Development of a measurement method of restoring force characteristics based on synchronized accelerometers

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This paper proposes a method of measuring restoring force characteristic by means of two synchronized accelerometers, in order to make urgent diagnosis for structures possibly damaged by an earthquake. Time integration and synchronization are key to accurately measure the relation. The method has been applied to a building for a few years, and it is shown that a restoring force characteristic can be measured. Numerical simulation is carried out to apply this method to structures which will be modeled as a multi-degree-of-freedom system.

Key Words : *urgent seismic damage diagnosis, restoring force characteristic, numerical integration*

1. Introduction

(1) Background

Infrastructural construction such as road construction, embankment works and irrigation works contributed to economical growth of Japan after world war . A lot of house and apartment buildings have also constructed. Nowadays these infrastructural and architectural structures exist in Japan. But all structures have limited life time. Generally infrastructural and architectural structures are needed to be repaired in appropriate time and if a structure is diagnosed not to function normally it is needed to be rebuilt. Structures which have been constructed after world war are deteriorating. Near future a lot of structures will become needed to be repaired in Japan.

By the way decline of the birthrate and aging of Jananese society have great influence on maintenance of these structures. The total populatuion of Japan is supposed to decline by 24% from 127 million in 2008 to 97 million in 2048 and the labor population of Japan is also supposed to decline by 38% from 76 million in 2008 to 47 million in 2048[1)]. A decrease in population and in labor population cause decline of tax revenue and decline of the number of engineers who are in charge of infrastructural and architectural structures. This means that maintenance of infrastructural and architectural structures constructed in Japan will have to be done by fewer engineers and budget than now. Automation and mechanization must be carried out in order to reduce cost and burden of engineers.

A degree of long-term deterioration of a structure is able to assume by a design specification, service length and environmental condition. But it is difficult to assume a degree of damage caused by a large earthquake and a terrorism in advance.

It is important to take countermeasure in advance in order to reduce damage and injury caused by earthquakes. Nowadays seismic retrofitting of infrastructural and architectural structures which do not meet earthquake standards are being conducted and new methods of construction are being developed.

A measure taken in advance is not enough. Some structures must not meet earthquake standards when a large earthquake occurs. Even if a structure meets the standards a big ground motion can damage the structure. It is needed to reduce discontinuity of social activity by diagnoses of structure inside disaster-stricken area just after earthquake occurs. Actually promptly checking a damage degree of each structure in case of large earthquake is an important topic. Nowadays visual inspection by experts and risk decision to architectural buildings[2] are to be conducted in order to grasp the state of disaster-stricken area in case of large earthquake. But these methos rely upon experience and subjective of experts. Supposing large earthquake caused in a trench, quickly grasping of the state of disaster-stricken area cannot be achieved by visual inspection. Setting measurement system in each structure and quantitively graspig damage degree of a structure based on data measured by this system is thought to be an alternative method.

There are two types of measurement in structural health monitoring. One is a measurement by sensors set densely in local area to grasp behavior of components. The other is a measurement by sensors set sparsely to grasp behavior of whole structure. It were needed to monitor all components if we wanted to detect damage of all components in the case of a major earthquake. But that is not realistic in a view point of cost-effectiveness.

On the other hand the sensor system which is set sparsely to grasp behavior of whole structure cannot detect the damage if the size of damage is too small to detect by visual inspection.

(2) Aim

We propose sensing system which measures restoring force characteristic of each layer and sensing system which can detect damage of each component. The former system can measure the degree of plasticity in case of major earthquake to estimate the degree of damage of each layer. In this paper we define restoring force characteristic as a relationship between load and displacement in the case of dynamic behavior of a structure. It is needed to solve equation of motion in order to calculate a behanior of a structure in the case of a major earthquake. If the behavior is elastic we can calculate analytical solution. But if the behavior is elasto-plastic we have to conduct numerical integration to calculate numerical solution. For this numerical calculation we have to prepare restoring force characteristic of target component in advance. A restoring force characteristic is usually measured by a shaking table test or a load test. In this reserach we construct a system which can measure restoring force characteristic of real structure. We conducted two works written below.

- implementation of time synchronization between several sensors
- improvement of the accuracy of numerical integration

With respect to the latter system it is ideal to conduct measurement by dense sensor network to detect a damage of a component. For this kind of measurement wireless sensor network is effective. Wireless sensor network consists of a number of sensor platform which include small sensors and wireless terminals. By using wireless sensor network we can collect measurement data acquired by dense sensor network in target space without complicated wiring. Sensor platforms developed up to now have a function of multi-hop communication nd a function of power saving and miniaturization of sensors was achieved. There is an example of a research in which the system was used for the observation of an ecosystem [3]. But this system is not practical for measurement of a structure. There are several ploblem in this system written below.

- This system does not have a function of time synchronization
- It is unable to determine positions of sensors automatically
- Accuracy of sensors implemented by default is not enough

So it is needed to improve this system to use for measurement of a structure. In this reasearch software development was conducted for "time synchronization".

2. An outline of measurement system and techniques

(1) An overall picture of measurement system

In this section an overall picture of measurement system which we propose in this paper wolud be shown. This measurement system consists of a sensing system which measures acceleration of each layer of a target structure to compute restoring force characteristics and a sensing system in which densly localized sensor platforms gathers local information of the structure to detect local damages. First measurement system of restoring force characteristics wholud be shown(**Fig. 1**). The aim of this system is diagnosis of a civil structure by measurement and evaluation of restoring force characteristics. A target of this system is a whole structure, an apartment structure and a bridge pier, for example. This system has an eye on construction of a system by which a person in charge of disaster management

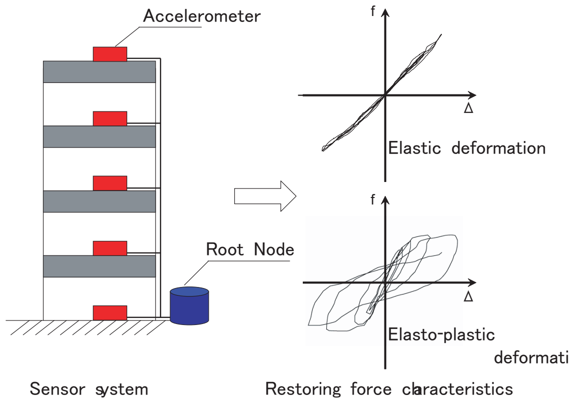

Fig.1 Measurement system of restoring force characteristic.

can grasp a state of disaster in an area of jurisdiction promptly. Simple shear model is supposed to be applied to analyze a target structure and sensor is supposed to be set every layer. It is needed to set multiple sensors on one layer to capture the behavior which contains a twist component, but in this research this view point whold not be considered.

There are some methods to evaluate restoring force characteristics. Visual inspection of restoring force characteristics by specialist of earthquake engineering is one method. This diagnosis is based on the reselt of shaking table tests which were held on the past. Automatic diagnosis by software is another method. In this method a database which contains results of shaking table test targeted at many component and structures wholud be constructed and based on this database software estimates a lebel of damage by measured restoring force characteristics. Under the existing circumstances the former method must be applied, but in the future practical implementation must be conducted to the latter method to realize rapid and cost-effective diagnosis.

After rough diagnosis had done by measurement system of restoring force characteristics, based on the information of the diagnosis an emergency measure whould be done. It is capable to make a plan of the emergency measure by visual inspection if the damage is large. But if the damage is diagnosed to be small it is difficult to determine the place and degree of the damage. Even if the damage is too small to have an effect on resilience of the structure increment of eccentricity of the target structure or chemical deterioration might occur. Then damage identification by detailed measurement which is conducted by densely located sensors. This paper proposed mobile measurement system which contains a number of wireless sensor platforms(**Fig. 2**).

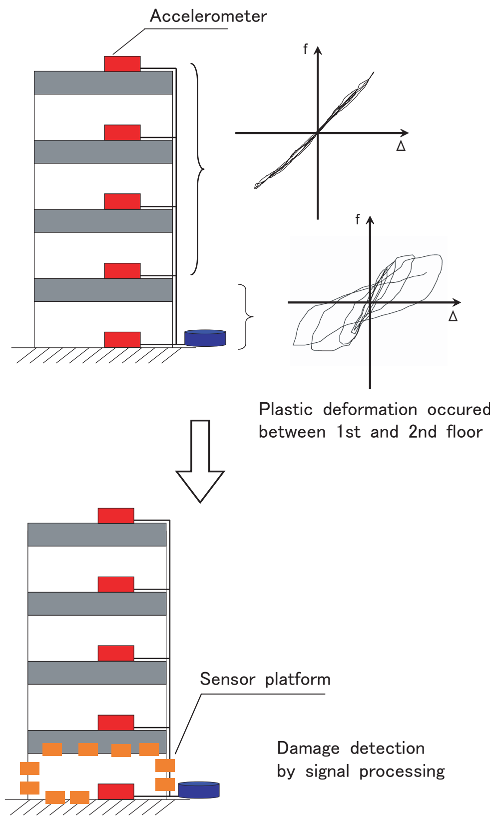

Fig.2 Fixed and mobile measurement system.

(2) Elemental technologies

Components of measurement sysytem is shown in **Fig. 3**. As is written in previous section, this measurement sysytem is consists of 2 systems, and these systems are divided to 4 layers, "Sensor", "Node", "System", and "Analysis". In this research, technologies on "Fixed system"-"Analysis" and "Mobile system"-"System" were developed. On "Mobile system"-"Analysis" elemental technologies for diagnosis of structures have not developed yet for this system. About elemental technologies on other layers, machines developped in other reseraches were used for this research. Elemental technologies for these layers whould be descrived in following chapters.

a) Measurement system of restoring force characteristics

The aim of this research is computation of restoring force characteristics from acceleration

Layer	Fixed system	Mobile system
Sensor	・Accelerometer for measurement of behavior of target structure	・Accelerometer (MEMS) for measurement of behavior of target structure ・GPS receiver／Acoustic
Node	・Measurement is conducted continuously (″Ring memory″ is used) ・Power is supplied by power cable	・Measurement is conducted only in case of earthquake ・Batteries are used for power supply ・″One-chip microcontroller″ is used for control
System	・Wired communication ・″Master node″ transmits a signal to ″Slave node″s to start continuous recording	・Micro power wireless communication ・Time synchronization must be conducted by wireless communication
Analysis	・Computation of restoring force characteristic only from acceleration ・Time synchronization by using vertical component	・Signal processing to detect a place and a size of a damage

Fig.3 Total structure of measurement system.(Two parts which are inplemented in this reserach are shown with under line.)

data set taken by multiple sensors. Function and accuracy required for this system would be shown below.

