MULTI-DIRECTIONAL DISPLACEMENT OF COMPLIANT SOIL

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This paper presents new bi-directional elements in earthquake displacement modeling: 2-D-compliance; 2-D-earthquake loading; and 2D-computation of displacements. Incorporating flexibility and multi-directionality into the Newmark model, compliance affects are found to greatly amplify or de-amplify displacements, relative to traditional rigid-block analyses. The multi-directional shear stress vector is *always* greater than the computed vector from traditional dip-slope models. Thus, traditional Newmark models always underestimate computed stresses on the failure plane, often resulting in underestimation of displacements.

***Key Words**: Earthquake, Newmark, displacement, compliance, 2D, multidirectional analysis, clay,*

1. INTRODUCTION

Permanent seismic deformation of soil occurs when driving stresses exceed the mobilized capacity of the soil to resist shearing. Soils are flexible and resonate in compliant response to earthquake motions. This is especially the case for thick and soft native soils, as well as for fill, landfills and tall soil embankments. The interaction of soil compliance and earthquake motion can significantly amplify or de-amplify permanent displacements. A rational assessment of permanent plastic displacements in soil requires consideration of motions within the sliding mass, whereas, the traditional Newmark-type model for computing seismic displacement assumes rigidity of the ground and cannot account for resonance affects on permanent displacements. Also, traditional Newmark models only assess dip directed displacements in response to dip-directed motions. In nature, slopes are subjected to potentially shearing loads in all directions and can displace in both strike and dip.

To modify the traditional Newmark formulation we incorporated two new elements into our displacement model. First, to account for the flexible nature of slopes we modified the rigid formulation to allow the displaced mass to deform elastically as a viscous-damped oscillator. The oscillations impart stresses to the shear plane and influence the amplitude of deformations. Second, the new model computes a shear stress on the failure plane from multi-directional earthquake-, and sliding mass compliant-motions in both strike and dip, as well as the dip-directed gravitational slope stress.

Incorporating flexibility and multi-directionality into our Newmark model, we find that when the natural resonant period of the landslide mass coincides with the mean earthquake period, displacements can greatly exceed those computed by traditional rigid-block Newmark analyses. On the other hand, when the soil resonant period is out of phase with the earthquake motion, or greatly exceeds it, resultant deformations can be greatly reduced. Multi-directionality is also important when assessing displacements on sloping ground. We found that the resultant multi-directional shear stress vector from earthquake and slope gravitation was *always* greater than the computed vector from traditional dip slope-only models. That is, traditional Newmark models always underestimate computed stresses on the failure plane.

This paper presents a multi-directional and compliant lumped mass Newmark approach for assessing permanent seismic displacements. The approach is formulated around yield criteria for normally consolidated young coastal and marine cohesive sediment. The dynamic sliding block method of Newmark[12)] is widely used to estimate permanent displacements within non-liquefiable slopes during earthquakes. The original formulation of Newmark's method assessed one-dimensional and one-directional (downslope) motion, assumed rigidity of the sliding ground, and was perfectly elasto-plastic with no strain softening. Newmark's method neglects the influence of the dynamic response of sliding mass on the subsequent motions and displacements.

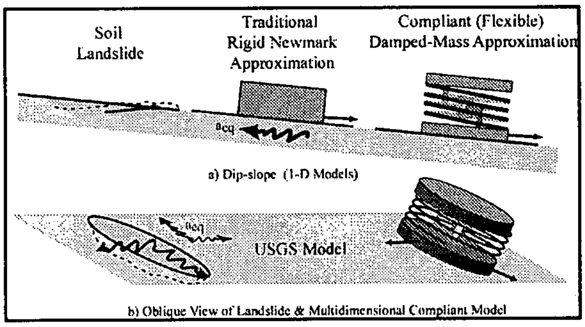

Figure 1. The model described in this study is on the lower right. The model approximates a landslide mass as

a partitioned mass with intervening spring and dashpot, subjected to strike and dip motions.

The original method also only utilized the sloping ground component of motion and, thus, neglected the strike component of motion. Subsequent work [2,4,5,7,8,13] expanded on Newmark's original work, introduced the compliance concept and developed simplified procedures for predicting the permanent displacements in earth dams.

