(第25回地震工学研究発表会講演論文集（1999年7月）)

Linear And Non-Linear Analysis Of Three Dimensional Pile-Soil System In Time Domain

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The pile-soil system is idealized by a finite irregular region joined to a regular semi-infinite far field. A hybrid of Finite Element and Thin-Layer Element methods are used for the dynamic analysis. The former is to develop the dynamic stiffness matrix of the irregular zone for both linear and non-linear formulation in time domain. The latter is to develop the dynamic stiffness matrix for the regular zone in frequency domain and a transformation to time domain is performed using recursive method. The whole system is later assembled for interaction analysis using incremental and time-integration schemes.

Key Words: *Thin Layer Element Method, Finite Element Method, Time Domain, recursive equation, dynamic stiffness coefficient, interaction analysis*

1. INTRODUCTION:

Because of the semi-infinite nature of soil, new solution techniques have been developed accounting correctly for the energy dissipated or introduced into the system because of wave propagation, which includes the effects of non-linear behavior of the soil. Many attempts in this way were made and considerable amount of work has been done in recent years to obtain improved solutions of the above mentioned problems with a particular interest in the seismic design of structures accounting for dynamic soil-structure interaction.

In this study we are concerned specifically with Pile-Soil interaction problems based on the substructure method.

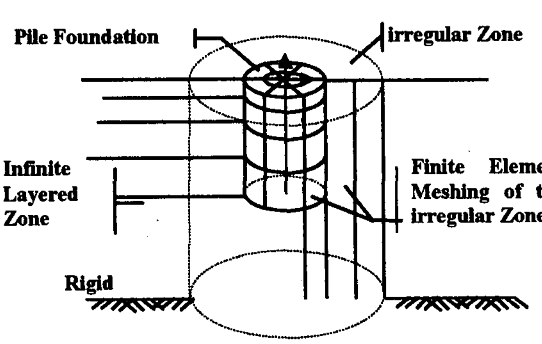


Fig. 1 Model concept and system coordinates of analyzed system

In the substructure approach the impedance is combined with the structural model to perform a dynamic analysis for a given free field loaded from a site response analysis, or for a dynamic load directly applied to the structure. The difficulty to obtain rigorous solutions for an embedded structure with complicated shapes in layered half space make it necessary to resort to a hybrid of Thin Layer Element and Finite Element methods to model the total soil-substructure system. As the regular unbounded soil on the exterior of the interaction horizon up to the infinity behaves linearly, it can be analyzed in the frequency domain. Applying Thin Layer Element Method (TLEM) results in the Dynamic Stiffness matrix in the frequency domain that must be transformed into time domain using a recursive method. The irregular substructure domain is analyzed using linear and non-linear Finite Element Method (FEM) in cylindrical coordinates. The total dynamic system with a non-linear structure can be analyzed and three sets of numerical simulated results are presented.

2. FORMULATION OF THE OUTER LAYERED ZONE:

(1) The Thin Layer Element Method:

(a) General wave equation:

The soil is composed of horizontal layers that are homogeneous, isotropic and linearly viscoelastic with material damping independent from frequency. The vertical boundaries to the irregular region represent the far field as a semi-analytic energy transmitting boundary [3].

The three-dimensional wave equation in cylindrical coordinates for an isotropic homogeneous elastic media can be written as:

$$\left.\begin{aligned}
\rho\frac{\partial u_r}{\partial t^2} &= (\lambda+2G)\frac{\partial\Delta}{\partial r}-\frac{2G}{r}\frac{\partial\omega_z}{\partial\theta}+2G\frac{\partial\omega_\theta}{\partial z}\\
\rho\frac{\partial u_z}{\partial t^2} &= (\lambda+2G)\frac{\partial\Delta}{\partial z}-\frac{2G}{r}\frac{\partial(r\omega_\theta)}{\partial r}+\frac{2G}{r}\frac{\partial\omega_r}{\partial\theta}\\
\rho\frac{\partial u_r}{\partial t^2} &= (\lambda+2G)\frac{1}{r}\frac{\partial\Delta}{\partial r}-2G\frac{\partial\omega_r}{\partial z}+2G\frac{\partial\omega_z}{\partial r}
\end{aligned}\right\}\quad(1)$$

where

$$\left.\begin{aligned}
&\Delta=\frac{1}{r}\frac{\partial(ru_r)}{\partial r}+\frac{\partial u_z}{\partial z}+\frac{1}{r}\frac{\partial u_\theta}{\partial\theta};2\omega_r=\frac{1}{r}\frac{\partial u_z}{\partial\theta}-\frac{\partial u_\theta}{\partial z}\\
&2\omega_z=\frac{1}{r}\frac{\partial(ru_\theta)}{\partial r}-\frac{1}{r}\frac{\partial u_r}{\partial\theta};2\omega_\theta=\frac{\partial u_z}{\partial z}-\frac{\partial u_z}{\partial r}
\end{aligned}\right\}\quad(2)$$

(b) Analytical model and general assumptions:

The displacements are expanded in the tangential direction using analytical functions and can be written as:

$$\begin{cases} u_r = \sum_{m=0}^{N} v_r^m(r,z)\cos m\theta + \hat{v}_r^m(r,z)\sin m\theta \\ u_z = \sum_{m=0}^{N} v_z^m(r,z)\cos m\theta + \hat{v}_z^m(r,z)\sin m\theta \\ u_\theta = \sum_{m=0}^{N} v_\theta^m(r,z)\cos m\theta - \hat{v}_\theta^m(r,z)\sin m\theta \end{cases} \qquad (3)$$

where the subscript m is the harmonic number and N the total number of harmonics required to represent the load. If the load is symmetric about the θ axis, the subscripted $\hat{v}$ is dropped out from the above equations. And we assume that only the Zeroth and First harmonic modes are necessary in the analysis to describe the loading. The forces are similarly expanded to tangential direction.

