beta}\}$ with N being the number of the modes used, we rewrite Eq. (a.3) as

$$u_3(x,t) = \sum_\beta U^\alpha(t) w^\alpha(x). \tag{a.4}$$

Substituting this u_3 into the Lagrangian, we have

$$\begin{aligned} \mathcal{L} &= \sum \frac{1}{2} M_{\alpha\alpha} \left(\dot{U}^\alpha\right)^2(t) - \frac{1}{2} K_{\alpha\alpha} (U^\alpha)^2(t) \\ &\quad + \sum \frac{1}{2} M_{\alpha\beta} \dot{U}^\alpha(t) \dot{U}^\beta(t) \\ &\quad - \frac{1}{2} K_{\alpha\beta} U^\alpha(t) U^\beta(t), \end{aligned} \tag{a.5}$$

where

$$\begin{aligned} &\{M_{\alpha\beta}, K_{\alpha\beta}\} \\ &= \int \left\{\rho \left(A w^\alpha w^\beta \right.\right. \\ &\left.\left. + I (w^\alpha)' \left(w^\beta\right)'\right), EI (w^\alpha)'' \left(w^\beta\right)''\right\} \, dx \end{aligned} \tag{a.6}$$

Note that the integration of $w^\alpha w^\beta$, $(w^\alpha)'\left(w^\beta\right)'$ and $(w^\alpha)''\left(w^\beta\right)''$ does not vanish for $\alpha \neq \beta$, and hence coupling of different U^α 's always happens.

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