Sensor Accelerometers would be used in this system as sensing layer. Because the aim is to measure the response acceleration of a structure in the case of major earthquake, the range of sensors must be set as -2000-2000[gal]. A servo type accelerometer which realizes linear relationship between acceleration and voltage is desirable and SN ratio of accelerometers must be high so that mechanical damages are detected by visual inspection of restoring force characteristics.

Node To acceleration measurement node an AD converter, a data storage, a communication terminal, a power source and (micro) controller muse be needed. Key parameters of an AD converter are sampling frequency and data length. It is needed to set these parameters to appropriate values. According to sampling theorem[4)] sampling frequency must be twice as high as natural frequency of a high order. A data strage must be selected on the view point of accurate recording of whole vibration data.

System Because this system consists of multiple sensors, it needs to have a function of time synchronization and a function of data collection. Sensors must be synchronized accurately to evaluate healthiness of a target structure by measured restoring force characteristics. As to a function of data collection measurement data in multiple sensors must be colledted to a master node without loss in this system.

Analysis Collected data is processed to compute restoring force characteristics after data collection. In this step a function which integrates acceleration data set to make displacement data set, a function which synchronizes sensors substitutively and a function which evaluates healthiness of a target structure by analysis of restoring force characteristics are needed for the system. As to a function of intefration it is needed to estimate the amount of error of integration in advance because the accuracy of integration has an effect on the accuracy of restoring force characteristics directly. As to a function of time synchronization it is important to take time synchronization between data set without communication because time synchronization by communication between sensors might be incapable in the case of a major earthquake. It must be evaluated how accurate the method of time synchronization without communication is in comparison with a method of time synchronization by communication. As to evaluation of restoring force characteristics automation is important. It is also important to construct a function which extracts indices of healthiness of a target structure by two dimensional data set consists of force data set and displacement data set.

b) Mobile measurement system

The aim of this system is to detect damage by measurement of densely arranged sensor platforms to a layer of which plastic deformation is detected by measurement system of restoring force characteristics. Functions required for this system whould be shown below.

Sensor Accelerometes are mainly used as sensors. Because an elastic wave caused artificially is assumed to be measured the range must be -10 - 10[gal]. It is desirable to use servo type accelerometers and SN ratio must be set properly.

Node To acceleration measurement node an AD converter, a data storage, a communication terminal, a power source and (micro) controller muse be needed. It is desirable to set these parameters properly to detect damages. According to sampling theorem[4)] sampling frequency must be twice as high as natural frequency of an elastic wave which is used for diagnosis. A data strage must be selected on the view point of accurate recording of whole test wave.

System Because this system consists of multiple sensors, it needs to have a function of time synchronization and a function of data collection in the same way as Measurement system of restoring force characteristics. This system is required to take time synchronization to detect damages by data set of elastic wave measured. As to a function of data collection measurement data in multiple sensors must be colledted to a master node without loss in this system in the same way as Measurement system of restoring force characteristics.

Analysis It is required to detect damages by processing of measurement data set. On this step it is needed to detect a place and a size of a damage by processing data sets which are measured by multiple sensors set on a component of a target structure. Automatic damage detection must be constructed for prompt evaluation of healthiness of a structure but implement of this function was not conducted in this research.

3. Development of elemental technologies for measurement system of restoring force characteristics

(1) Sensor system

3 accelerometers were combined to 3-axis accelerometer for the use of sensor layer. Specs of components of the machinary were shown in **Fig. 4**. In an analog circuit used in this research a filter circuit was set between an accelerometer and an AD converter This 6th order butterworth filter cuts frequency component more than 50[Hz] and an output signal was inputed to an AD converter. A merit of setting a filter in an analog circuit is exclusion of an effect of aliasing. A demerit is to input different signal from initial signal to an AD converter because a high-frequency component was cut and because an effect of nonlinear phase on the input signal was added.

Functions	Spec
Accelerometer	Servo type
Frequency characteristic	DC-50Hz (-3dB)
Maximum acceleration	±2200Gal
Resolution	less than 1Gal
Measurement directions	3 components(X, Y, Z)
AD resolution	24bit
Sampling frequency	2000Hz
Trigger level	5Gal
How to start measurement	Waked up by DO signal from master node
Error of time synchronization	0.6ms
Clock	Inner clock (corrected by GPS)
Sampling length	120s
Capacity of recording	6 waves (inside the accelerometers) 150 waves (inside the BOX computer)

Fig.4 A system of restoring force measurement.

(2) Node layer

In this chapter elemental techniques of measurement system of restoring force characteristics whould be shown.

AD converter As to AD converter sampling ration and bit number are important parameters. In this research digital sigma type AD converter whose resolution is 24 bit is adopted. Sampling ratio was set to 2000[Hz]. On the comparison of usual seismic measurement these parameters are too large but this realizes determination of optimal sampling ratio for structural healthiness by data analysis. A sensor node for this experiment consisits of a CPU, an exclusive interface IC and an AD converter in order to make CPU burden for the sake of control of AD converter 10%.

Memory and Communication equipment Implementation was done to record acceleration on the time of earthquake occurrence. A trigger of recording was exceedance of output of at least one accelerometer 5[gal]. A ring memory was adopted as a memory to make start time of recording acceleration data set backdated 30[s]. It was capable to record only 6 data sets by high-speed memory SRRAM24Byte because the size of one data set of acceleration for one node was 2.88[MByte]. So data was corrected by RS485(105600[bps]) communication to record in memory card 512[MByte]. As a result it became capable to record 50 data sets.

(3) System layer

a) Time synchronization between sensor platforms

In this chapter time synchronization method was implemented to "Mica2" for practical use of sensor platforms. In this implementation measurement of restoring force characteristic of each component of a structure was took in a consideration. Sensor platforms is explained in chapter 1. Specifications of "Mica2" is described in **Table 1**. wireless module which was used in this research was implemented in "Mica2" by default. RBS(Reference Broadcast Synchronization) [5], in which sensor nodes are synchronized by reference signals is a famous method of time synchronization for sensor network. In this research a method in which sensor nodes are synchronized by time stamps sent from specified sensor node.

System composition The following itemization shows structural elements of synchronized sensor networks.

- function generator
- "Clock node" (Mica2)
- "Sensor node" (Mica2)
- "Receiving node" (Mica2)

Table1 Main specifications of "Mica2"

One-Chip MicroComputer	
Chip	ATmega 128L
CPU	7.3MHz
RAM	4kB
Flash	128kB
ADC	28.6kHz 10bit
RF Transceiver	
Chip	Chipcom CC1000
Frequency	315MHz
FSK data rate	38.4Kbaud
Microphone	
Chip	WM-62A
SNR	more than 58dB
Sounder	
Chip	PS14T40A
Frequency	4.5kHz

Synchronized measurement of acceleration is conducted by this system.

"Clock node" transmits time stamps regularly(1〜10[Hz]) by using pulse signals send from function generator as trigger signals.

This time stamps contain sequential numbers equivalent to time. These sequential numbers are used when measurement data are synchronized.

All "Sensor node" start measurement by using receipt of first time stamp as a trigger of measurement. "Sensor node" measures acceleration data in assigned sampling rate and measured data is recorded in memory equipped on Mica2.

"Sensor node" sends packets contains time data and measurement data shown in **Fig. 5** on the timing of receipt of time stamps

"Receiving node" receives these packets and transfers them to the server.

Concrete information equipped in Nth packet is shown like below.

- ID of sensor node
- Nth and N+1th time stamps($TS(N)$, $TS(N+1)$)
- "a top margin"," a bottom margin"(M_H , M_T)
- the number of measurement data (n)
- arragement of measurement data

A margin means the number of count which a timer fired from receipt of time stamp to first sampling of acceleration data. This is used for correcting sampling time of data. Length of time which the margin indicates corresponds is shorter than the interval of data sampling. Sampling time is computed based on the number of time

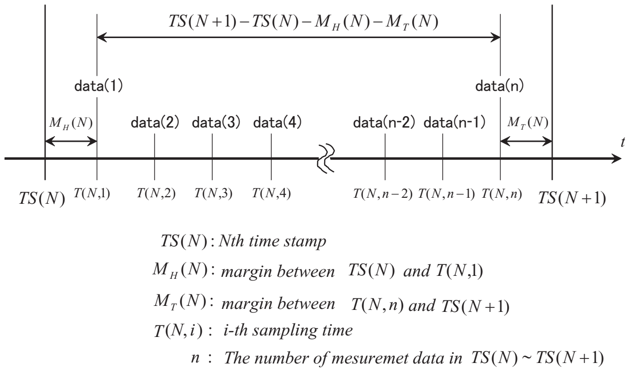

Fig.5 Component of Nth packet.

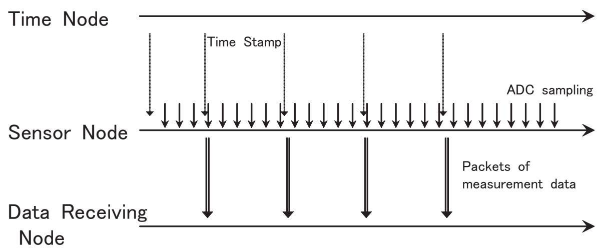

Fig.6 Time flow of synchronized measurement.

stamp, a margin and the number of sampling data (**Fig. 5**). A time series from start of measurement to data collection is shown like **Fig. 6**. As mentioned before, receipt of time stamp triggers the formation and transmission of data packets.