2. DISPLACEMENTS IN COMPLIANT SOIL

Coupled analysis factors the stresses resulting from dynamic motions within the flexible sliding mass into the computation of displacements at the base of that mass. Our formulation is similar to that of Kramer and Smith[6], though modified to incorporate bi-directionality. In the original formulation[12], the driving force on the potential slide block, F_D, incorporates the inertial force of

$$F_D = -\frac{\gamma h}{g}\ddot{u}_b + \frac{\gamma h}{g}\sin\alpha$$

the sliding mass at the slide surface and the static slope gravitational force.

(1)

The effective stress-normalized shear stress on a unit thick and infinitely long soil mass is:

$$\frac{\tau}{\gamma h} = \ddot{u}_b \frac{\gamma}{\gamma'}\cos^2\alpha + \sin\alpha \qquad (2)$$

To modify the rigid Newmark formulation for compliance, we partition the soil column by the mass ratio, M_r, that is the ratio of the mass of the upper portion of the soil column (M_1) and the total mass, M_t, above the potential slide plane:

$$M_r = \frac{m_1}{m_o + m_1} = \frac{m_1}{M_t} \qquad (3)$$

A schematic diagram of the partitioned mass is presented in Figure 1. Shear stiffness and damping elements are incorporated between the partitioned soil mass. With their inclusion, the induced load on the failure plane incorporates the base mass inertial force (eq), the static slope gravitational force (α), and the internal forces induced by the shear stiffness (k) and viscous damping elements (c) of the soil mass:

$$\begin{aligned} F_D &= F_{eq} + F_\alpha + F_k + F_c \\ &= -(1 - M_r)\frac{\gamma h}{g}\cos\alpha \cdot \ddot{u}_b + \frac{\gamma h}{g}\cos\alpha\sin\alpha + c\dot{u}_1 + ku_1 \end{aligned}$$

(4)

The forces in equation 4 are appropriate for down-slope (dip) directed motion. In the slope-parallel (strike) direction F_α has no value.

Loads on the slide plane require a computation of the motions within the soil mass to determine the spring and dash-pot forces acting of the lower and upper masses. The equation of motion in the upper mass is:

$$m_1\ddot{u} + c\dot{u}_1 + ku_1 = -m_1(\ddot{u}_b + \ddot{u}_0) \qquad (5)$$

We use the Newmark beta-method[10,11] to finite difference the computed motions between the upper and lower mass.

3. MULTIDIRECTIONAL ANALYSIS

In multi-directional dynamic-displacement analysis, we resolve a resultant load vector for the combined earthquake motions in the strike and dip directions, and the 2-D soil viscoelasticity. A comparison between the way resultant stress vectors are computed by the original Newmark rigid displacement model, and our compliant multi-directional model is shown in Figure 2. Looking down the confining stress axis at the elliptical soil yield surface, it can be seen that the rigid model (Figure 2a) utilizes only the downslope gravitational stress, and dip-stress associated with the earthquake motion. In the traditional model, the sum of the two combined dip-directed loads fall within the yield stress of the soil, and so a stable soil mass would remain at rest (i.e, $\tau_y > \Sigma\tau_{dip}$). On the other hand, with the additional strike component of motion, and the stresses associated with strike- and dip- directed spring and dashpot forces, Figure 2b, the stress vector falls outside the yield surface. In this case, a stable mass would decouple from the underlying accelerating soil column and accumulate permanent displacements (i.e, $\tau_y < [\Sigma\tau_{dip}^2 + \Sigma\tau_{strike}^2]^{0.5}$).

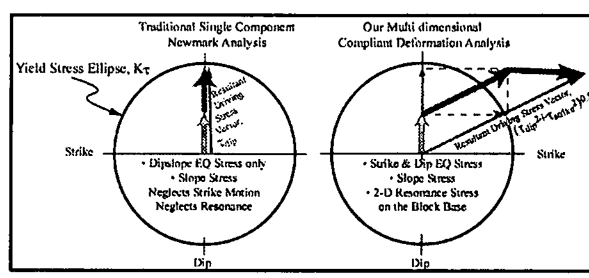

Figure 2. Stress vectors for traditional Newmark analysis (a, left) and multi-directional compliant analysis (b, right), plotted on the yield circle for soil, looking down the isotropic consolidation line. Traditional models neglect strike-earthquake motions and 2-D compliance.

