A SIMPLE AND EASY CALCULATION METHOD TO SEISMIC RESPONSE OF EMBANKMENT CONSIDERING HORIZONTAL AND VERTICAL INTERACTION

Yoshiya HATA1, Koji ICHII2, Seiji KANO3, Takashi TSUCHIDA4, Liming LI5 and Norihiko YAMASHITA6

1Researcher, R&D Center, Nippon Koei Co., Ltd. (2304 Inarihara, Tsukuba, Ibaraki 300-1259, Japan) E-mail:hata-ys@n-koei.jp
2Associate professor, Graduate school of eng., Hiroshima university (1-4-1 Kagamiyama, Higashihiroshima Hiroshima 739-8527, Japan) E-mail:ichiikoji@hiroshima-u.ac.jp
3Research associate, Graduate school of eng., Hiroshima university (1-4-1 Kagamiyama, Higashihiroshima Hiroshima 739-8527, Japan) E-mail:skano@hiroshima-u.ac.jp
4Professor, Graduate school of eng., Hiroshima university (1-4-1 Kagamiyama, Higashihiroshima Hiroshima 739-8527, Japan) E-mail:ttuchida@hiroshima-u.ac.jp
5Manager, Dept. of Civil Eng. & Eng. Mechanics, Columbia university (500 West 120th street, New York 10027-6623, U.S.A.) E-mail:liming@civil.columbia.edu
6Associate professor, Faculty of civil eng., Kobe city college of technology (8-3 Gakuenhigashimachi, Nishi ward, Kobe, Hyogo 651-2194, Japan) E-mail:yamasita@kobe-kosen.ac.jp

Conventional seismic design methods for embankments do not consider the effect of vertical ground motions; however, the level of vertical seismic motions in recent earthquakes was powerful and could have affected the horizontal response of structures located within the zone of shaking. A method to consider the dynamic interaction between horizontal and vertical seismic response is proposed. The proposed method is based on the equation of motion of the SDOF model and uses a cross spring for the interaction between horizontal and vertical movement of the mass. Based on a dynamic centrifuge test, it was shown that the proposed cross-spring model was able to suitably evaluate the seismic response of the embankment both in the horizontal and vertical directions.

Key Words: embankment, earthquake, SDOF model, interaction, dynamic centrifuge model test

1. INTRODUCTION

Strong vertical seismic motion has been regarded as the one of the significant characteristics of the 2004 Mid Niigata earthquake and the 2008 Iwate-Miyagi nairiku earthquake. For example, at the Takezawa site of Yamakoshi village where a huge ground disaster occurred during the 2004 Mid Niigata Earthquake, the peak ground acceleration in UD (up-down) direction (1059gal) was greater than that of the motion in EW (east-west) directions (722gal)1. A comparison of the horizontal and vertical peak ground accelerations is shown in the Fig.1, and it can be seen that the magnitudes of the vertical peak accelerations are about half those of the horizontal peak accelerations. Thus, it could be unreasonable to neglect the influence of vertical motion on structural damage3. However, the effect of vertical motion is not sufficiently accounted for in the seismic design of embankments3.

Though the seismic resistance of embankments to the effects of vertical seismic motion has been examined to a limited extent, much remains unelucidated3. For the evaluation of seismic response, a mass-spring model has been used separately against horizontal and vertical seismic motion at the same time. A method to calculate the response acceleration as a composed input using the results of the separately conducted response analyses was
proposed in the past. In order to consider the horizontal and vertical response of embankment simultaneously, the authors proposed a new numerical analysis method for the seismic response of embankments. In this model, an embankment was represented by a single mass-spring and the seismic responses were assumed to be rocking motions. However, there were some problems with this model because the vertical seismic response was not adequately taken into consideration. For example, the horizontal seismic response could occur even when only vertical shaking is applied as the input motion, but this effect cannot be considered in the model.

In this study, a simple calculation method for seismic response considering the interaction between the horizontal and vertical shaking of an embankment is proposed.

