Discussion on the availability of combining SPAC method and seismic interferometry in wavefield dominated by microtremors

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In the SPAC method, the underground structure is assumed to be horizontal layers, which confines the accuracy of the estimation. On the other hand, the seismic interferometry theory makes it possible to identify the differnce of ground structure between each two sites by taking the ratio of power spectra respectively. Hence, we propose a method combining the conventional SPAC method and the concept of Green's function, which is known as the seismic interferometry. In order to show the availability of this combination, wavefield dominated by microtremors is simulated in which the SPAC method could be conducted effectively. Then the availability of seismic interferometry proposed previously is examined under the simulated wavefield and the need to take the ratio of the power spectra instead of using the power spectra itself directly is confirmed.

Key Words: *Green's function, SPAC method, cross spectrum, ground structure, layered medium*

1. INTRODUCTION

Since Aki[1] proposed a new approach to estimate phase velocities of surface waves, spatial auto-correlation (SPAC) method has been a very useful tool to estimate ground structure because of its simple post-process. After that, many reseachers both in and out of Japan continued to publish papers on practical adaption of Aki's theory to microtremor exploration. However, in all those improved methods, the layers under surface can only be assumed to be horizontal through the SPAC method while in fact, the layers are likely to be inclined slightly with certain angle. Hence, it is expected to obtain more detailed information of ground structure such as inclination by making better use of the records.

In recent years, the seismic interferometry theory[2][3] has also been widely used to estimate ground structure. It is proved that in an elastic medium the Fourier transform of azimuthal average of the cross correlation of motion between two sites is proportional to the imaginary part of the exact Green's function between these sites[4]. Hence, it becomes possible to calculate the ratio of imaginary part of different Green's function by taking the ratio of corresponding cross correlation to analyze ground structure more particularly because Green's function indicates intrinsic property of the medium. Actually, seismic interferometry is conditionally consistent with the SPAC method[5] which offers the base of introducing seismic interferometry to SPAC method.

SPAC method requires the multiplification calculation of Fourier transformation of records at two sites of center of an array and a one site on the circular array. By taking the ratio of power spectral between two different sites, it is hoped to obtain the ratio of imaginary part of Green's function according to seismic interferometry theory correspondingly and analyze the difference of ground structure through the ratio. More information such as the inclination of layers could be obtained.

Since this new concept has been proposed[7][8], some problems has been pointed out and the availablity of the combination remains to be proved. Firstly, the ratio of power spectra is used to calculate the ratio of imaginary part of Green's function whicn means the wavefield is supposed to consist of mainly body wave[9] [10]. However, the SPAC method requires the wavefield to be dominated by microtremors. It seems to be paradox but it is believed that seismic interferometry theory itself satisfies wavefield of full wave[11]. It is hoped that by taking the ratio of

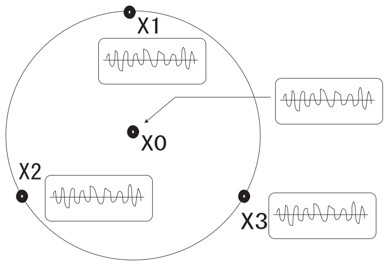

Figure 1. The equilateral-triangle array

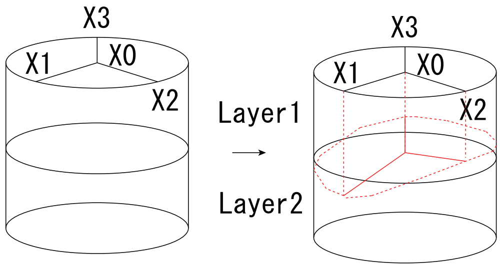

Figure 2. An example describing how the new method is expected to raise the accuracy of estimating ground structure.

power spectra between two sites, the surface wave content will be extinguished and the body wave content remains. Secondly, under the assumption of body wave being dominating, it is said that power spectra itself of each site could be used to analyze out the peak frequency of the ground structure (in simple case, the first layer) which tend to say that there is no need to take the ratio of them. Nevertheless, in wavefield dominated by microtremor and with the inclination of layers small enough, it is hard to extract useful information from each power spectra alone and to compare between them.

