Modal identification methodology from Ambient Vibration Monitoring

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In order to identify the modal properties of a structure based on the ambient vibration monitoring, a numerical methodology and a full scale test are presented. The method is based on the auto- and cross- correlation functions and a frequency domain analysis. The methodology was tested in a 5 story steel frame structure constructed at the Disaster Prevent Research Institute, Kyoto University, Japan, using ambient vibration testing. The dynamic response was measured with simultaneous sensors in every floor. The results were compared with FEM results and with previous results obtained from monitoring under forced vibration. The system identification obtained under ambient vibration tests are in very good agreement with the forced vibration and the analytical results..

Key Words : *ambient vibration, system identification, monitoring, steel frame structure*

1. INTRODUCTION

Full-scale testing is the most reliable method to determine the dynamic properties of a structure, which then can serve as basis for validating and/or updating analytical models. Furthermore, the dynamic characteristics extracted from the dynamic response measurements are also essential for structural safety evaluation and structural health monitoring. He *et al*, 2005[1)].

Dynamic tests are usually subdivided into two groups: (a) forced vibration tests and (b) ambient vibration tests. The main problem associated to the performance of forced vibration tests in large structures (ex. bridges, high buildings and dam stems) is the difficulty to excite, with sufficient energy and in controlled manner, their most significant modes of vibration. In the ambient vibration, testing a structure can be adequately excited by wind, traffic, and human activities and the resulting motions can be measured with highly sensitive instruments. Expensive and cumbersome devices to excite the structure are therefore not needed. Consequently, the overall cost of the measurements conducted on a large structure is reduced. Ventura *et al*, 2002[2)].

This paper describe a numerical methodology in order to identify the modal properties of a structure based on the ambient vibration monitoring. The method differs from Natural Excitation Technique (NEXT)[3)] in view that after calculate auto- and cross- correlation functions, the methodology uses a frequency domain instead of time domain. This methodology was tested to obtain the modal parameters of a 5 story steel frame structure constructed at the Disaster Prevent Research Institute, Kyoto University, Japan, using ambient vibration testing techniques. Two series of ambient vibration were conducted on October 21st, and November, 29th. Details about the first analysis, that was done without simultaneous monitoring, are given in Kuroiwa & Iemura[4)]. In this paper only the simultaneous results are analyzed, taking into account that the natural frequencies are obtained using cross spectral analysis. The results were compared with FEM results and with previous results obtained from monitoring under forced vibration. Bae, G., 1999[5)].

2. METHODOLOGY

Considering that the monitored data are simultaneous, is possible to calculate the cross- and auto-correlation functions from the time histories, in order to determine the natural frequencies, by use of the following transfer function, T_{xy},

$$T_{xy} = \frac{P_{xy(f)}}{P_{xx(f)}} \qquad (1)$$

In the particular case of frame structure, P_{xy} corresponds to the cross spectrum of the ground data and each floor data, and P_{xx} corresponds to the power spectrum of the ground data.

The next step of the modal identification is to determine the mode shapes based on the monitored data. From this step, the methodology differs from the Natural Excitation Technique, which uses time domain modal identification scheme to estimate the modal parameters by treating the correlation functions as free vibration responses. In the following methodology, the mode shapes are obtained from frequency domain.

(1) All mode shape analysis has to be done based on the displacements. In case of the obtained data from sensors are acceleration or velocity, integration methods have to be applied.

(2) From the displacements data in time domain, for each floor, are obtained the auto-spectral density. Taking into account that the natural frequencies were already obtained considering the cross correlation between ground and each floor, during the mode shape analysis, only the auto-spectral from 1^{st} to the last floor, in case of building, are necessary.

(3) From the Power Spectral Density Functions, corresponding to the determined frequencies, are defined the value S_{yy} that are used to determine the positive modal matrix elements ϕ, according to the following equation, Bendat[6)]

$$\phi_{i(y_j)} = \left[S_{y_j y_j (f_i)} \right]^{1/2} \qquad (2)$$

where S_{yy} is the output auto-spectral density value at the i^{th} normal-mode frequency and the j^{th} location.