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2. MODELING OF EMBANKMENTS

The equation of motion of the embankment shown in the Fig.2(a) is given as equation (1) using the natural frequency of embankment in the horizontal direction \( f_H \). It is the horizontal response SDOF model.

\[
\ddot{x} + 4\pi \xi f_H \dot{x} + 4\pi^2 f_H^2 x = -\ddot{X}
\]

where \( \xi \) is the damping ratio against horizontal response, \( x \) is the horizontal displacement, and \( \ddot{X} \) is the input earthquake acceleration in the horizontal direction.

The equation of motion in the vertical direction for the embankment shown in the Fig.2(b) is given as equation (2) using the natural frequency \( f_V \) and damping ratio \( \xi_V \).

\[
\ddot{y} + 4\pi \xi_V f_V \dot{y} + 4\pi^2 f_V^2 y = -\ddot{Y}
\]

Where \( y \) is the vertical displacement and \( \ddot{Y} \) is the input earthquake acceleration in the vertical direction. Note that the dynamic interaction between the horizontal and vertical seismic response are not considered in equations (1) and (2).

The authors formerly modelled the embankment by using the rotary response SDOF model as shown in the Fig.2(c), which can take the horizontal and vertical component of the earthquake acceleration into consideration simultaneously. The following equation of motion was proposed using the equilibrium of the moment, based on the assumption that the embankment rotates around the position of the rotary spring.

\[
\ddot{\theta} + 4\pi \xi_H f_H \dot{\theta} + 4\pi^2 f_H^2 \theta = -\frac{\ddot{X} - \ddot{Y}}{H_E} + \frac{\ddot{Y}}{H_E}
\]

Where \( \theta \) is the seismic response rotation angle and \( H_E \) is the equivalent height of embankments in the static condition. However, the following problems remained in this rotary response SDOF model.

1) The model considers the seismic response of embankments only in the horizontal direction, and vertical seismic response is evaluated only as the vertical component of the rotary motion. In other words, the seismic response of embankments in the vertical direction is not being fully taken into consideration.

2) Horizontal seismic response does not occur when the input earthquake acceleration is vertical only. In other words, the interaction between the horizontal seismic response and the vertical seismic response is not considered.

3) Since the spring rotation is calculated based on the natural frequency in the horizontal direction only, the natural frequency in the vertical direction is not considered and thus the seismic response in the vertical direction cannot be adequately evaluated.

In this study, a model is proposed in which an embankment is modeled as a mass-spring type model. The mass is connected to two springs representing horizontal and vertical seismic response. An illustrated explanation of the proposed model is shown in the Fig.3. The variables shown in Fig.3 and Fig.4 are defined as follows, \( X \) is the horizontal displacement of the ground surface, \( Y \) is the vertical displacement of the ground surface, \( x \) is the horizontal seismic response of the embankment, \( y \) is the vertical seismic response of the embankment, \( m_E \) is the effective mass of the embankment, \( H_E \) is the equivalent height of the embankment, \( k_H \) is the spring constant against the horizontal (shearing) seismic response and \( k_V \) is the spring constant against the vertical (stretching) seismic response. The equivalent length \( L_E \) of the
embankment after seismic deformation. The potential energy can be expressed as
\[ L_E = x^2 + (H_E + y)^2 \] (4)

Using the Taylor expansion is used, an upper equation can be rewritten as follows.
\[ L_E = H_E + y + \frac{x^2}{2H_E} \] (5)

Next, the kinematic energy will be derived. The kinematic energy of the vibration model is defined as follows.
\[ T = \frac{1}{2} m_E \left( \dot{x}^2 + \dot{y}^2 + \dot{\theta}^2 \right) \] (7)

The potential energy against the shearing spring is given as follows.
\[ U^H = \frac{1}{2} k_H x_i^2 \] (8)

The potential energy against the stretching spring is found by using the equation (3) as follows.
\[ U^V = \frac{1}{2} k_V \left( y + \frac{x^2}{2H_E} \right)^2 \] (9)

Therefore, the potential energy of the vibration model can be defined by adding equation (8) and equation (9).
\[ U = U^H + U^V \] (10)