In this paper, the concept of SPAC method, interferometry and the combination of them are firstly proposed comprehensively. Then, in order to solve the two problems mentioned above, we use finite-difference method to simulate some 2-layered simple layered medium under the wavefield dominated by microtrmors. Next, SPAC method is applied to certain array of observation sites to examine if this wavefield is effective for SPAC method. Finally, the availability of seismic interferometry would be analyzed and the need to take the ratio of power spectra will be shown.

2. COMBINATION OF SPAC METHOD & SEISMIC INTERFEROMETRY

In this chapter, the basic knowledge of SPAC method and seismic interferometry will be introduced and the new concept of combining them will be proposed.

(1) SPAC method

Given an equilateral-triangle array as shown in Fig.1 and vertical time series records at four sites, the azimuthal-average of spatial auto-correlation coefficients $\rho(\omega,r)$ can be calculated as:

$$\rho(\omega,r) = \frac{1}{3}\sum_{j=1}^{3}\frac{S_{0j}(\omega)}{S_{00}(\omega)S_{jj}(\omega)}, \tag{1}$$

where ω and r are the angular frequency and radius of the circular array, respectively. $S_{jj}(\omega)$ and $S_{0j}(\omega)$ are the power spectra of vertical component of microtremors at site j ($j = 0,1,2,3$) and the cross spectra of the vertical records between site j and 0. The site 0 is center of the array and sites 1, 2, and 3 are located on the circle.

Practically, $S_{jk}(\omega)$ is obtained by the product of Fourier transformation of records at two sites as

$$S_{jk}(\omega) = F(\omega,X_j)F^*(\omega,X_k), \tag{2}$$

where X_j is a location of site j and * stands for the complex conjugate.

Afterwards, a dispersion curve can be obtained and the property of under-surface layers are estimated using the curve.

(2) Green's function from correlation

It has been demonstrated[6][3] that in an elastic medium, the averaged cross correlation of motions at sites 1 and 2, whose locations are X_1 and X_2, can be written as:

$$\begin{aligned} &< F_\ell(\omega,X_1)F_m^*(\omega,X_2) > \\ &\quad = -2\pi E_s k^{-3}\Im[G_{\ell m}(X_1,X_2,\omega)], \end{aligned} \tag{3}$$

where ℓ and m indicate one of three directions, (x,y,z). k and E_s are wave number of shear wave and the averaged energy density of shear wave, respectively. $<\ \cdot\ >$ stands for an expectation and $\Im[\,\cdot\,]$ for the imaginary part.

(3) Combination of SPAC method and Green's function

In the simpliest case using an equilateral-triangle array, ℓ and m are both set as vertical direction, which is z, and sites 1 and 2 as the same sites. Hence, it becomes

$$\begin{aligned} &< F_z(\omega,X_1)F_z^*(\omega,X_1) > \\ &\quad = -2\pi E_s k^{-3}\Im[G_{zz}(X_1,X_1,\omega)] \end{aligned} \tag{4}$$

Therefore, let us take the ratio of the power spectra at center of the array and an azimuthal site so that the corresponding ratio of imaginary part of Green's function can be obtained as

$$\frac{S_{jj}(\omega)}{S_{00}(\omega)} = \frac{\Im[G_{zz}(X_j,X_j,\omega)]}{\Im[G_{zz}(X_0,X_0,\omega)]} \tag{5}$$

and it is expected to analyze the difference of ground structure between every two sites such as the inclination. The process is indicated simply in Fig.2.