(4) The phase angles between the simultaneous data have to be obtained in order to identify the positive and negative elements of the modal matrix, and consequently, the full mode shapes. The phase angle between two signals is given by

$$\phi_{xy}(f) = \tan^{-1}\left[\frac{Q_{xy}(f)}{C_{xy}(f)} \right] \qquad (3)$$

Where, Q_{xy} is the coincident spectral density function and C_{xy} is the quadrature spectral density function, both related to the cross-correlation function between signals x and y. In the following example, the values of x were taken as reference in the 1^{st} floor data, and the y values the 2^{nd} to 5^{th} floor.

3. FULL SCALE TEST

(1) Description of frame model

Test frame is five-story steel structure shown in **Fig. 1** and the elevation view, with cross section of members are shown in **Fig. 2**.

This structure was modeled with two-dimensional beam elements. All members with rigid connections. As for the external columns, 1^{st} and 2^{nd} floors, the total geometric moment of inertia was considered 4.054×10^{-4} m^4 and from 3^{rd} to 5^{th} floors, 2.560×10^{-4} m^4. As for the internal columns, 1^{st} and 2^{nd} floors, 1.199×10^{-3} m^4 and from 3^{rd} to 5^{th} floors, 8.026×10^{-4} m^4.

The floors are consisted of two steel beams and a reinforced concrete slab. In view of the difficulties to evaluate the degree of interaction between slab and beam, as first analysis, the total inertia was considered, 1.15×10^{-3} m^4 , which represents the total geometric moment of inertia of the only two steel beams. In the second analysis, was considered full interaction between the composite members. Considering the effective width of concrete, w_{eff} = 1.0 m, and thickness h = 0.125 m, the inertia of the equivalent section was considered equal to 2.31×10^{-3} m^4. The Young's modulus of concrete and steel was considered respectively 1.8×10^{10} N/m^2 and 2.1

Fig. 1 View of five story steel frame

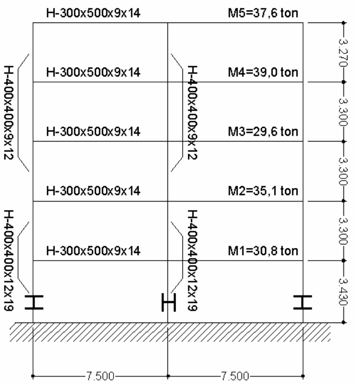

Fig. 2 Elevation view

x 10^{11} N/m^2.

The total mass per floor, including equipments, was considered according to Bae (1999) as following: 1st floor, M=37600 kg; 2nd floor, M=39000 kg; 3rd floor, M=29600 kg; 4th floor, M=35100 kg; 5th floor, M=30800 kg.

(2) Instrumentation

The instruments used for the dynamic measurements of the structure were uni- and tri-axial sensors (**Fig. 3**) with velocity output data (VCT Corp, models UP-255S/ UP-252), cables, A/D

Fig. 3 Sensors

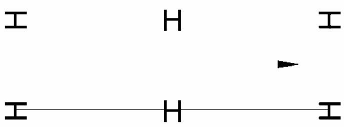

Fig. 4 Typical floor plan and sensor location

converter, amplifier and a laptop computer for the data acquisition and data storage.

The data was recorded for a period of 30 minutes per floor (5 minutes per set up) at 100 samples per second. Only one day was needed to complete the totality of the ambient vibration measurements.

Every floor and the ground level were monitored simultaneously. In order to capture the translational mode in the East-West direction, the sensors were located as shown in **Fig. 4**.

(3) Modal Identification

The technique described were used to perform modal identification. The software Matlab, release 13, was used to obtain the power spectral densities.

a) Natural Frequencies

The graphics in **Fig. 5** show the transfer functions between the cross spectrum of ground and every floor.

The results of natural frequencies obtained from ambient vibration test are presented in **Table 1**, and they are compared with the results obtained from forced and controlled vibration by Bae[5], and with the finite element model.