Finally, the equation of motion of this model will be derived. Usually, the equation of motion is solved by taking Lagrangian \( \mathcal{L} = T - U \) with the generalization coordinate \( q \).
\[ \frac{\partial}{\partial t} \left( \frac{\partial \mathcal{L}}{\partial \dot{q}} \right) - \frac{\partial \mathcal{L}}{\partial q} = 0 \] (11)

However, this equation of motion can be solved by the next formulation because in this process the kinematic energy \( T \) does not contain \( q \), and potential energy \( U \) did not contain \( \dot{q} \).
\[ \frac{\partial}{\partial t} \left( \frac{\partial T}{\partial \dot{q}} \right) - \frac{\partial U}{\partial q} = 0 \] (12)

Here, generalized \( q \) is \( x \) and \( y \), and the differential equation is given as follows. The first term of left side in equation (12) becomes the next equations.
\[ \frac{\partial T}{\partial \dot{x}} = m_E (\ddot{x} + \dot{X}) \] (13)
\[ \frac{\partial T}{\partial \dot{y}} = m_E (\ddot{y} + \dot{Y}) \] (14)

The second term of the left side in equation (12) is shown as the next equations.
\[ \frac{\partial U}{\partial x} = k_H x + k_V y \] (15)
\[ \frac{\partial U}{\partial y} = k_H y + k_V x \] (16)

Where, the parametric coefficients \( \alpha \) and \( \beta \) in equations (15) and (16) are shown as follows.
\[ \alpha = \frac{x}{H_E} \left( y + \frac{x^2}{2H_E} \right) \] (17)
\[ \beta = \frac{x}{H_E} \left( y + \frac{x^2}{2H_E} \right) \] (18)

Substituting equations (13), (14), (15), (16), (17) and (18) into equation (12), the following differential equation can be derived.
\[ m_E \ddot{x} + k_H x = -m_E \ddot{X} - k_V \alpha \] (19)
\[ m_E \ddot{y} + k_V y = -m_E \ddot{Y} - k_V \beta \] (20)

Here, \( \alpha \) and \( \beta \) in equations (19) and (20) are the parameters which were not considered in the equation of the usual single degree of freedom type model, and they express the effect of interaction between the horizontal and vertical seismic response of the embankment. It is clear that the two equations of motion were independent when the parametric coefficients \( \alpha \) and \( \beta \) are not considered, and that equations (19) and (20) correspond to the equation of motion of the usual single degree of freedom type model. The following equations of motion are derived from equations (19) and (20) when damping is taken into consideration.
\[ m_E \ddot{x} + c_H \dot{x} + k_H x = -m_E \ddot{X} - k_V \alpha \] (21)
\[ m_E \ddot{y} + c_V \dot{y} + k_V y = -m_E \ddot{Y} - k_V \beta \] (22)

Here, \( c_H \) and \( c_V \) are viscous damping constants for the horizontal and vertical seismic responses respectively, and they are defined as follows.
\[ c_H = 2 \xi_H \sqrt{m_E k_H} \] (23)
\[ c_V = 2 \xi_V \sqrt{m_E k_V} \] (24)

\( \xi_H \) and \( \xi_V \) are damping ratios for the horizontal and vertical seismic responses of the embankment vibration model. The horizontal and vertical spring constants of the embankment model, \( k_H \) and \( k_V \), in equations (21), (22), (23) and (24), are defined as follows.
\[ k_H = 4 \pi^2 f_H^2 m_E \] (25)
\[ k_V = 4 \pi^2 f_V^2 m_E \] (26)

Substituting equations (23), (24), (25) and (26) into equations (18) and (19), the following equations are derived.
\[ \ddot{x} + 2 \xi_H \omega_H \dot{x} + \omega_H^2 x = -\ddot{X} - \omega_H^2 \alpha \] (27)
\[ \ddot{y} + 2 \xi_V \omega_V \dot{y} + \omega_V^2 y = -\ddot{Y} - \omega_V^2 \beta \] (28)