Table 1 Two-layered models of ground structure

Layer	P-wave velocity	S-wave velocity	Density	Thickness
	[m/s]	[m/s]	[t/m^3]	[m]
1	800	400	1.5	120
2	1800	1200	2.0	∞

In the simplist equilateral-triangle case, three ratios $\frac{S_{11}(\omega)}{S_{00}(\omega)}$, $\frac{S_{22}(\omega)}{S_{00}(\omega)}$, $\frac{S_{33}(\omega)}{S_{00}(\omega)}$ according to Eqn.5 can be obtained to analyze the difference between the corresponding two sites. Moreover, it is convenient and efficient to calculate this ratio because the power spectra is just the intermediate result in the process of SPAC method.

3. The VALIDITY OF SEISMIC INTERFEROMETRY IN THE WAVEFIELD DOMINATED BY MICROTREMORS

Since this new concept has been proposed, some preliminary research in order to confirm the availability of it has been done[7][8]. Through a smulation examination by finite difference method, the most compelling conclusion is drawn that Eqn.(5) is well satified in case of shallow ground structure[8]. However, until now, the SPAC method has not been taken consideration into the examination yet. Eqn.(5) is proved to be valid in diffused wavefield which is not necessarily an available one for SPAC method.

Besides, in terms of the assumption of SPAC method and Eqn.(5), the SPAC method requires the wavefield dominated by microtremor while the retrieval of Green's function from power spectra demands body wave's domination[10][9]. The assumption seems to contradict with each other. Though, it is worth noticing that seismic interferometry itself is a full wave theory[11]. Though the power spectra should embody mainly surface wave, the ratio between them has the potential to contain information of body wave while the surface wave component diminishes through the division.

(1) A wavefield available for SPAC method

In order to show that SPAC method and seismic interferometry are both effective in the wavefield dominated by microtremor, firstly a wavefield available to apply SPAC method should be created. We still use Sakai's program[13] which is based on finite difference method as the tool to do simulation.

We assume the ground structure to be horizontally 2-layered medium, namely, no difference between each two sites. The inclined layered medium will be used afterward when adding the seismic interferometry theory. The property of each layer is shown in Table 1. To be ideal, we create the wavefield by exerting randomly distributed sources at random time surroundingly (as far away from the array

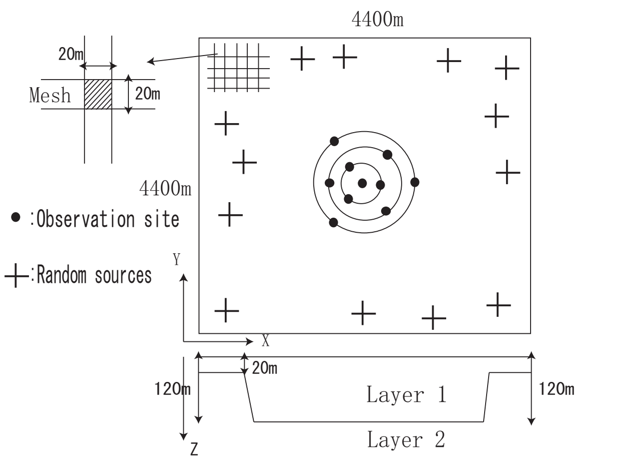

Figure 3. Plan and section plan of horizontally layered medium

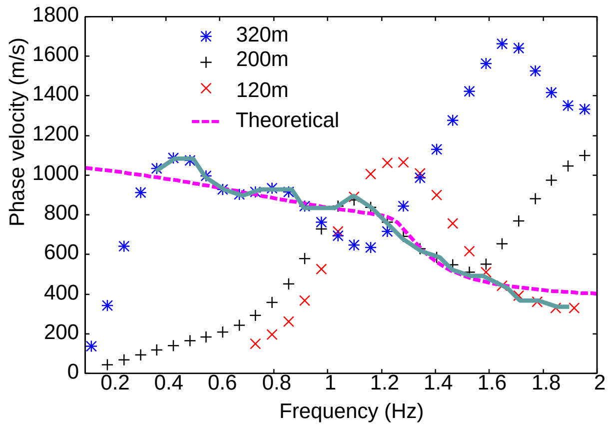

Figure 4. Dispersion curve

as possible). Each source is on the surface and with random force orientation. The source time function is a delta-like signal. The scope of simulation is $220 \times 220 \times 40$ with each mesh size of $20 \times 20 \times 20m$. The three observation arrays are set to be equilateral triangle with radius of 120*m*, 200*m* and 320*m* respectively. The plan and section plan are shown in Fig.3. The boderline of the first layer is set to be like a basin so as to help surface wave generate.