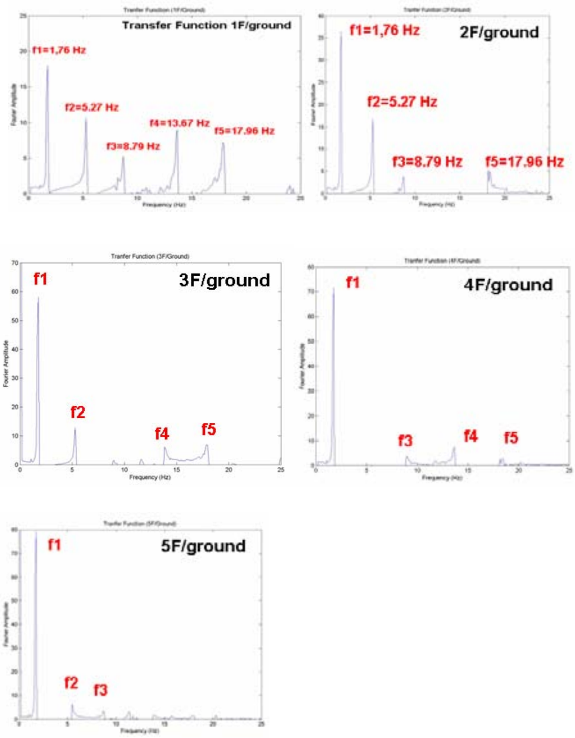

Fig. 5 Transfer functions 1st. – 5th floor

Table 1. Comparison between ambient vibration, forced vibration and finite element model results

Natural Frequencies (Hz)				
Mode	AVM	Forced	FEM_1	FEM_2
1	1.76	1.69	1.69	1.98
2	5.27	5.18	5.22	5.92
3	8.79	8.72	9.26	10.19
4	13.67	13.60	13.26	13.90
5	17.96	17.80	17.75	18.17

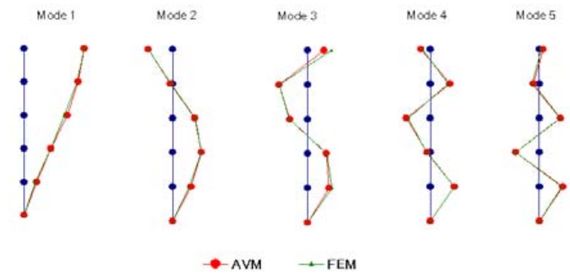

Fig. 6 Comparison of Mode Shapes

b) Mode Shapes

b.1. The output data from sensors were velocity. In order to obtain the displacements in time domain, Trapezoidal Integration Method was applied.

b.2. From displacement data of 1st. to 5th. Floor, the auto-spectral density function were obtained.

b.3. In the frequency domain, corresponding to 5 natural frequencies (i^{th} normal-mode frequency) already defined, were obtained the auto-spectral density values for every floor (j^{th} location). Normalizing the results, the following matrix, with only positive values were obtained

$$M = \begin{bmatrix} 1 & 0.85 & 0.57 & 0.39 & 0.17 \\ 0.9 & 0.09 & 1 & 0.63 & 0.25 \\ 0.72 & 0.77 & 0.61 & 1 & 0.89 \\ 0.45 & 1 & 0.65 & 0.14 & 1 \\ 0.21 & 0.63 & 0.75 & 1 & 0.99 \end{bmatrix} \quad (4)$$

b.4. By application of **Equation 3**, the phase angle matrix were obtained:

$$\theta = \begin{bmatrix} 2.41 & 179.98 & 1.39 & 179.68 & 16.94 \\ 2.06 & 179.53 & 178.9 & 3.47 & 171.10 \\ 2.18 & 0.71 & 179.71 & 176.89 & 1.79 \\ 1.32 & 0.27 & 3.29 & 179.12 & 178.45 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (5)$$

Elements with value close to 180, we assume as negative value, and close to 0, as positive. **Fig. 6** shows the mode shapes obtained based on Ambient Vibration and Finite Element Model.