Multi-directionality has a profound influence on dynamic displacement analysis for two reasons. First, on sloping ground the static shear stress resides above the isotropic consolidation line, so any motion that includes a strike component and exceeds the yield circle must induce both strike and dip-directed displacements on sloping ground. Even polarized strike-directed motions on sloping ground must displace the soil partially downslope. Second, it can be seen that neglecting the strike motion in computing the total stress vector underestimates the total applied shear stress on the potential failure plane. Traditional downslope Newmark displacement models underestimate the loads exceeding the soil yield stress and thus, typically, underestimate the computed displacements. That traditional 1-D models must underestimate shear stress on a failure plane is true for both level and sloped ground.

We compute the Pythagorean resultant stress vector as follows:

$$\tau_d = \left(\left(\Sigma\tau_{dip}{}^2\right)+\left(\Sigma\tau_{strike}{}^2\right)\right)$$
$$= (\tau_\alpha + \tau_{eq-dip} + \tau_{k-dip} + \tau_{c-dip})^2 \qquad (6)$$
$$+(\tau_{eq-strike} + \tau_{k-stike} + \tau_{c-strike})^2$$

and the azimuth, θ, associated with that vector is:

$$\theta = \tan^{-1}\left[\frac{\sum \tau_{strike}}{\sum \tau_{dip}}\right] \qquad (7)$$

When the total applied stress exceeds the yield stress then that difference is broken into the corresponding strike and dip differential acceleration components using the computed azimuth, and permanent strike and dip accelerations, velocities, and displacments are computed.

4. MODEL RESULTS & PERMANENT DISPLACEMENT RESPONSE SPECTRA

Parting along a potential slide plane begins when the ground acceleration exceeds the resisting yield acceleration of the soil mass. The ground beneath the over-riding slide mass accelerates faster than the upper soil column can resist without permanent shear. Permanent relative displacements accumulate in the upper soil column as it falls behind the accelerating lower soil column. Relative displacements will continue to accumulate until the acceleration of the ground beneath the slide mass falls below the yield acceleration and the velocity of the slide mass matches that of the lower soil column.

It is easier to drive a soil block downslope with up-dip accelerations in the underlying ground, than the reverse. However, a strong acceleration pulse can preferentially drive a soil block in any direction, even upslope. Soils are flexible and resonate when excited by transient earthquake motions. Resonance affects are important in displacement computation for thick or soft soils like San Francisco or Osaka bay muds, and for landfills and tall engineered-soil embankments. The interaction of earthquake motions and soil resonance in our compliant model can significantly amplify or de-amplify the predicted permanent displacements relative to the traditional Newmark displacement model. Normalizing the permanent displacements computed by a flexible model with those of a rigid model allows for the quantification of resonance-associated amplifications. A critical parameter effecting this amplification in flexible soil systems is the site period of the potential slide mass. The period can be calculated for an infinite slope by the equation $T=2\pi(h/V_s)\cdot M_r^{0.5}$. For embankments, the site period can be calculated by the equation $T=4\pi(h/V_s)\cdot a_n^{-1}$, given tabulated values of a_n [1].

Permanent Displacement Response Spectra (PDRS) plots the amplification ratio of the permanent displacements (flexible system/rigid system) against either the site period, or the ratio of the site period and mean motion period. PDRS is similar to the 'slope spectra'[6]. An example of a PDRS plot is Figure 3.

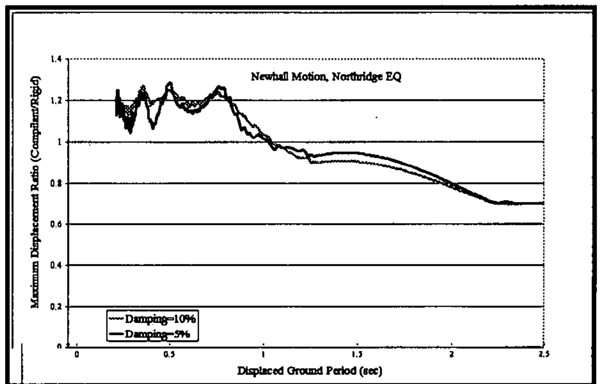

Figure 3. PDRS plot for constant Bay Mud soil conditions, but varied site period subjected to the Newhall motion from the Northridge Earthquake.