This measurement system is capable of real-time collecting measurement data sampled by synchronized multiple sensors.

Verification An experiment shown below was conducted in order to verify the accuracy of time synchronization of this system. Measurement was conducted by synchronized measurement system in which pulse signals generated by an identical function generator were inputed into two Sensor nodes simultaneously. In this experiment sampling rate was set as 500Hz. The times of rising points of pulse signal(event times) were computed as to two Sensor nodes from measurement data sets and difference of them were also computed (refer to **Fig. 7**) .

There is a difference between actual event time and detected event time in case of discrete sampling (**Fig. 7**). It is rare for them to coincide. Because of this theoretical reason a maximum value of an error of time synchronization is able to be 2[ms] in this experiment in which sampling rate was 500[Hz]. Differences of detected event times for about 450 events were computed and histogram was made. The result was shown in

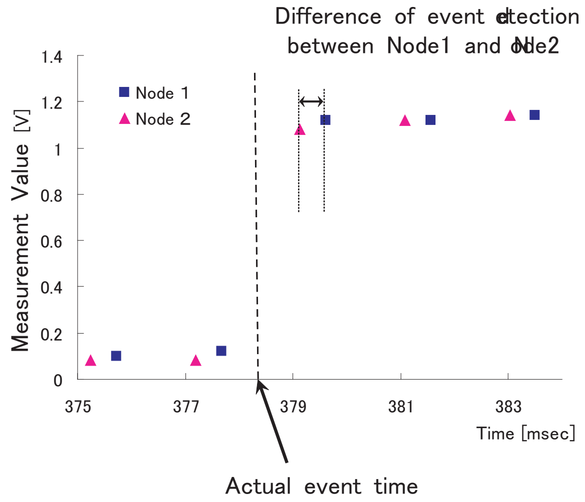

Fig.7 Sample of measurement data (near event time)

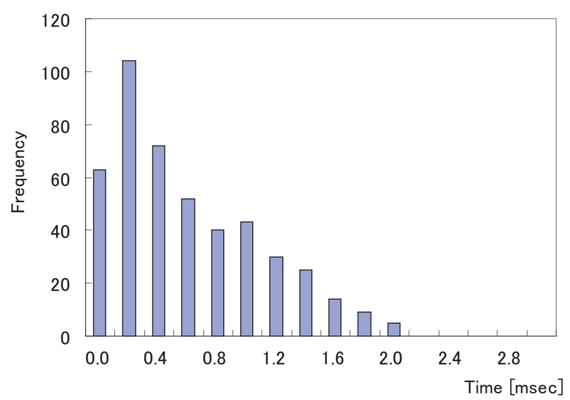

Fig.8 The difference between two detected event times

Fig. 8. The difference of event times was shown to be less than 2[ms] for all events. This verified that the error of time synchronization of this system is less than the resolution of this experiment. It is able to conduct another experiment in case of higher sampling rate. In this case the difference of event times would exceed time resolution of sampling of the experiment if the resolution of sampling exceeds the resolution of time synchronization of this system. Because this system is planed to measure response behavior of civil structures in case of an earthquake it is enough to verify the accuracy of time synchronization of 2[ms].

(4) Processing layer

a) Integration of acceleration

Past researches Many methods in which acceleration data set was integrated to displacement data set were proposed. "Linear acceleration method" in which correction by polynomials was conducted after integration in time domain is a famous method[6]. A method of integration in which trapezoidal approximation is used is also generally used [7].

If displacement data set was computed only by integration of acceleration data set unreal fluctuation of displacement was occured in almost all cases. So it is needed to correct in a process of integration. A method in which high-pass filter was conducted to acceleration data set [8] and a method in which a sequential line was deducted from velocity data set [9] are widely used. The latter method is based on a fenomena in which base line of acceleration fluctuates in measurement time. As causes of this fenomena change of slope of ground by severe vibration and change of inner condition of sensor nodes are considered[10]. This is because length of permanent displacement which is represented by result of integration fluctuates.

A method in which filter with a recursion formula is used in order to correct displacement data set on-site was constructed [11]. Integration methods like this have limitation in accuracy of amplitute and phase although it is enable to correct measurement data in real time.

There is a past research in which integration in frequency domain is implemented. In this reserach permanent displacement is evaluated by using real part of fourier spectrum and permanent displacement was accurately reappered in case of short length measurement. The aim of this research is not an evaluation of permanent displacement but an evaluation of plastic deformation in dinamic vibration.

Integration in frequency domain Error value which is computed by taking difference between integrated displacement data set and actual displacement data set must be small. An error in measuremrent of acceleration and an error in integration rose as causes of error in displacement. Causes of error in measurement acceleration are shown below.

- a slope of base line because of a slope of foundation
- noise of a sensor probe
- noise of a analog circuit
- quantization error

Causes of error in integration are shown below.

- extension of noise in low frequency component

- accumulation of approximation error

In this research integration in frequency domain[12) 13)] was used. In this method calculations shown below is conducted twice.

$$X_k = \sum_{n=1}^{N} x_n \cdot \exp(-2\pi i \frac{kn}{N}) \tag{1}$$

$$Y_k = A_k \cdot X_k \tag{2}$$

$$y_n = \sum_{k=1}^{N} Y_k \cdot \exp(2\pi i \frac{kn}{N}) \tag{3}$$

coefficients multiplied to frequency component of acceleration data set are shown below.

$$A_k = \frac{1}{i\omega_k} \quad \omega_k = \frac{2\pi k}{N} \tag{4}$$

If this method is used without filtering noise in low frequency component increases to produce low-frequency trend in displacement data set. So generally integration is conducted after low-cut filtering. Because this reserach uses a method using DFT, filter is multiplied to coefficientsA_k to restrain increase in noise in low frequency component caused by integration.

4. Verification by shaking table test

(1) Outline and aim of an experiment

An outline of an experiment would be experimented in this chapter. An experiment using 1-axis shaking table was conducted in order to verify the accuracy of integration of acceleration data set. A movement of shaling table was measured by accelerometer and laser displacement sensor and displacement data set computed from acceleration data set and displacement data set measured by the laser sensor were compared. A servo type accelerometer and laser type displacement sensor were set in shaking table(**Fig. 9**). A servo type accelerometer was same as a sensor node developed as a sensor layer of measurement system of restoring force characteristic.

sine wave(1[Hz]∼60[Hz]) and measured seismic wave were inputed to shaking table and two accelerometers and displacement sensor were used.

(2) Properties of accelerometer

If the amplitude of frequency domain of acceleration data set is represented as $A(\omega)$ fourier amplitude of displacement data set computed by twice integration from acceleration data set is represented as $A(\omega)/\omega^2$. Ratio of these two data set was computed. If fourier amplitude of displacement data set is represented as $U(\omega)$ this

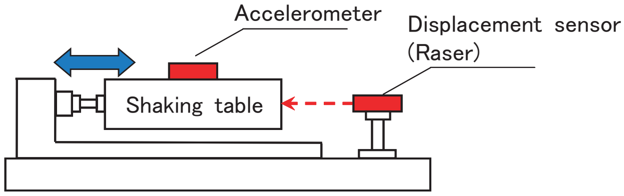

Fig.9 Shaking table test.

computation is shown as below.

$$R(\omega) = \frac{A(\omega)}{U(\omega) \cdot \omega^2}. \tag{5}$$

Ratio of amplitude R is shown in **Fig. 11**. From this figure the ratio R is away from 1 in low frequency domain. This is because characteristic of instruments and expansion of error in fourier amplitude of acceleration in low frequency domain. A low-cut filter was applied in order to eliminate the error in low frequency domain caused as a result of integration. According to **Fig. 11** a cut-off frequency was set as 0.2[Hz]. Ratio of fourier amplitude of displacement data set as a result of twice integration from acceleration data set and fourier amplitude of measured displacement data set for several frequencies was shown in **Fig. 12** In this cimputation sine wave was used as an input wave.

(3) Verification of accuracy of time integraion

Measured acceleration was integrated setting cut off frequency as 0.2[Hz] and 0.1[Hz]. Difference between the result of integration and measured displacement data set was computed. Two seismic waves were used as input waves. An amplitude of one wave is small and the other was big. Difference of displacement data set in case that cut off frequency was set as 0.2[Hz] was shown in **Fig. 13**. In this figure 5 period trend was observed. This is because the appearance of low frequency component in measured displacement caused by conduction of low cut filter. So low frequency cimponent was eliminated from difference data set setting a cut off frequency as 0.2[Hz]. The result of this is shown in **Fig. 14** and **Fig. 15**. For the two types of waves the value of 3 σ were 0.06[cm] and 0.05[cm] and the maximum values were 0.16[cm] and 0.10[cm]. Difference of displacement data set in case that cut off frequency was set as 0.1[Hz] was shown in **Fig. 16** and **Fig. 17**. In these figures low frequency components of error were eliminated. For the two types of waves the value of 3 σ were 0.09[cm] and 0.07[cm] and the maximum values were 0.16[cm] and 0.11[cm].

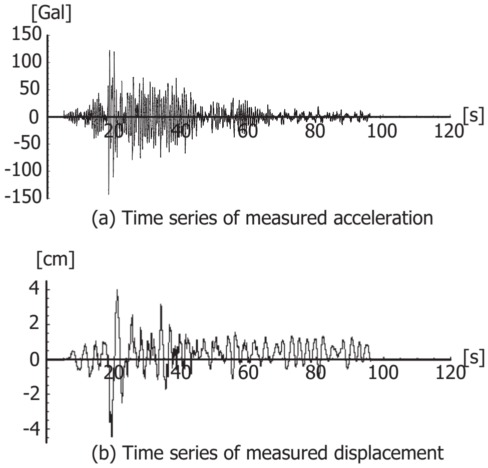

Fig.10 Wave forms of measured data set.