According to equations (27) and (28), the equivalent height \( H_E \) of the embankment is a very important variable.
in assessing the vertical-horizontal interaction, which is controlled by the coefficient \( \alpha \) and \( \beta \). The equivalent height \( H_E \) of the embankment can be found using the relationship between the vibration mode and the mass as follows\(^9\):

\[
H_E = \sum_{p=1}^{n} m_p \varphi_{pq} \left( \sum_{i=1}^{n} h_i \right)
\]  

(29)

Where \( m_p \) is the mass in the \( p \) layer and \( \varphi_{pq} \) is the response amplitude of the \( q \)th vibration mode. \( h_i \) is the height for one layer when the embankment height \( H \) was divided into \( n \) horizontal layers as defined by the following equation.

\[
H = \sum_{i=1}^{n} h_i
\]  

(30)

The whole effective mass \( m_E \) of the embankment is given in the same way as follows:

\[
m_E = \sum_{i=1}^{n} m_i
\]  

(31)

The natural frequency in the horizontal and vertical direction, \( f_H \) and \( f_V \), and the equivalent height \( H_E \) of the embankment are usually chosen to express the first mode in the MDOF model based on engineering judgment. If the equivalent height \( H_E \) can be decided suitably, this model can be applied to various embankment shapes.

### 3. EXAMPLE OF NUMERICAL ANALYSIS

The natural frequency of an embankment in the horizontal direction \( f_H \) ranges from 2.0Hz to 4.3Hz\(^{10}\). The mean of that range, 3.15Hz, has been assumed in the following calculation. The natural frequency \( f_V \) of the embankment in the vertical direction is assumed to be 1.4 times the natural frequency \( f_H \) in the horizontal direction\(^{11}\). Therefore, \( f_V \) is 4.41Hz. The embankment is assumed to have a symmetrical trapezoid shape with a shear wave velocity of 100m/sec. The breadth of the embankment crest is 8m and the gradient of the embankment slope is 1:1.8. The embankment height \( H \) was 9m based on the previously defined equation for natural frequency of embankments in the horizontal direction\(^{12}\). The density of embankment was assumed to be uniform and using equation (29) the equivalent height \( H_E \) of the embankment model in the first vibration mode was found to be 3.64m. Both the horizontal and vertical damping ratios \( \xi_H \) and \( \xi_V \) were assumed to be 5\%. The NS (north-south) and UD components of the motion observed in the 1995 Hyogoken-nambu earthquake were used as the input earthquake motions. This numerical calculation is a linear seismic response analysis and spring constants in the horizontal and vertical directions remained constant throughout the calculation.

The list of calculation cases is provided in Table 1. Each case corresponds to a different combination of calculation model and input motion. The four aforementioned SDOF models were adopted as the calculation models. Three patterns of the input motion were adopted: (1) horizontal motion only, (2) vertical motion only, and (3) both horizontal and vertical motion.

The time histories of the absolute response accelerations in the horizontal and vertical directions have been summarized in Fig.5.
Fig. 5 Time history of absolute response acceleration in the horizontal and vertical directions.
The peak acceleration in the horizontal direction for Case 3 is 2108 gal (Fig.5(c)), is about the same as that of the usual simple SDOF model case (Case 1: 2115 gal). On the other hand, the peak acceleration in the vertical direction in Case 3 is 9 gal, which is induced by the vertical seismic response of rotary motion of the mass. Because the seismic response in the vertical direction is not being taken into consideration in Cases 3 - 5, the acceleration time histories of the input earthquake motions and output earthquake motions (responses) for the UD component are almost the same and the peak accelerations in the outputs are greatly lower than those of a standard case (Case 2: 1179 gal) It can thus be said that the seismic response of the embankment cannot be suitably simulated by the rotary response SDOF model.