For simplicity, we use the vertical component of the record with duration of 10s and time step of 0.004s. Then the SPAC method metioned before is applied and the dispersion curve of 3 arrays is drawn as Fig.4 shows. The line is the theoretical phase velocity of rayleigh wave. The frequency range is limited because of the precision limit of Sakai's program[13] itself. We can see that the phase velocity disperses which means the wave field is

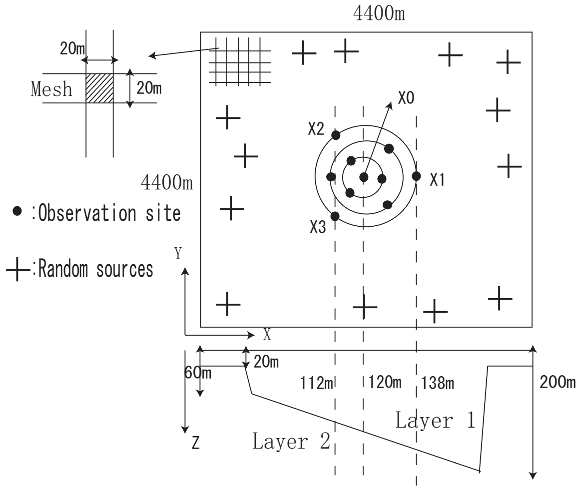

Figure 5. Plan and section plan of inclined layered medium

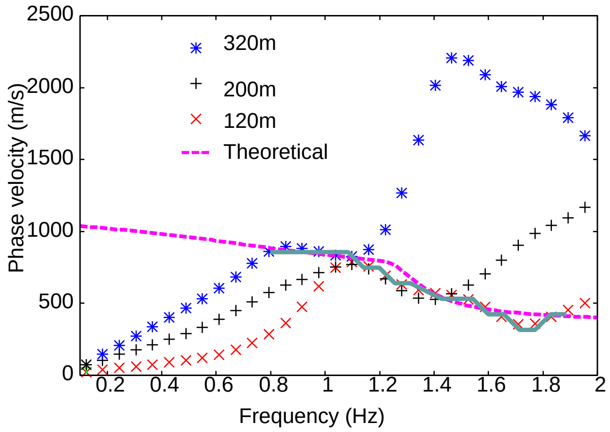

Figure 6. Dispersion curve

dominated by surface wave. Besides, the result from SPAC method coincides with theoretical one quite well which shows the wavefield is available to apply SPAC method.

(2) The availablity of seismic interferometry in the wavefield dominated by microtremor

As the way of creating wavefield available for SPAC method has been proved to be right, for the second simulation, the way to exert sources is the same while the model of ground structure is modified to be a inclined one. Because is is concluded the sensitivity and error will be large for deep structure[8], the first layer's thickness is set to be small value of around 120m. The plan and section plan of simulation are shown in Fig.5.

Then, in the same way, the SPAC method is applied using the records observed at the same 3 arrays of sites to see how it behaves when the layers are not horizontal but slightly inclined. The result is shown in Fig.6. It can be seen that the theoretical dispersion curve still matches with the result from simulation very well. Hence, even in slightly inclined layered medium, the SPAC method is still available.

