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FEM-FDM Coupled Method for Saturated Soil Analysis Considering Large Deformation

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An improvement of the FEM-FDM coupled scheme is presented for dynamic analysis of fully saturated soil considering large deformation. In the space domain, the equilibrium equation of fluid saturated soil is discretized by the finite element method, as well as the continuity equation is discretized by the finite difference method within a rectangular mesh which is difference from that used by the finite element method. The finite difference method used in a difference mesh would not suffer from numerical problems when the initial mesh of the finite element method is not rectangular or the mesh is heavily distorted for large deformation. The proposed method is applied to a one-dimensional elastic consolidation problem and an embankment problem in order to verify its usefulness.

Key words: *large deformation, saturated soil, finite element, finite difference, numerical method*

1. Introduction

The saturated soil consists of solid grain particles and voids that are filled with water. It can be considered as a mixture of binary phase deformable medium of solid grains and water, each of which is regarded as a continuum and follows its own motion equations. The general theoretical frame-work of the binary porous medium was first developed by Biot [1]. Based on Biot's theory, the incremental finite element method for dynamics was derived by Zienkiewicz et al. [2] and other researcher [3], in which geometrical and material non-linear behavior can be included. We notice that the dynamics of the porous medium mixture has been applied to liquefaction analysis of saturated soil based on large deformation theory [4].

In References [5-6], a FEM-FDM coupled scheme was proposed for liquefaction analysis of saturated sand to reduce the total degrees of freedom and to avoid the shear locking under the undrained condition. But the finite difference method used for the spatial discretization of the continuity equation would suffer from numerical problems when the initial mesh is not rectangular or the mesh is heavily distorted when large deformation occurs. In this paper, after the FEM-FDM coupled scheme considering large deformation is presented for dynamic analysis of fully saturated soil, an improvement of the FEM-FDM method is proposed to overcome this problem and a program code is developed on the basis of LIQCA program [5-6].

2. Governing Equations

Based on the updated Lagrangian method and the u-p approximation formulation, the acceleration of fluid phase can be neglected, and the local equilibrium equation of motion for total mixture of soil skeleton and fluid phase is simplified as

$$\frac{\partial \sigma_{ij}}{\partial x_j} + \rho b_i - \rho \ddot{u}_i = 0 \tag{1}$$

where σ_{ij} is the Cauchy total stress in the combined solid and fluid mixture, u_i is the displacement of the solid skeleton, ρ is the density of the assembly and b_i is the body force.

For the pore fluid, the local equilibrium equation of motion can be written as

$$\frac{\partial (np)}{\partial x_i} - n\rho^f b_i + n\gamma^f k^{-1}\dot{w}_i + n\rho^f \ddot{u}_i = 0 \tag{2}$$

where n is the porosity, p is the pore pressure (taken positive when compressive), ρ^f is the density of the pore water, k is Darcy permeability coefficients, γ^f is the unit weight of the fluid and w_i describes the fluid displacement relative to the skeleton of soil.

Defining the excess pore pressure p_E by

$$\frac{\partial(np - np_E)}{\partial x_i} = n\rho^f b_i \quad (3)$$

Then the equation (2) can be written as

$$\dot{w}_i = -\frac{k}{\gamma^f}\left(\frac{1}{n}\frac{\partial(np_E)}{\partial x_i} + \rho^f \ddot{u}_i\right) \quad (4)$$

According to the law of mass conservation, the local form of the continuity equation can be obtained. For the soil skeleton, we have

$$\frac{\partial(\rho^s(1-n))}{\partial t} + \frac{\partial(\rho^s(1-n)v_i)}{\partial x_i} = 0 \quad (5)$$

where ρ^s and v_i is the density and velocity of the solid particles.
For fluid phase, we get

$$\frac{\partial(n\rho^f)}{\partial t} + \frac{\partial(\rho^f(nv_i + \dot{w}_i))}{\partial x_i} = 0 \quad (6)$$

From equation (5) and equation (6), the following equation can be obtained after some manipulation [5][8].