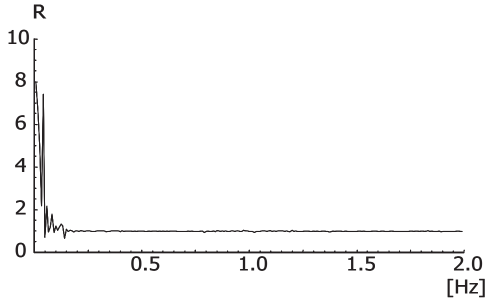

Fig.11 The ratio of fourier amplitude (seismic wave).

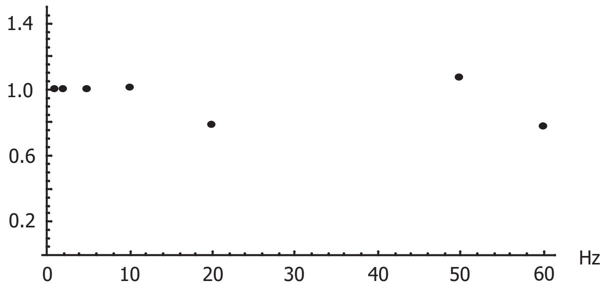

Fig.12 The ratio of fourier amplitude (sine wave).

5. Observation of seismic vibration of a real structure

(1) Outline of measurement

Monitoring of strong-motion of a modern apartment was conducted to verify whether realistic restoring force characteristic of civil structures are properly observed in case of middle level earthquakes. Three 3-axis accelerometers have been set on a platform of 1st floor, a ceiling of 1st floor and a platform of 11th floor of a 11-story RC made apartment and strong-motions

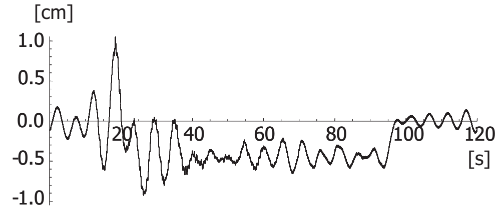

Fig.13 The error of integration (Cut off frequency : 0.2[Hz]).

Fig.14 Low cut component(more than 0.2[Hz]) of the error of integration (Cut off frequency : 0.2[Hz], Big amplitude).

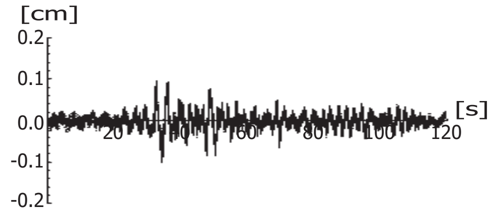

Fig.15 Low cut component(more than 0.2[Hz]) of the error of integration (Cut off frequency : 0.2[Hz], Small amplitude).

Fig.16 Low cut component(more than 0.1[Hz]) of the error of integration (Cut off frequency : 0.1[Hz], Big amplitude).

have been observed[14]. The outline of this system is shown in **Fig. 18**. Three sensor nodes are connected by communication cables and time series data of acceleration is corrected when earthquakes occurs to generate trigger signals. The error of time synchronization is less than ± 0.6[ms]. Detail of measurement methods are described in Section 3. Sensor nodes are connected on platforms and ceilings with bolts. Acceleration data sets are corrected to a local host and transmitted

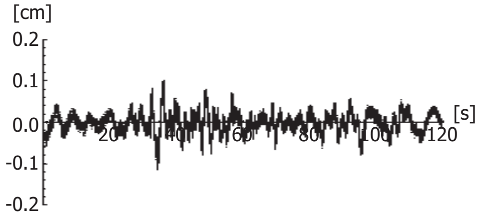

Fig.17 Low cut component(more than 0.1[Hz]) of the error of integration (Cut off frequency : 0.1[Hz], Small amplitude) .

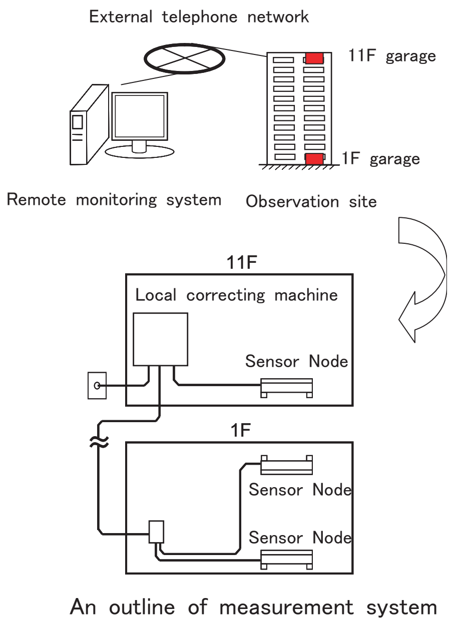

Fig.18 An outline of measurement system.

through network to a office. Sampling frequency and data length are set as 2000[Hz] and 24[bit] respectively.These values are overspec for the sake of monitoring of seismic vibration. Especially if the final goal of this research is on-site diagnosis of structures damaged by seismic ground motion, it is needed to reduce size of measurement data. Discussion about these values were descrived in a past research[13], in which it is concluded that 200[Hz] and 16[Bit] are appropriate. In following chapters time series of acceleration measured on platform of 1th floor and 11th floor were used to compute restoring force characteristics of this structure.

(2) Estimation of restoring force characteristics of the target structure

a) Restoring force characteristics

Displacement data sets were computed from acceleration data sets with a low-cut filter cutting low-frequency component less than 0.2[Hz] in order to draw restoring force characteristics of a target structure. Data sets used in this research were measured in a period from Aug 2005 to Dec 2007. Relative displacement and acceleration were computed by equations shown below.

$$(\Delta u_n, a_n) = (u_n^{11} - u_n^1, a_n^{11}) \tag{6}$$

In this equations superscripts "1" and "11" represent being measured in 1st floor and in 11th floor respectively. Example restoring force characteristics are shown in **Fig. 19**. In these three cases values of maximum acceleration in 11th floor are 334, 23, 2[Gal]. The earthquake in which 334[Gal] was observed is maximum earthquake in measurement period. According to these figures linear restoring force characteristics are computed for earthquakes in which values of response accelerations are from several Gal to several hundred Gal.

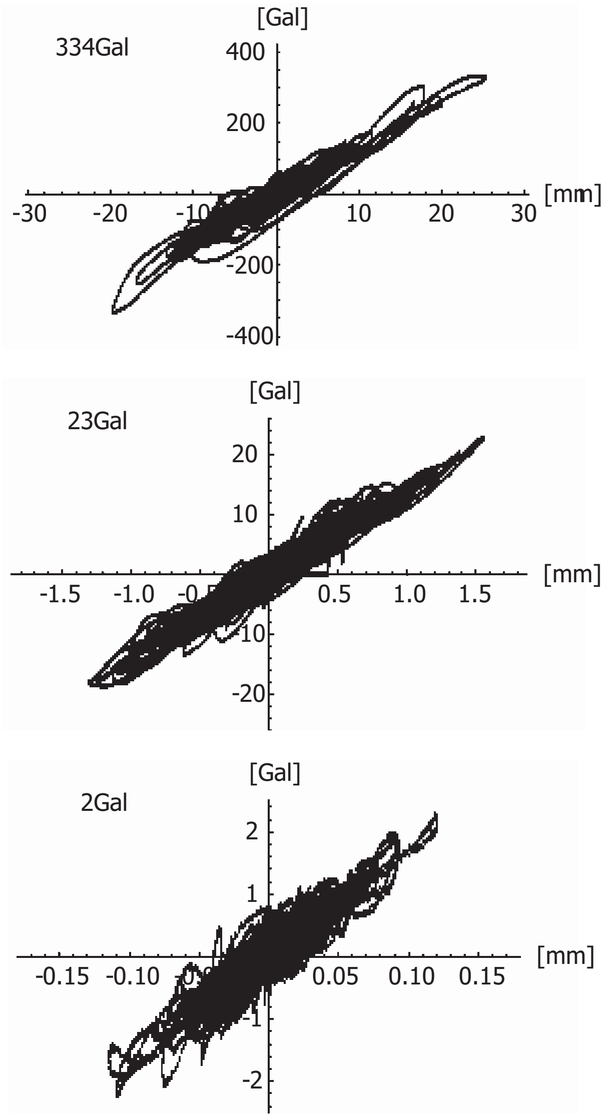

Fig.19 Example of restoring force characteristic computed from measured data.

The restoring force characteristic in main vibration direction is shown in **Fig. 19**. To compute restoring force characteristics in main vibration direction, direction in which relative displacement was maximized was calculated and EW and NS components were converted in this direction. In case that seismic vibration is large linear restoring force characteristics are shown in EW and NS components being same as **Fig. 19** But in case that seismic vibration is small, linearity of restoring force characteristics computed by EW and NS components are not clear rather than restoring force characteristics in main vibration direction. Quantitative analysis using measured restoring force characteristics was conducted. Values of slope of restoring force characteristics were computed for all observed earthquakes. Least-square method was used to calculate slope values. Restoring force characteristics are represented like $a = k\,\Delta u$ if time series of calculated relative displacement Δu and acceleration measured a are represented like $\{\Delta u_n, a_n\}$ and if values of slope of restoring force characteristics are represented like k. So k would be calculated by minimizing values of error which contains k as a variable.

$$E(k) = \sqrt[2]{\frac{1}{N}\sum_{n=1}^{N}(a_n - k\,\Delta u_n)^2} \qquad (7)$$

In this equation N is the number of data measured. From this equation an equation shown below is drawn out.

$$k = \frac{\sum_{n=1}^{N} a_n\,\Delta u_n}{\sum_{n=1}^{N}\Delta u_n^2} \qquad (8)$$

Values of errors of Equation(7) E are calculated by using k. E was Normalized by being divided by norm of time series of acceleration $\sqrt[2]{\frac{1}{N}\sum a_n^2}$.

$$\Delta E_{rel} = \frac{E(k)}{\sqrt[2]{\frac{1}{N}\sum_{n=1}^{N} a_n^2}} \qquad (9)$$

Values of maximum response accelerations in all cases, values of errors E and values of relative errors ΔE are shown in **Table 2**. Units of maximum values of response acceleration and error of them are $[Gal]$. **Fig. 20** is a scatter diagram between the value of error and the value of slope k and **Fig. 21** is a scatter diagram between the value of relative error and the value of slope k. According to these diagram, an actual value of k seems to be about 150. In **Fig. 20** there is not clear trend in the value of error and slope. But in **Fig. 21** the value of slope is near 150 in case that the value of relative error is small. Accordingly as an index which represents reliability of linear restoring force characteristic computed relative error is more appropriate than error. If the value of relative error is about 0.3 the value of slope of restoring force characteristic could be coomputed in the accuracy of $\pm 10\%$.