(4) Validation

The validation of the described method to determine the mode shape was checked, using the time history data obtained directly from the measurements, in order to uncouple the modes of vibration. The total displacement of a cantilever column is the total sum of the modal components, Clough 1975[5)]:

$$v = \phi_1 Y_1 + \phi_2 Y_2 + \ldots + \phi_n Y_n = \sum_{n=1}^{N} \phi_n Y_n \quad (6)$$

For any modal component, the displacements are given by the mode shape vector ϕ_n, multiplied by the modal amplitude Y_n.

Given the mode shape vectors and the total displacements (measured), the objective of the validation is to obtain the modal amplitude functions, Yn, in view of compare the periods of theses functions with the natural period obtained.

The solution is obtained solving the following linear system in every instant $t = tn$:

$$\begin{bmatrix} v_{DOF1} \\ v_{DOF2} \\ v_{DOF3} \\ v_{DOF4} \\ v_{DOF5} \end{bmatrix}_{t=t_n} = [M]_{5x5} \cdot \begin{bmatrix} Y_{1(t=t_n)} \\ Y_{2(t=t_n)} \\ Y_{3(t=t_n)} \\ Y_{4(t=t_n)} \\ Y_{5(t=t_n)} \end{bmatrix} \quad (7)$$

The Y functions obtained are shown in **Figure 7**.

The average periods and frequencies for each mode are given in the **Table 2**.

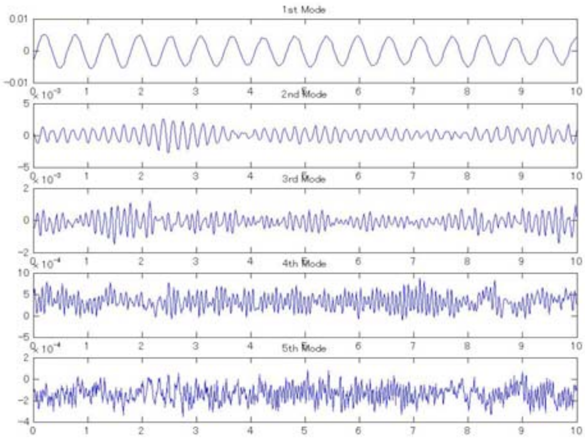

Fig. 7 Modal amplitude functions x time

Table 2 Validation Test – natural periods and frequencies

Mode	Average Period (s)	Average Frequency (Hz)
1st.	0.57	1.75
2nd.	0.18	5.55
3rd	0.118	8.45
4th	0.073	13.69
5th	0.054	18.37

4. CONCLUSION

The results of the ambient response of five story steel frame were presented. All monitoring was done with simultaneous recording data. By using the methodology based on Cross Spectral Analysis, were clear identified the 5 fundamental lateral frequencies and respective mode shapes.

The results were compared with the values from forced and controlled vibration and with the finite element models. In the FEM_1 results, with ignored interaction between concrete slab and steel beam, were closer to the tests results. In general the differences between the FEM_1 frequencies and AVM were very small. Only for the 3rd mode shape, the difference was about 5 %, but this difference occurred also between FEM_1 and the forced vibration test, showing that some updating should be done on the finite element model.

The mode shapes and natural frequencies obtained were checked by uncoupling the recorded time histories data. The results were in very good agreement with the results obtained from AVM.

ACKNOWLEDGEMENT

The authors would like to express sincere gratitude to Professor Y. Suzuki of Kyoto University, for providing the use of the structure and the instrumentation.

Thanks to Ms. A. Shirayama for providing all documentation available about the 5 story-steel frame, and also for helping during the experiment.

The authors would like to thank Professor J. Kiyono of Kyoto University for providing the portable sensor and express gratitude to Mr. H. Goto for kindly explanations on how to manage the seismometer.

We also thank Professor A. Toyooka, Mr. T. Shitanishi, Mr. M. Higuchi, Mr. T. Watajima, and Mr. M. Yokita for invaluable helping during the experiment days.

Our gratitude to Professor A. Igarashi for invaluable commentaries about the validation of modal shapes methodology.

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(Received April 6th, 2007)