In the case of the cross-spring model, the peak acceleration of the vertical seismic response due to the contribution of a horizontal seismic input is 45 gal (Fig.5(f), Case 6), and the peak acceleration of the horizontal seismic response due to the contribution of a vertical seismic input is 2 gal (Fig.5(g), Case 7). Thus, it can be seen that the interaction between the horizontal shaking and the vertical shaking has been considered. In this case, the vertical seismic response due to the contribution of horizontal seismic response is larger than the horizontal seismic response due to the contribution of the vertical seismic response. The peak accelerations of the horizontal response shown in Case 6 and the vertical response shown in Case 7 are almost the same as the standard peak horizontal and vertical accelerations shown in Cases 1 and 2, respectively. When the both horizontal and the vertical inputs were considered in Case 8 (Fig.5(h)), the horizontal seismic response was almost same as that of the standard case (Case 1). However, the peak acceleration in vertical shaking is 1203 gal, which is far beyond the standard case (Case 2) and can be attributed to the contribution of the horizontal seismic response to the vertical response. Therefore, the cross-spring model has the possibility to simulate the seismic response of embankments more suitably than other types of SDOF models.

4. THE EXAMINATION OF APPLICATION

The applicability of the proposed cross-spring model was examined by means of a centrifuge model test. The geometry of the model used in the dynamic centrifuge test is illustrated in Fig.6. The model is for an embankment with a height of 20 m, a slope of 1:1.8, and a scale of 1/50. The material properties for embankment model are given in Table 2. The model embankment was constructed of a sandy soil with a density corresponding to a degree of compaction \( D_c \) of 90%. The embankment model was made with 4cm layers of soil compacted to the specified density. The sandy soil was the mixture of Toyoura sand and kaolin clay with a dry density ratio of 9:1. Water was added to the dry material mixture in order to achieve the optimum moisture content. Based on CD test results, the material is known to have negative dilatancy characteristics. Silicon rubber was used at the boundary between the embankment model and the side wall of soil tank to mitigate the shaking transmitted from the tank frame.

The experiment was conducted by the following process. First, the static stress condition was reproduced in the centrifugal force condition (50 g). Then, the model was shaken with a small amplitude white noise wave that would allow for the estimation the natural frequency of the embankment. Next, the natural frequency of the embankment model was estimated from the Fourier spectrum of the observed acceleration at the location of the focused accelerometer (Fig.6) where the average confining pressure, \( \sigma_{cr} \), was 70.5 kPa. The natural frequency of the embankment model was estimated to be 2.4 Hz in the horizontal direction and 3.3 Hz in the vertical direction. Finally, horizontal shaking was applied without vertical shaking. The NS component of the motion observed at the Kobe JMA in the 1995 Hyogoken-nambu earthquake (Peak acceleration = 818 gal) was used as the input earthquake motion. Note that the peak acceleration was adjusted to be 460 gal in consideration of the capabilities of shaking table device. As shown in Fig.6, the seismic response acceleration in both the horizontal and the vertical directions and the horizontal and the vertical seismic displacements at the shoulder were measured. The observed deformation of the embankment after the shaking is shown in Fig.7.

<table>
<thead>
<tr>
<th>Table 1 The list of calculation cases.</th>
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<tbody>
<tr>
<td>Case No.</td>
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<tr>
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<tr>
<td>Case 1</td>
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<td>Case 2</td>
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<td>Case 3</td>
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<td>Case 4</td>
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<td>Case 6</td>
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<tr>
<td>Case 7</td>
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<tr>
<td>Case 8</td>
</tr>
</tbody>
</table>

The observed deformation of the embankment after the shaking is shown in Fig.7.
Table 2 The material properties for the embankment model.

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Value</th>
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<tbody>
<tr>
<td>Sand (90%)</td>
<td>Silt (4%)</td>
</tr>
<tr>
<td>Clay (6%)</td>
<td></td>
</tr>
<tr>
<td>Maximum grain size (0.425 mm)</td>
<td></td>
</tr>
<tr>
<td>Maximum dry density (1.75 t/m³)</td>
<td></td>
</tr>
<tr>
<td>Optimum moisture content (11.8%)</td>
<td></td>
</tr>
<tr>
<td>Wet unit weight (17.1 kN/m³)</td>
<td></td>
</tr>
<tr>
<td>Cohesion (1.39 kPa)</td>
<td></td>
</tr>
<tr>
<td>Internal friction angle (33.8°)</td>
<td></td>
</tr>
</tbody>
</table>

Observed accelerations at the bottom of soil tank are shown in the Fig.8. Because the observed acceleration in the vertical direction was very small, the vertical input motion is neglected in the following calculation.