$$\frac{\partial \dot{w}_i}{\partial x_i} + l_{ii} + \frac{n}{K^f}\dot{p}_E = 0 \quad (7)$$

where l_{ij} is the symmetric rate of deformation tensor and K^f the bulk modulus of the fluid phase.
From equation (4), we obtain

$$\begin{aligned}\frac{\partial \dot{w}_i}{\partial x_i} = -\frac{k}{g\rho^f}\Bigg\{&\left(\rho^f \dot{l}_{ii} + \left(\frac{\partial^2 p_E}{\partial^2 x_i}\right)_i\right) \\ &+ (K^f - p_E)\frac{\partial(\ln n)}{\partial x_i}\frac{\partial(\ln \rho^f)}{\partial x_i} \\ &- K^f\left(\frac{\partial(\ln \rho^f)}{\partial x_i}\right)_i^2 + p_E\left(\frac{\partial^2(\ln n)}{\partial x_i^2}\right)_i\Bigg\}\end{aligned} \quad (8)$$

where g is the gravitational constant.

If the gradients of $\ln(n)$ and $\ln(\rho^f)$ are so small that the quadratic terms in the above expressions can be ignored and satisfy the following

$$\left(\frac{\partial^2(\ln n)}{\partial x_i^2}\right)_i = 0 \quad (9)$$

Then the equation (8) can be expressed by

$$\frac{\partial \dot{w}_i}{\partial x_i} = -\frac{k}{g}\left(\dot{l}_{ii} + \frac{1}{\rho^f}\left(\frac{\partial^2 p_E}{\partial^2 x_i}\right)_i\right) \quad (10)$$

Finally, substituting equation (10) into equation (7), the continuity equation in the final form can be obtained as

$$-\frac{k}{g}\dot{l}_{ii} - \frac{k}{\gamma^f}\left(\frac{\partial^2 p_E}{\partial^2 x_i}\right)_i + l_{ii} + \frac{n}{K^f}\dot{p}_E = 0 \quad (11)$$

It is obvious that the equations (1) and (11) together with the constitutive law will define a coupled set of equations in which u_i and p_E are the only unknown variables.

3. Constitutive Model

In this study, we employ the Jaumann stress rate as an objective measure of stress rate for the constitutive relation to consider the large deformation problem. It is adopted in the present formulation

$$\dot{\sigma}_{ij}^J = \dot{\sigma}_{ij} - \sigma_{ik}\omega_{jk} - \sigma_{jk}\omega_{ik} \quad (12)$$

where $\dot{\sigma}_{ij}$ is the rate of stress, ω_{ij} is the skew symmetric spin tensor.

A general relationship between the objective stress rate and the deformation rate can be written as

$$\dot{\sigma}_{ij}^J = D_{ijkl}\, l_{kl} - \dot{p}\delta_{ij} \quad (13)$$

where $\dot{p}$ is the rate of pore pressure, δ_{ij} is the Kronecker delta and D_{ijkl} is the Eulerian elastic-plastic tensor of the solid skeleton.

In Reference [7], an effective cyclic elastic-plastic constitutive model was proposed by Oka et al. based on a non-linear kinematics harding rule for saturated sand. In the present study, this model is extent so that it can be applied to fit the finite strain theory.

4. Improvement of FEM-FDM Coupled Method

If it is assumed that the time increment keep small enough in each step of dynamic analysis, then during the step we have approximately

$${}^{t+\Delta t}\rho^f = {}^t\rho^f \quad \text{and} \quad {}^{t+\Delta t}n = {}^t n$$

By the updated Lagrangian method, the weak formulation of equation (1) can be obtained as

$$\begin{aligned}&\int_{{}^tV} {}^t\rho {}^t\ddot{u}_i \delta v_i d{}^tV + \int_{{}^tV}\left(\int_{t_1}^{t_2} \dot{S}_{ij}dt\right)\delta\dot{E}_{ij}d{}^tV \\ &= \int_{{}^tA}({}^tT_i + dT_i)\delta v_i d{}^tA \\ &+ \int_{{}^tV} {}^t\rho({}^tB_i + dB_i)\delta v_i d{}^tV - \int_{{}^tV} {}^t\sigma_{ij}\delta\dot{E}_{ij}d{}^tV\end{aligned} \quad (14)$$

where S_{ij} is the second Piola-Kirchhoff stress tensor and E_{ij} is the Lagrangian strain tensor.