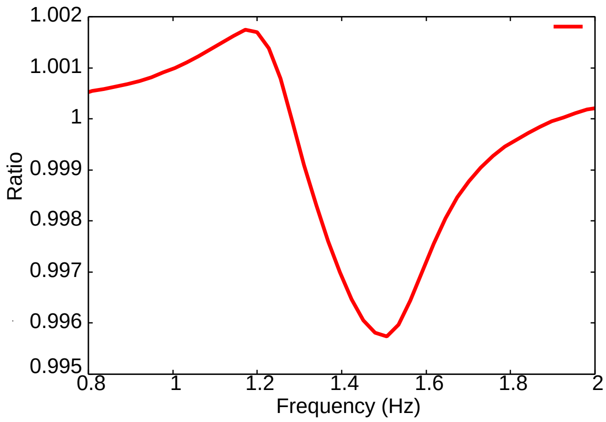

Figure 7. The theoretical ratio between site X1 and X2

Then, in order to examine if Eqn.(5) is satisfied, all the sites on the largest array are chosen as the examination sites as Fig.5 shows. For example, the site X1,X0 and X2(X3) corresponds to the depth of the first layer of 128m, 120m and 112m, respectively and the ratio is calculated between X1&X0, X1&X2, X1&X3, X0&X3 and X0&X2. The same thing is done for the other two arrays which means finally we obtain totally $5 \times 3 = 15$ ratios.

We use Hisada's program[14][15] to calculate the theoretical ratio of imaginary part of Green's function (the right hand side of Eqn.(5)) while the ratio of power spectra (the left hand side of Eqn.(5)) is easily calculated from the intermediate result (the power spectra of each site) just now. We use the frequency at which the ratio got the peak value (critical frequency) as the indicator to compare between them instead of comparing all over all range of frequency[8].

Because of the property of ground structure, even for the pair of sites (X1&X2) with the biggist difference in ground structure, the theoretical ratio of imaginary part of Green's function is quite small even at the critical frequency as shown in Fig.7. Besides, with the first layer's thickness varies in the small range, the shape of the ratio also changes quite slow. If we compare them one pair by another pair, the peak of simulation ratio will not be obvious for a part of pairs. In addition, the domination of microtremor and the system error of theoretical ratio[8] will exert bad influence on the result. Hence, we add all the ratios into one figure to see an comprehensive image. Accordingly, we choose 3 representative theoretical ratio in one figure to represent its rough moving scope. Finally, we compare them two as shown in Fig.8. We can see that for the theoretical ratio, there are two peaks which correspond to the two highest and densist peaks in simulatio ratio. They matches quite well which indicates that Eqn.(5) still works to certain degree in the wavefield dominated by microtremors.