A comparison of the acceleration time history that was observed at the focal point (Fig.6) and the calculation result from the cross-spring model is shown in the Fig.9. An equivalent height $H_e=10$ m and a damping ratio $\xi=5\%$ in both the horizontal and the vertical direction were used in the calculation. As can be seen in Fig.9(b), in spite of using only horizontal input, a vertical seismic response with a peak acceleration of about 20 gal was measured. Considering the fact that the calculation method is based on a SDOF model, it can be said that the results are not a bad estimation of the observations in the horizontal and the vertical directions. A comparison of the Fourier spectra for the experimental results and the calculation results is shown in the Fig.10. As evidenced by Fig.10(a), the agreement is not so bad in the horizontal direction. However, in the vertical direction, the observed peak at 2.8 Hz and other lower frequency peaks are not evaluated by the model. To investigate the reason for this, acceleration without shaking was recorded and its Fourier spectrum is shown in Fig.10(b). It can be seen that the observed vibrations with frequencies lower than 2.2 Hz in the vertical direction can be regarded as the effects of the noise.

A comparison of the experimental results and the calculation results in the nonstationary spectrum of the horizontal shaking is presented in Fig.11. According to this figure, the two spectra are similar in a broad perspective, and the calculation results from the cross-spring model can be regarded as a good estimation of the phenomena. Note that in the calculation result the responses in the frequency range of 3 to 4 Hz are far smaller than those observed. This is because the cross-spring model models only the first response mode in the horizontal and vertical directions. In other words, the seismic response of the higher order vibration modes (e.g. 2nd and 3rd vibration mode) cannot be considered in the proposed model.

A comparison of the nonstationary spectrum in the vertical direction is presented in Fig.12. The calculated results show a certain agreement with the observations for the frequency range of 3 to 5 Hz. The effect of the aforementioned noise could be the reason for the discrepancies at frequencies lower than 2 Hz. Moreover, the fact that the cross-spring model cannot consider the high order vibrational modes could be the reason for the discrepancies at frequencies higher than 5 Hz. In the Fig.12 it can be seen that the experimental spectrum shows nonstationary responses of 2.0 Hz and 2.8 Hz from 9 to 10 sec, which differ from the natural frequency of 3.3 Hz in the vertical direction. The seismic response due to the seismic sliding of the embankment could be the reason for these nonstationary responses.
Though the proposed cross-spring model is a SDOF mass-spring model, the seismic response of the embankment in the first vibration mode can be estimated adequately. Additionally, this model can consider the horizontal and vertical dynamic interaction. This model is based on the assumption of linear response. When the amplitude of the input motion increases, the shear modulus will decrease and the damping ratio will increase as the soil’s response becomes non linear. Although these non-linearity effects remain for future study, it is understood that these effects are small in the experimental results as evidenced by the accelerogram in Fig.9 and the nonstational spectrum in Fig.11. This is because the degree of compaction is high (Dc = 90%) and the shear wave velocity is large (Vₕ = 195 m/sec) in this case.

5. CONCLUSION

In this study, a new simple SDOF method for predicting the seismic response of an embankment considering horizontal and vertical dynamic interaction was proposed using the equation of motion. A dynamic centrifuge test was carried out to examine the applicability of the proposed ‘cross-spring’ model.

1) The proposed cross-spring model can evaluate the first vibration mode of seismic response of an embankment both in the horizontal and vertical directions simultaneously. The calculated results show good agreement with the test results.

2) The proposed model can consider the dynamic interaction between horizontal and vertical directions. Based on the calculations, the vertical seismic response resulting from the contribution of the horizontal seismic response is larger than the horizontal seismic response resulting from the contribution of the vertical seismic response.

Further detailed examinations, such as the consideration of the nonlinearity in the cross-spring model, are expected in future studies.

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