Table2 Values of error and relative error of restoring force characteristics for all earthquakes.

Date	Response Acceleration [Gal]	Slope [s^{-2}]	Relative Error	Error [Gal^2]
2005/08/16	334.14	130.10	0.24	9.33
2005/10/19	29.16	133.35	0.20	1.03
2005/10/22	17.53	137.68	0.29	0.83
2005/11/01	11.00	141.82	0.21	0.26
2005/12/02	22.88	140.07	0.24	0.96
2005/12/05	15.67	147.77	0.18	0.49
2005/12/17	42.28	149.12	0.21	1.21
2006/01/18	16.71	152.23	0.29	0.71
2006/03/28	3.58	135.67	0.30	0.17
2006/03/29	7.41	136.40	0.47	0.35
2006/04/02	12.63	157.14	0.17	0.21
2006/04/10	11.69	143.20	0.44	0.40
2006/07/06	9.03	152.79	0.31	0.31
2006/09/09	10.69	158.16	0.27	0.41
2007/03/25	3.16	84.47	0.37	0.22
2007/03/31	10.41	147.69	0.28	0.32
2007/04/03	1.77	5.77	0.95	0.18
2007/04/05	14.06	167.01	0.29	0.41
2007/04/11	1.42	2.48	1.00	0.17
2007/04/14	2.30	150.67	0.39	0.12
2007/05/29	6.60	162.42	0.54	0.28
2007/06/16	3.35	125.91	0.37	0.17
2007/07/16	11.57	122.71	0.38	0.77
2007/11/26	26.00	155.49	0.21	0.73
2007/12/25	16.41	160.69	0.27	0.64

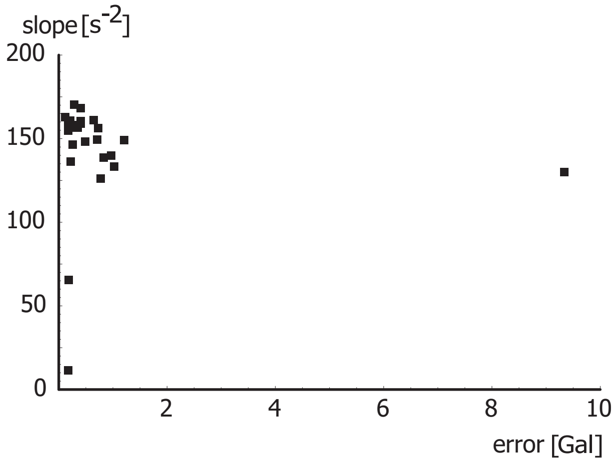

Fig.20 The relationship between the value slope and error of restoring force characteristics.

Time serties of measured acceleration were converted to frequency domain to calculate natural frequencies of 1st mode. A transition of the natural frequency in a period of monitoring is shown in **Fig. 22**. According to this figure there is no trend in the transition. According to a past research as to a transition of natural frequency of RC structure[15], daily transition of natural frequency because of a change in temperature is about 0.02 ∼0.04[Hz] throuthout the year and rearly daily transition gets over 0.06[Hz]. The width of tran-

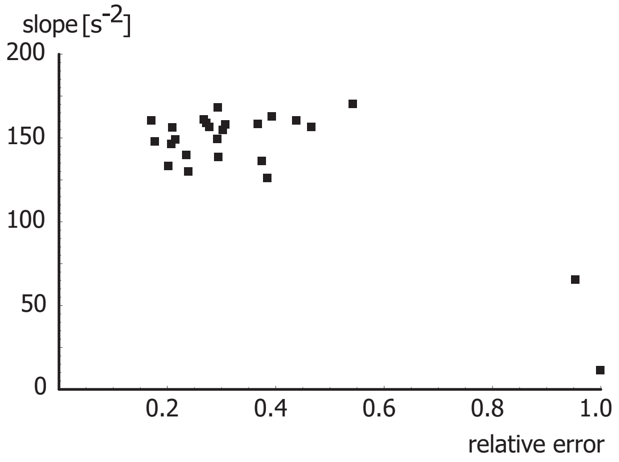

Fig.21 The relationship between the value slope and realtive error of restoring force characteristics.

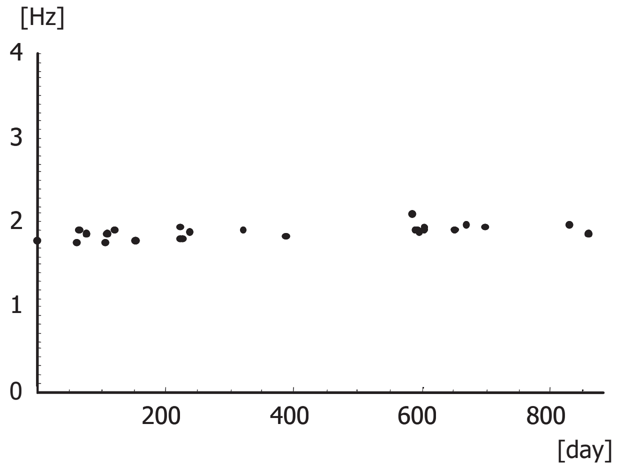

Fig.22 The transition of natural frequency.

6

sition of natural frequency taken in this research is few times as large as that of this past research. This is because natural frequencies were calculated not by ambient vibration but by seismic response.

b) Consideration

In this research linear restoring force characteristics are calculated even if the maximum value of displacememnt is less than 1[mm]. As is shown in last chapter, displacement data sets computed from acceleration data sets contains about 1[mm] error in a shaking table test. Accordingly it is doubtful that restoring force characteristics of actual structures in case of small earthquakes were properly measured by same type accelerometers. As a matter of course the width of the error of restoring force characteristics measured in case of a large earthquake shown in **Fig. 19** is several [mm]. It is consistent. But as to restoring force characteristics measured in case of small earthquakes, a lag from a straight line is less than 1[mm]. As a reason values of lags of restoring force characteristics from straight lines it is considered that maximum value of acceleration is small. If aximum value of acceleration is small, error of integration became small to compute displacement data sets in which the value of error is small. Outside of this reason it is suggested that there should be correlation between acceleration data sets measured and displacement data sets computed. If acceleration data sets has errors whose slope is k, a lag from a straight line becomes small as a result. The reason of correration in error is integration in frequency domain with low-sut filter. Further investigation is needed.

6. Evaluation of restoring force characteristics by numerical calculations

In last chapter whole structure is regarded as one component in modeling. By this modeling it is enable to treat a wall or a pillar as one component. Considering this point of view, in this chapter a method to measure restoring force characteristics which show behavior of springs connecting particles of target structures whould be investigated by numerical calculation using MDOF model.

(1) Formularization

How to formularize a method to measure restoring force characteristics by N piece of springs whould be presented by using NDOF model as an example. Behavior of response of a target structure is supposed to restrict in one direction. Displacmement of αth particle is supposed to be represented by x^α and restoring force of a spring connecting α-1th and αth particle is supposed to be represented by F^α respectively. Based on these assumption, equation of motion of αth particle would be shown like below.

$$m^\alpha\,\ddot{x}^\alpha(t) + F^\alpha(t) - F^{\alpha+1}(t) = -m^\alpha\,g(t) \quad (10)$$

If a fixed number of a spring is set as k^α, F^α in linear domain is shown like below.

$$F^\alpha = k^\alpha(x^\alpha - x^{\alpha-1}) \quad (11)$$

In this equation, x^0 is corresponding to input seismic vibration g and F^{N+1} is zero.

If acceleration of αth particle whould be represented by a^α, restoring force is formularized like below according to equation of motion of Nth particle.

$$F^N(t) = -\,m^N\,a^N(t) \quad (12)$$

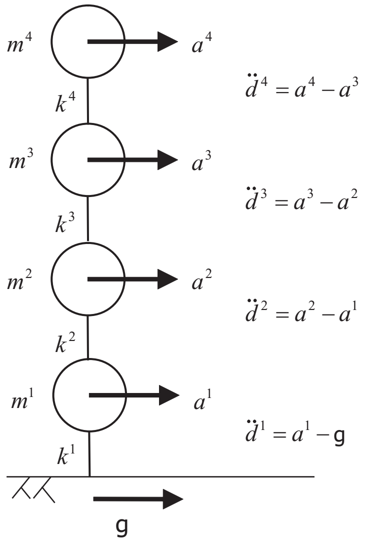

Fig.23 MDOF model.