(2) The neccessity of taking the ratio between

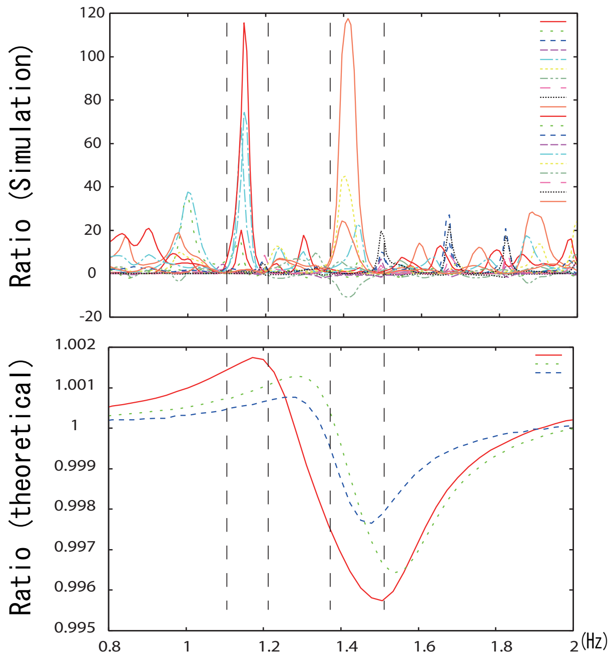

Figure 8. The comprehensive comparison between theoretical and simulation ratio

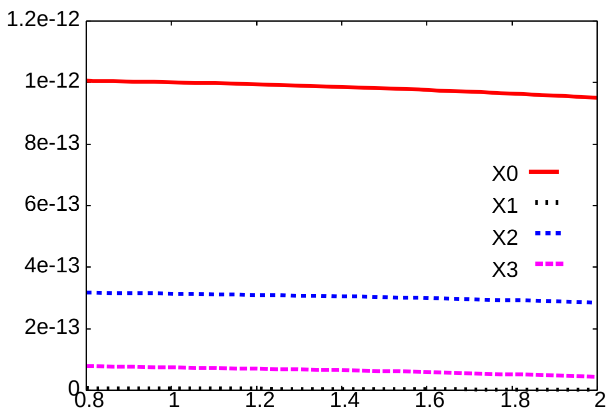

Figure 9. The power spectra of X0, X1, X2 and X3

two power spectra

This problem becomes easy to solve now. Fig.9 shows the power spectra of X1, X0, X2 and X3. It is obvious that there are almost no peak of power spectra over all the frequency range. In other words, it is hard to find the frequency at which the power spectra arrives at the peak. It is because that the microtremor takes on the main part of the power which makes it difficult to extract reflection response out.

However, by taking the ratio of X1 and X0, for example, the peak turns up and the peak coincides with the theoretical ratio of imaginary part of Green's function as Fig.10 shows. It indicates that the ratio contains the information of body waves. Therefore, taking the ratio of power spectra between two sites is necessary in order to get the critical frequency. This critical frequency denotes the differece between the reflection response of two sites.

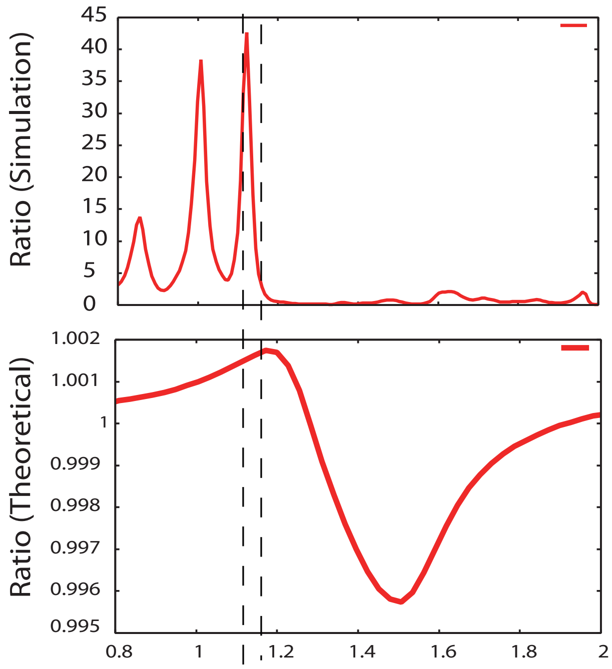

Figure 10. Comparison after taking the ratio of power spectra

4. CONCLUSIONS

We introduced the new method of combining SPAC method and seismic interferometry briefly and do the discussion mainly on the availability of seismic interferometry in wavefield dominated by microtremor. There are several conclusions.

(1) The SPAC method is available when the layers underground are slightly inclined in spite of the assumption of horizontally layered medium.
(2) Sesimic interferometry still works in the wavefield dominated by microtremor. In some case like the examination in this article, because of the property of ground structure, the peak value of theoretical ratio $\frac{\Im[G_{zz}(X_j,X_j,\omega)]}{\Im[G_{zz}(X_i,X_i,\omega)]}$ is quite small which causes the critical frequency hard to identify practically. Hence, we can make all the ratios in one figure to identify the most obvious peaks and do analysis. More work will be done on this in the future.
(3) It is necessary to take the ratio between power spectra of 2 sites because power spectra alone contains mainly microtremors and is hard to analyze.

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