The second Piola-Kirchhoff stress rate tensor is assumed that approximately equal to the Truesdell stress rate tensor, then the second Piola-Kirchhoff stress rate tensor can be expressed as

$$\begin{aligned} \dot{S}_{ij} &= \dot{\sigma}^{J}_{ij} + \psi_{ijkl}\, l_{kl} \\ &= D_{ijkl}\, l_{kl} + \psi_{ijkl}\, l_{kl} - \dot{p}\delta_{ij} \end{aligned} \qquad (15)$$

where ψ_{ijkl} is the 4th-order tensor in terms of the current stress components.

The weak formulation of equation (11) is written as

$$\begin{aligned} &-\int_{^tV} {}^{t}\rho_f\, {}^{t+dt}_{\ \ t}\dot{l}_{ii}\, \delta p_E\, d^tV \\ &-\int_{^tV} \left(\frac{\partial^2 ({}^{t+dt}_{\ \ t}p_E)}{\partial x_i^2}\right)_i \delta p_E\, d^tV \\ &+\int_{^tV} \frac{{}^t\gamma_w}{k}\, {}^{t+dt}_{\ \ t}l_{ii}\, \delta p_E\, d^tV \\ &+\int_{^tV} \frac{{}^tn\, {}^t\gamma_w}{kK_f}\, {}^{t+dt}_{\ \ t}\dot{p}_E\, \delta p_E\, d^tV = 0 \end{aligned} \qquad (16)$$

Using a FEM-FDM coupled scheme in which is proposed in References [5-6], equation (14) is discretized in the space domain by the finite element method, and equation (16) is discretized in the space domain by the finite difference method in a same element mesh.

If the initial mesh were not rectangular or the mesh were heavily distorted when the large deformation occurs, the expressions of difference for the second term in equation (16) do not give exact values for the partial derivative of excess pore pressure.

For the improvement in this paper, we discretize equation (14) by the finite element method in a mesh which can be named by FEM_Mesh and discretize equation (16) by the finite difference method in another different mesh which can be named by FDM_Mesh. The FDM_Mesh is different from the deformed mesh FEM_Mesh and can be a rectangular grid in which is beneficial to discretize equation (16) by the finite difference method. Therefore the second term in equation (16) is approximated by a expression of difference in which the error can be of high order. The procedure of this method is explained briefly in the following steps.

(1) Updated the coordinates of the old mesh FEM_Mesh and FDM_Mesh (according to displacements over the previous solution step) to form the reference ones.

(2) A rectangular new FDM_Mesh is generated in the region in which is covered by the reference FEM_Mesh.

(3) Interpolate or extrapolate the relevant quantities such as pore pressure, strain and stress values at the new mesh element using the respective values in the reference mesh element.

(4) Analysis of next set of increments.

The cycle (1)~(4) is repeated until the desired step is reached.

Adding the Rayleigh damping and using the Newmark's β method for the time domain integration, we obtain the final formulations from equation (14) and equation (16) for large deformation analysis of the saturated soil using the FEM-FDM coupled method.