On the other hand velocity and displacement of a string located between N-1th and Nth particles are computed by using a^N and a^{N-1} which are recorded in measurement.

$$\dot{d}^N(t) = \int_{t_0}^{t} a^N(t) - a^{N-1}(t)\,dt \qquad (13)$$

$$d^N(t) = \int_{t_0}^{t} \dot{d}^N(t)\,dt \qquad (14)$$

Accordingly, drawing $(d^N, -a^N)$ on a plane straight line whose slope is k^N/m^N would present as restoring force characteristics of a string connecting N-1th and Nth particles. By repeating this operation from upper strings, restoring force characteristics of a string connecting αth and α-1th particles would be computed. For example from equation(10) following equation would be introduced.

$$F^\alpha(t) = -\sum_{\beta=\alpha}^{N} m^\beta \left(\ddot{x}^\beta(t) + g(t)\right) \qquad (15)$$

If a mass of a particle is known, restoring force is given by $\sum_{\beta=\alpha}^{N} m^\beta a^\beta$. If a mass of a particle is unknown, assuming particles have same mass, restoring force is given by $A^\alpha = \sum_{\beta=\alpha}^{N} a^\beta$. Relative displacement is given by the following equation.

$$d^\alpha(t) = \int_{t_0}^{t}\int_{t_0}^{t} a^\alpha(t) - a^{\alpha-1}(t)\,dt\,dt \qquad (16)$$

By drawing $(d^\alpha, -A^\alpha)$ on same plane, restoring force characteristics are measured. Similarly to SDOF model, if a string of MDOF model reaches nonlinear domain, restoring force characteristics drawing on $(d^\alpha, -A^\alpha)$ also become nonlinear. It is enable to evaluate occurrence and level of damage by observation of nonlinear behavior of restoring force characteristics

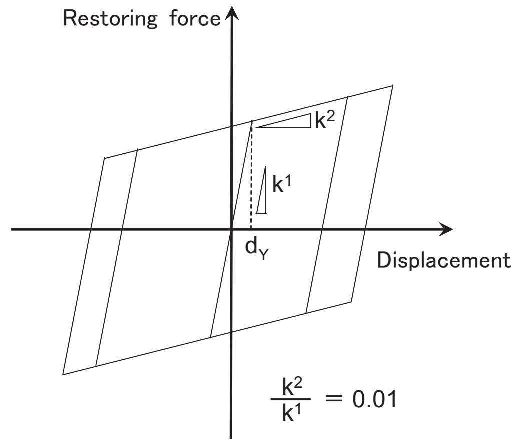

Fig.24 Stiffness of a string.

(2) Arrangement of accelerometers

It is nessesary to locate accelerometers on all particles to measure restoring force characteristics of all strings. But it is not nessesary to locate accelerometers on all particles in order to evaluate plasticity which is represented by sift of restoring force characteristics from straight line. In this chapter numerical calculations would be conducted by MDOF model in which accelerometers are set on every several particles. The aim of these calculations are investigation of minimum number of accelerometers for evaluation of plasticity. As a stiffness model of a string, bi-linear model is used. Numerical calculation is conducted setting ratio of initial stiffness and stiffness in plasticity as 0.01 (**Fig. 24**). In implementation of numerical calculation [16] is referred.

In numerical calculation seismic vibration measured in accelerometer set on 1st floor of actual structure in Iwate-Miyagi nairiku earthquake(2008) is used. Amplitude of seismic response was regulated so that plastic deformation in a string between ground and 1st particle would occur by about 50%. **Fig. 25** shows input seisimic vibration in Iwate-Miyagi nairiku earthquake(2008). This vibration was observed in an accelerometer set on 1st floor of the target building.

a) Numerical calculation by 2DOF model

At first accelerometers are assumed to be set on ground and 2 particles. 2DOF is assumed. Values

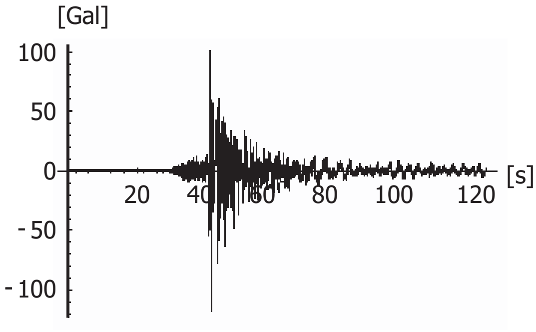

Fig.25 Input seismic vibration (Iwate-Miyagi nairiku earthquake(2008)).

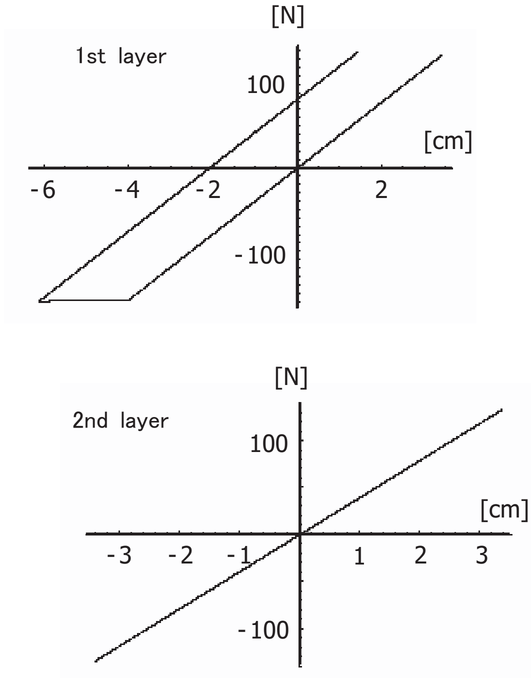

Fig.26 Actual restoring force characteristic.

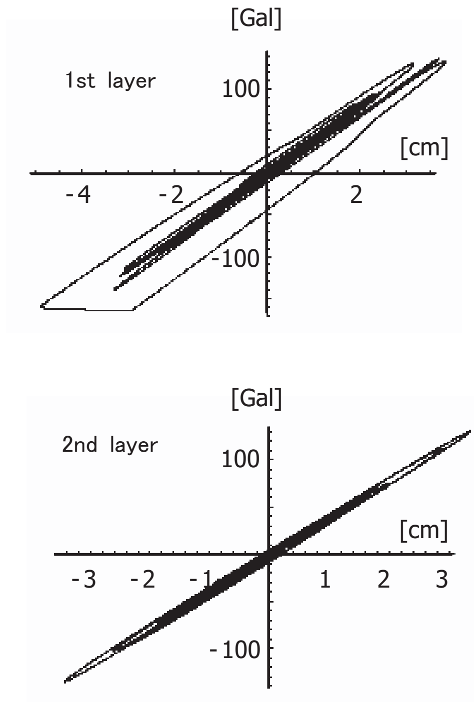

Fig.27 Measured restoring force characteristic.

of mass, damping ratio and initial stiffness of all layers are set as 100[kg], 0.02, 25.1[kg/s^2]. duration time, initial displacement, initial velocity and yield displacement are set as 120[s], 0[cm], 0[cm/s] and 4.0[cm]. To evaluate the accuracy of numerical calculation, restoring force characteristic composed by acceleration of a particle and displacement of a string is drawn on **Fig. 26**. This restoring force characteristic was regarded as actual restoring force characteristic. From this point forward restoring force characteristic given by solution of numerical calculation would be represented as "Actual restoring force characteristic". Based on measurement method of restoring force characteristics, restoring force characteristic computed from displacement calculated from measured acceleration is shown in **Fig. 27**. From these figures it is seen that plastic deformation of a string connecting ground and 1st particle occurs by about 50%.

Next a situation that accelerometers are set only on ground and 2nd particle was assumed. In this assumption 2DOF model is converted to 1DOF model. **Fig. 28** shows restoring force characteristic computed by average of restoring force of a string and measured elative displacement. **Fig. 29** shows restoring force characteristic computed by acceleration value of 2nd particle and realtive displacement computed from time integration. Although plastic deformation of a string connecting ground and 1st particle occurs it is difficult to evaluate plasticity of the structure from this Figure.

Regarding that response of 2DOF model is consisted of 2 types of mode, extraction of 1st mode is attempted. As is seen in **Fig. 30**, there is some amount of component except for 1st mode in fourier spectrum of acceleration and relative displacement. Relationship between acceleration and relative displacement is shown in **Fig. 31**. Extraction of 1st mode($\pm$ 0.5[Hz]) was conducted to create data used in this figure. It is enable to detect plasticity by observation of appearance of pararell lines and elliptic curve. Mode decomposition is not appropriate in nonlinear domain. But by force it is considered to observe plastic-

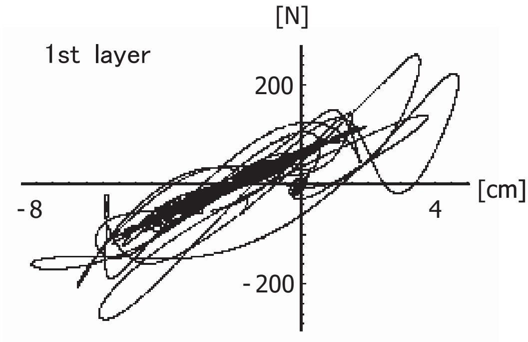

Fig.28 Actual restoring force characteristic(regarding 2DOF model as SDOF model).

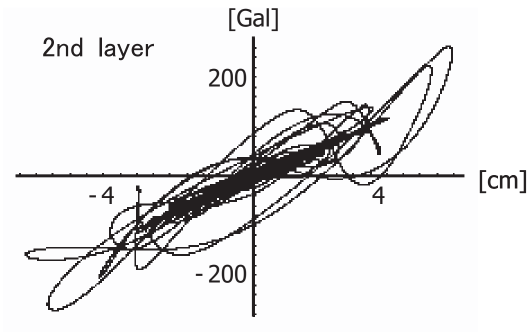

Fig.29 Measured restoring force characteristic(regarding 2DOF model as SDOF model).