The two profiles in Figure 3 are PDRS for the Newhall motion using Bay Mud soil properties given 5% and 10% damping. To compute these profiles, displacement trajectories were determined for soil columns of varied site period, and then the maximum displacement was normalized against the simple rigid model solution. The spectra was determined by varying the stiffness (V_s) and thus site period. When the first mode natural resonant period of the sheared flexible mass coincides with the mean earthquake period at 0.79 sec the displacements can exceed those computed by traditional rigid-block Newmark analyses by approximately 25%. There are also significant amplifications at frequency multiples of the first mode of the site (T/2; T/3; T/4, etc.). These amplifications are the peaks of the PDRS plot.

The alternate way to plot the PDRS is to set the abscissa as the ratio of the site period and mean motion period (here termed 'normalized period'). The point at 0.79s would shift to a dimensionless value of 1.0, but otherwise the shape of the spectra would remain the same. An example of a PDRS plot with normalized periods is in Figure 4. This plot shows maximum displacement results from a suite of soil columns of varied stiffness subjected to the same sinusoidal harmonic motion. Here, it can be seen the peak amplifications occur when the T_{soil}/T_{motion} ratio is approximately 1, 0.5, and 0.33. Earthquake motions are broad-banded, whereas harmonic motions are fixed in frequency. When harmonic motions coincide with the site period the resonance affects can dramatically amplify displacements.

When the soil column resonant period is out of phase with the earthquake motion, or exceeds it, resultant deformations can be reduced. These de-amplifications are the valleys of the PDRS plot. Figure 3 and 4 have low points in the range where T_{soil}/T_{motion} is less than 1, but these valleys still tend to be amplified relative to the rigid system. For T_{soil}/T_{motion} >1.2-1.3 the motions tend to be deamplified relative to the rigid system. In general way we can predict the range of likely site periods and the amplitudes where the rigid model either underpredicts or overpredicts maximum permanent displacements based on the normalized period . Rational assessment of permanent deformations in soil requires consideration of where the site period will be with respect to the likely earthquake motion. PDRS effectively maps the influence of soil-motion coupling on permanent seismic displacements.

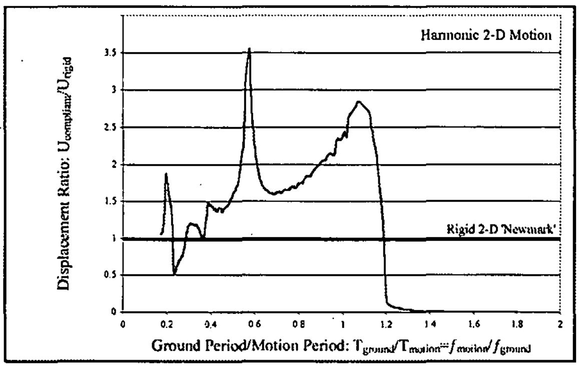

Figure 4. PDRS plot for Bay Mud soil conditions, but varied site period subjected to harmonic motion

5. CONCLUSIONS

There are several noteworthy findings of the bi-directional compliant model. First, strike motions contribute to dip directed deformations on all sloping ground due to the non-isotropic static slope stress, and even on level-ground due to the increased amplitude of the resultant 2-D stress vector. Secondly, compliant stresses influencing block accelerations must be considered when computing the total-induced stress vector at the shear surface. Conventional Newmark analyses presented in the geotechnical literature neglect these two important effects.

This paper describes a critical state-based multidirectional compliant method for assessment of the amplitude of permanent plastic deformations in fine-grained soil deposits during earthquakes. Incorporating flexibility and multi-directionality into the Newmark model, we find that Permanent Displacement Response Spectra (PDRS) characterizes the compliance affects that amplify or deamplify displacements. The addition of a multi-directional shear stress vector is critically important, as it is found that the 2-D vector is *always* greater than the computed vector from traditional dip-slope models. Thus, traditional dip-slope Newmark models always underestimate computed stresses on the failure plane, that often lead to under-computation of permanent displacements.

6. ACKNOWLEDGEMENTS

Pacific Gas and Electric Company funded of this project through the PG&E-USGS CRADA. In particular, Woody Savage, Robert White, Norm Abrahamson, and Lloyd Cluff are thanked for their guidance. USGS Chief Scientists Mary Lou Zoback, James Dieterich, and Michael Carr, and especially JMC Coordinator Daniel Ponti were instrumental in initiating and coordinating this task. Kobe University, Japan is thanked for academic and financial support of Dr. Ryuzo Ozaki during his Post-Doctoral appointment at the US Geological Survey in Menlo Park California in 1999 and 2000.

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