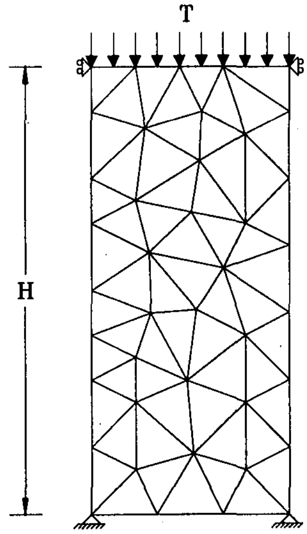

Fig. 1. FEM_Mesh of One-dimensional consolidation

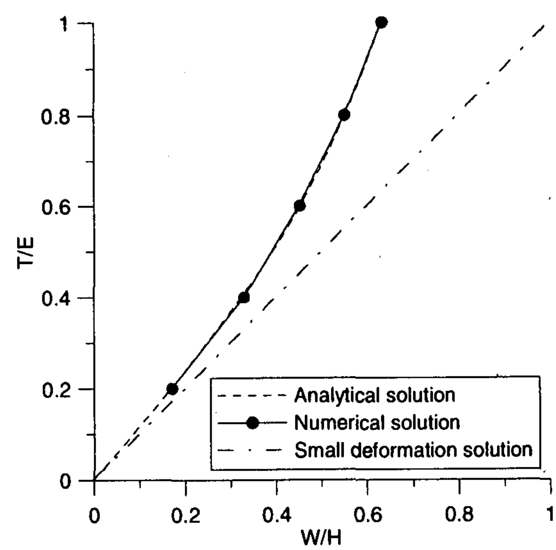

Fig. 2. Vertical settlements W vs. the load level

5. Numerical Examples

On the basis of LIQCA program [5-6], a program code was developed to analyze the large deformation dynamic response of saturated soil according to the procedure mentioned above. A one-dimensional example and a two-dimensional example are now presented to illustrate the foregoing method.

The first one is the problem of the one-dimensional elastic consolidation as shown in Fig. 1. It is analyzed as a ten-meter deep ground, fully saturated by water, infinitely extended in horizontal direction, and subjected to a step load applied at the ground surface, with drainage allowed only through the top surface. A porosity of 0.3, a specific permeability of 0.01 m/s, an elastic modulus of the ground E of 1Gpa, and a zero Poisson ratio are adopted. Gibson et al. [9] developed an analytical theory in which accounted for finite strain for this one-dimensional problem. The final displacements in which are obtained from the developed program and the theoretical solution are drawn vs. the applied load in Fig. 2.

The second example regards the elastic-plastic large deformation response of an embankment (See Fig. 3.) subjected to the horizontal and vertical strong motion record at Port Island during Hyogoken-Nambu earthquake. The responses at node A are shown in Fig. 4.

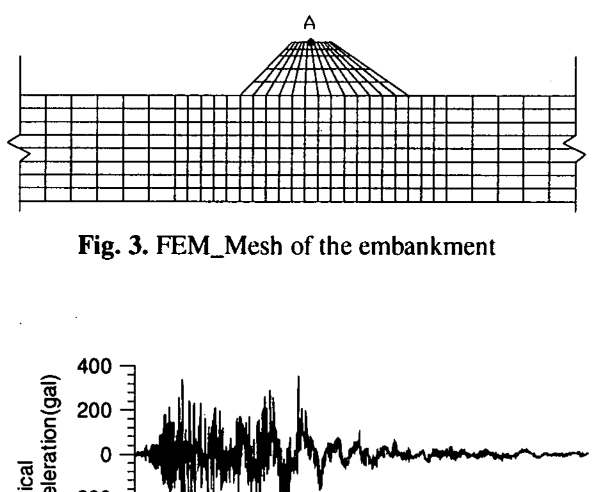

Fig. 3. FEM_Mesh of the embankment

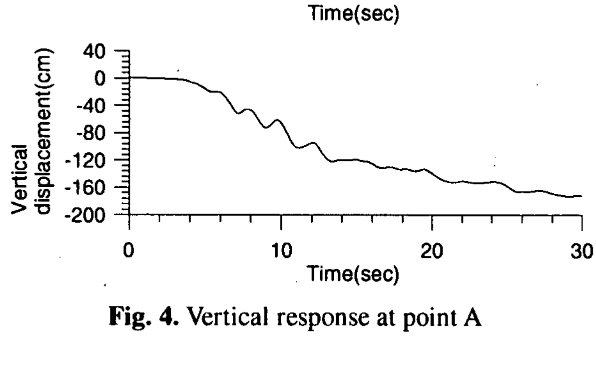

Fig. 4. Vertical response at point A

6. Conclusions

In this paper, the improvement of FEM-FDM coupled scheme is presented for the saturated soil analysis related to large deformation, and a program code is developed based on the LIQCA program. In the space domain, the continuity equation is discretized by the finite difference method within a rectangular mesh in which is difference from that used to discretize the equilibrium equation by the finite element method.

Good agreement was found between the analytical solution of a one-dimensional finite strain elastic consolidation and the numerical solution. The FEM-FDM coupled analysis was also applied for an embankment problem.

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