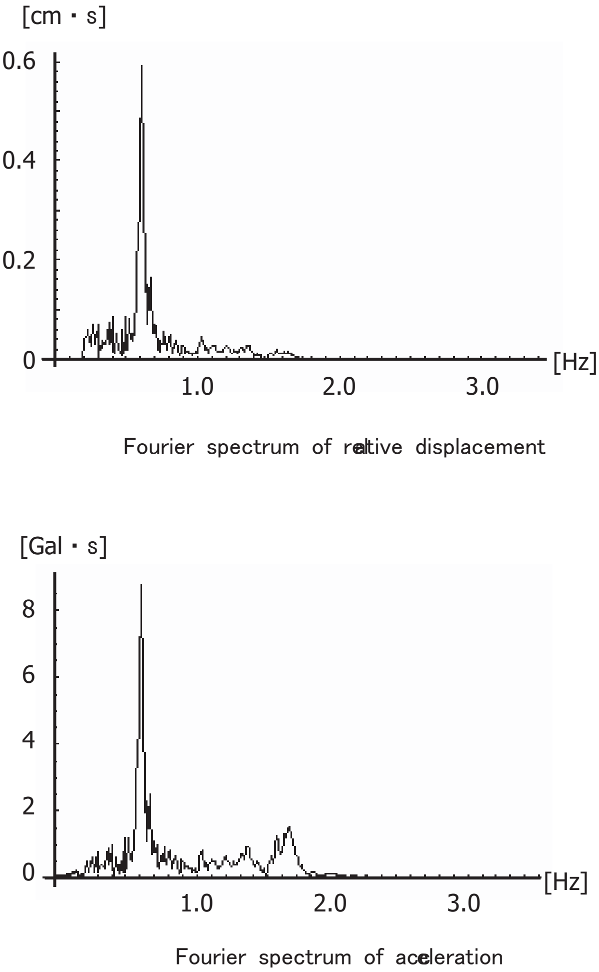

Fig.30 Fourier spectrum of acceleration and relative displacement.

ity in restoring force characteristic by extracting 1st mode. According to nature of input seismic vibration, 2nd mode surpasses 1st mode. In this case this method cannnot operate appropriately. **Fig. 32** shows restoring force characteristic drwan by relative displacement whose 2nd mode was extracted. Because time intagration being conducted in frequency domain, influence of eternal displacement does not appear. So restoring force characteristic has extreme error. But it is enable to evaluate plasticity of the model. For reference, relationship between acceleration data set and realtive displacement data set near the occurrence point of plasticity are shown in **Fig. 33**. It is clear that restoring force characteristic shift from straight line in the the occurrence point of plasticity.

b) Numerical calculation by 4DOF model

Numerical calculation by 4DOF model was conducted in the same way. Values of mass, damping ratio and initial stiffness are set as 50[kg], 0.02 and 25.1[kg/s^2] and duration time, initial displacement, initial velocity and yield displacement are set as 120[s], 0[cm], 0[cm/s] and 4.0[cm]. Relationship between restoring forces of strings connecting particles and relative displacement computed by numerical calculation is shown in **Fig. 34**. Plastic deformation occured by about 50% in 1st layer. Except for 1st layer behaviors of strings are linear. Restoring force characteristic based on relative displacement given by time integration of acceleration is shown in **Fig. 35**. For all types of model in the same manner as 2DOF model it is enable to evaluate plasticity by restoring force characteristics if measurements are conducted in all particles. As a following trial, numerical calculation was conducted in the case that measurements were conducted every other particles. For this calculation restoring force characteristics based on average of restoring force and relative displacement computed by numerical calculation is shown in **Fig. 36**. From this figure it is seen that evaluation of plasticity is capable by observation of result of numerical calculation. Accordingly if fine approximation of relative displacement and restoring force is conducted, same re-

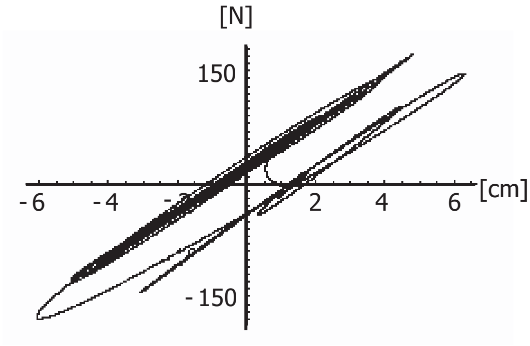

Fig.31 Actual restoring force characteristic(1st mode).

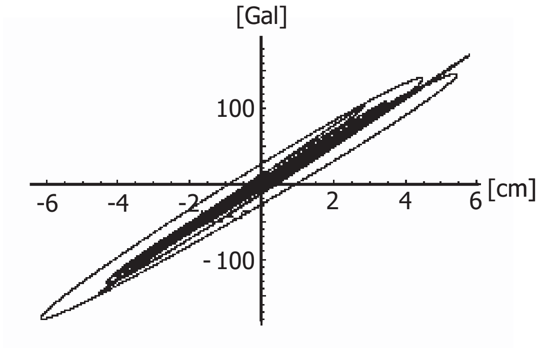

Fig.32 Measured restoring force characteristic(1st mode).

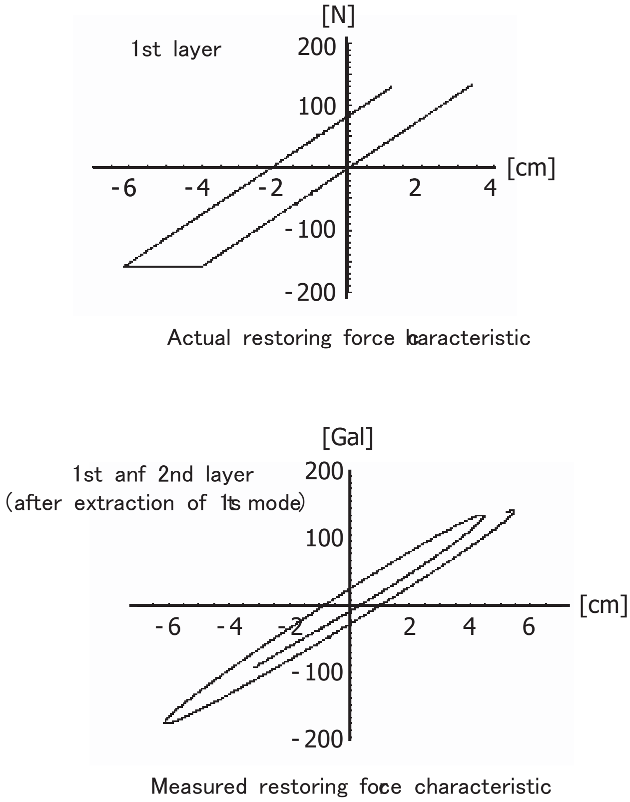

Fig.33 Restoring force characteristic in the occurrence point of plasticity.

sult would be acquired from result of measurement. Restoring force characteristics based on acceleration of 2nd particle and relative displacement given by time integration is shown in **Fig. 37**. Shape of restoring force characteristic which is the result of composition of 3rd and 4th layer is straight line whereas shape of restoring force characteristic which is the result of composition of 1st and 2nd layer has some width. This shape is the result of plasticity.

As a following step, frequency domain corresponding to 1st mode was extracted. restoring force characteristic based on 1st mode ($\pm 0.5[Hz]$) is shown in **Fig. 38**. For all restoring force characteristics shapes are straight lines, but restoring force characteristics which is composition of 1st and 2nd layer has nonlinearity. As to the occurrence point of plasticity, comparison between actual restoring force characteristic and restoring force characteristic as a result of measurement is shown in **Fig. 39**. It is confirmed that measured restoring force characteristic shifts from straight line in the occurrence point of plasticity. This suggests that ninlinearity of restoring force characteristics show plasticity.

7. Conclusion

In this research development of key technologies, verification by experiments and investigation by numerical calculations were conducted, in order to develop measurement system for restoring force characteristics of civil structures Main conclusions would be descripted in following paragraphs.

Proposal for measurement system of restoring force characteristics In this research measurement system in which acceleration data sets are corrected by multiple accelerometers set on a civil structures was proposed. By observation of restoring force characteristics computed from acceleration data sets, damage level of the target structure could be estimated. This method realizes estimation of damage level of 1st floor or "Dannotoshi layer" on which it is easy to occur plastic deformation by setting accelerometers on the target regions. Moreover a method of local damage detection by densely located networs sensing system was proposed.

Development of key technologies Key technologies of sensor layer, node layer, system layer and processing layer were developed in order to measure appropriate restoring force character-

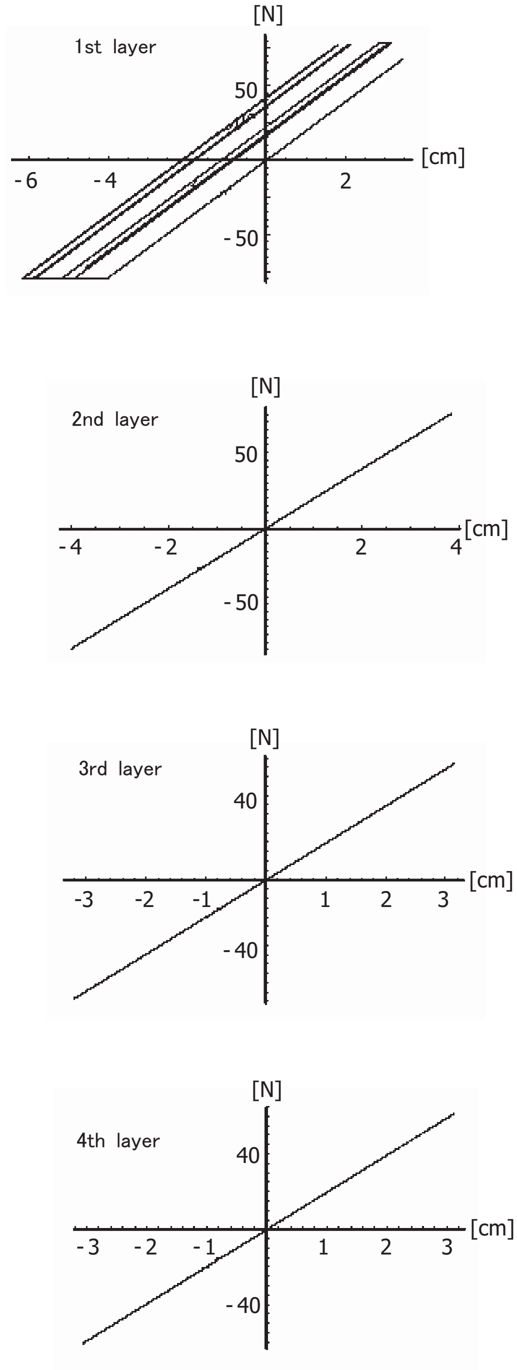

Fig.34 An actual restoring force characteristic (4DOF model).

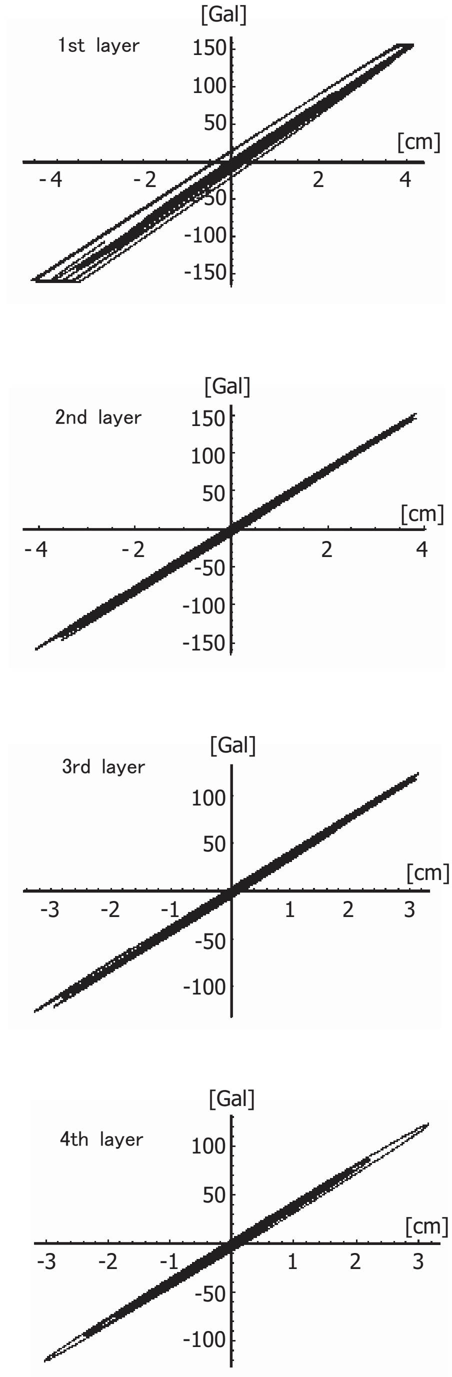

Fig.35 A measured restoring force characteristic (4DOF model).

istics. As a key technology of a node layer, AD conversion in high rate sampling was adopted. As a key technology of a system layer, time synchronization by wired communication and data correcting system were adopted. As a key technology of a prosessing layer, a method of time synchronization by using vertical component of acceleration data set and utilization of integration method of frequency domain were proposed. Moreover a method of time synchronization by wireless communication was implemented in sensor platforms and it is confirmed that these platforms could be synchronized at most in 2[ms].

Verification by shaking table test Performance of acceleration measurement system and accuracy of time integration were verified by shaking table test. As a result it is confirmed that amplitude characteristic is fine in natural frequency band of a target structure. It is also confirmed that maximum error of time integra-

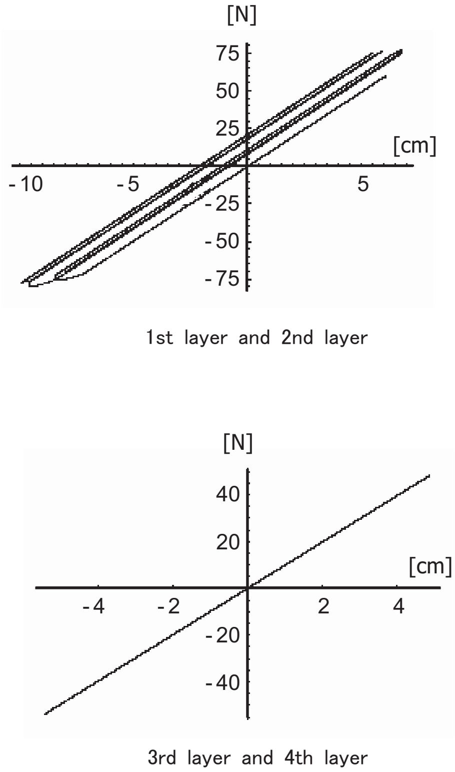

Fig.36 An actual restoring force characteristic considering 4DOF as 2DOF.

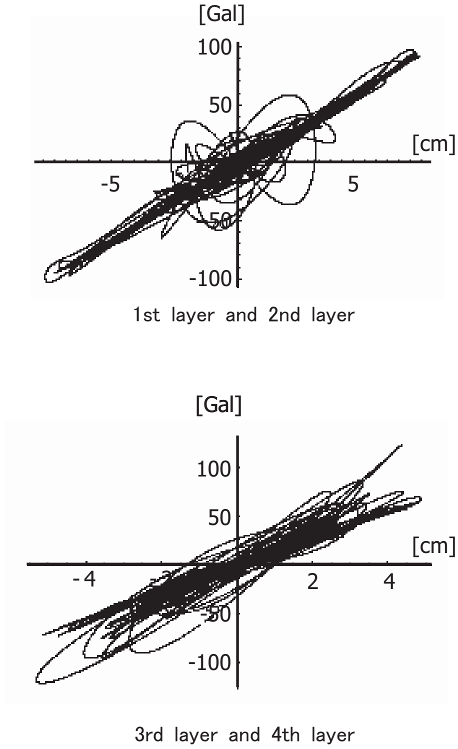

Fig.37 A measured restoring force characteristic considering 4DOF as 2DOF.

tion is about 1[mm].

Measurement of an actual structure A final goal of this research is measurement of restoring force characteristics of actual civil structures as accurate as that of experimental structure measured in shaking table test. During the observation period, seismic vibrations were appropriately corrected for about 30 cases. As a result of data processing, slope of restoring force characteristics computed from measurement data has constant value during the observation period. This result corresponds to a fact that natural frequency has constant value. Sampling frequency and data length are 2000[Hz], 24[bit] respectively in actual measurement. Same analysis in lower specs concluded that it is enough to measure in 200[Hz], 16[bit] in actual use.

Investigation by numerical calculation Numerical calculations were conducted for a structure which is modeled in shear components, in order to investigate whether restoring force characteristics could be reappeared only by acceleration data sets corrected on every layers. As a result of the calculations, restoring force characteristics computed only from acceleration data sets have same shape like correct one which is a result of numerical calculation for a structure with stiffness which is represented by bi-linear model. Numerical calculations for cases that accelerometers were set on every several layers were conducted. Regarding 2DOF structure as SDOF and computing restoring force characteristics of the structure, it becomes clear that plastic deformation could be distinguished if 1st mode is extracted.

Future works As to distinguish of plastic deformation by observation of restoring force characteristics, the following points are considered as future works.

- Development of a method to compute restoring force characteristics objectively
- Devising a method to compute indeces which represents damage level of target structures by using restoring force characteristics
- Development of a system which computes damage level of target structures by responses of the structures automatically

As to sensor network, the following points are considered as future works.

- Development of a sensor which has an accuracy needed for measurement of appropriate restoring force characteristics
- Development of wireless modules which can communicate in a circumstance with many walls
- Development of an algorithm which enables

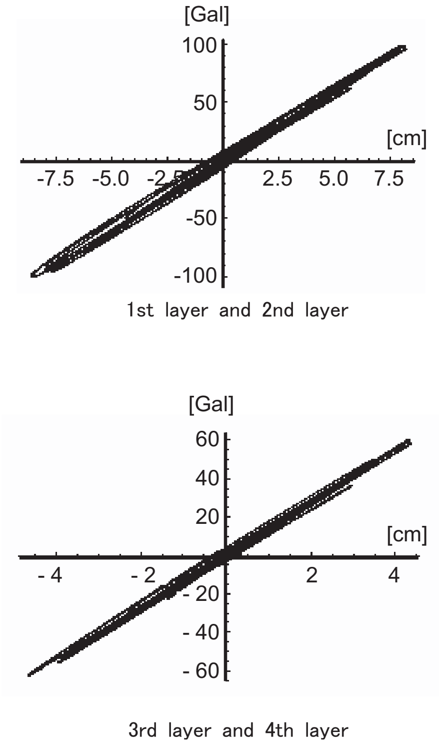

Fig.38 A 1st mode component of actual restoring force characteristic.

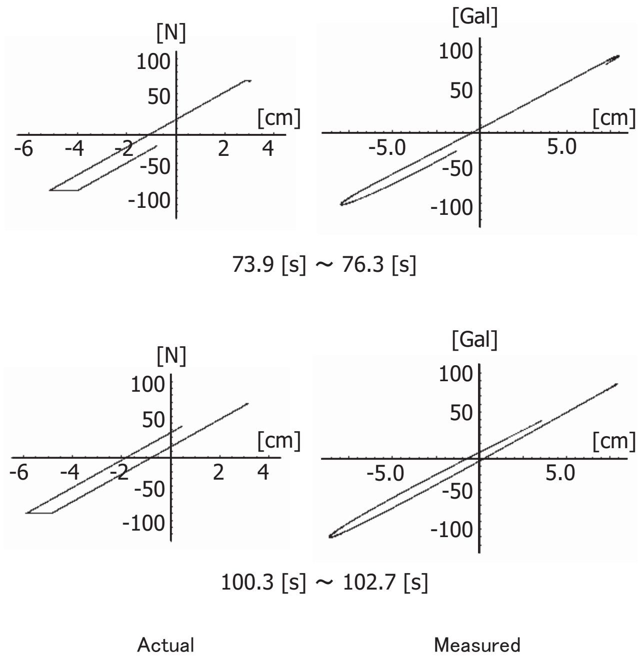

Fig.39 A 1st mode component of measured restoring force characteristic.

time synchronization with plenty of sensors in wide area

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