The general solution of the above for a harmonic excitation with a frequency ω can be written as:

$$\overline{V}_j = [N_H]_j \overline{F}_j \qquad (4)$$

where $\overline{V}_j = \{v_r, v_z, v_\theta\}^T$ is the displacement vector expanded to radial direction for a nodal connection of a layer j with the irregular zone. $\overline{F}_j$ are functions of depth. And the expansion matrix is written as follows:

$$[N_H]_j = \begin{bmatrix} H_{m,r}^{(2)}(kr) & 0 & mH_m^{(2)}(kr)/r \\ 0 & kH_m^{(2)}(kr) & 0 \\ mH_m^{(2)}(kr)/r & 0 & H_{m,r}^{(2)}(kr) \end{bmatrix} \quad \text{where}$$

$H_m^{(2)}(kr)$ is the Hankel function of order m of the second kind.

$H_{m,r}^{(2)}(kr)$ is the first derivative of $H_m^{(2)}(kr)$ with respect to r.

k is the wave number.

(2) Equation Of Motion

The equation of motion of the complete n-layered far field of analysis system can be written as:

$$([\mathbf{A}]k^2 + i[\mathbf{B}]k + [\mathbf{G}] - \omega^2[\mathbf{M}])\{\delta\} = 0 \qquad (5)$$

Since the eigenvalue problem of Eq. (5) is independent of the order m of the Fourier expansion to radial direction, it can be shown that any arbitrary three-dimensional harmonic displacement field in a layered soil can be expressed as a superposition of Love and Rayleigh waves. Hence is always possible to express the displacements in the far field in terms of eigenfunctions corresponding to the natural modes of wave propagation in the layered half space [8].

where $\{\delta\}$ is a vector of 3n components assembled from the displacements $\{\delta^R\}$ of the Rayleigh wave and $\{\delta^L\}$ of the Love wave. The detailed form of the matrices of Eq. (5) can be found in Ref. [2].

(3) Formulation Of The Energy Transmitting Boundary

Using Eq. (3) and (4), the complete force system acting on the cylindrical vertical boundary $r=r_o$ can take the form:

$$\{\overline{P}_o\} = [\mathbf{S}(\omega)]\{\overline{v}_o\} \qquad (6)$$

$$[\mathbf{S}(\omega)] = r_o(i\,[\mathbf{A}][\mathbf{W}][\mathbf{K}][\mathbf{V}]^{-1} + [\mathbf{D}] + [\mathbf{E}]) \qquad (7)$$

where $[\mathbf{W}]$ is a 3nx3n matrix which the columns are formed from the weighted eigenvectors $\{\overline{v}_o\}$, and $[\mathbf{K}]$ is a 3nx3n matrix which has the eigenvalue k at its diagonals. The detailed forms of the matrices of Eq. (5) are to be found in Ref. [2].

3. TRANSFORMATION TO TIME DOMAIN

(1) Rigorous Formulation

The transformation of the dynamic stiffness matrix of Eq. (7) from frequency to time domain can be done using direct or recursive methods. The former is generally done through Fourier transformation of the dynamic stiffness coefficients from the frequency to the corresponding ones in time domain.

$$S(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} S(\omega)e^{i\omega t}d\omega \qquad (8)$$

The above formulation costs high in computation time and memory since it requires a rigorous numerical solution because for every time step a Fourier transformation (FFT) is necessary. In addition to this concept of evaluating the interaction forces in the frequency domain, the amplitudes of the displacements can be calculated recursively using only the amplitudes of the N previous time step. The Fourier transform is thus avoided. The recursive evaluation is, in principle, rigorous. This scheme will lead to a significant reduction in the computational effort and storage requirement when interpolation in the frequency domain is applied.

(2) Interaction Forces In Time Domain

(a) Derivation

The recursive evaluation of the dynamic stiffness matrix in time domain $[\mathbf{S}(t)]$ of Eq. (8) can be done starting from the dynamic stiffness matrix in frequency domain $[\mathbf{S}(\omega)]$. Each dynamic stiffness coefficient in frequency domain $S(\omega)$ of $[\mathbf{S}(\omega)]$ can be approximated as a ratio of two-polynomials in $i\omega$ [11] using a curve fitting technique based on the least-square method:

$$S(\omega) \cong \frac{p_o + p_1 i\omega + p_2(i\omega)^2 + \cdots + p_{M-1}(i\omega)^{M-1}}{1 + q_1 i\omega + q_2(i\omega)^2 + \cdots + q_{N-1}(i\omega)^{N-1}} \qquad (9)$$

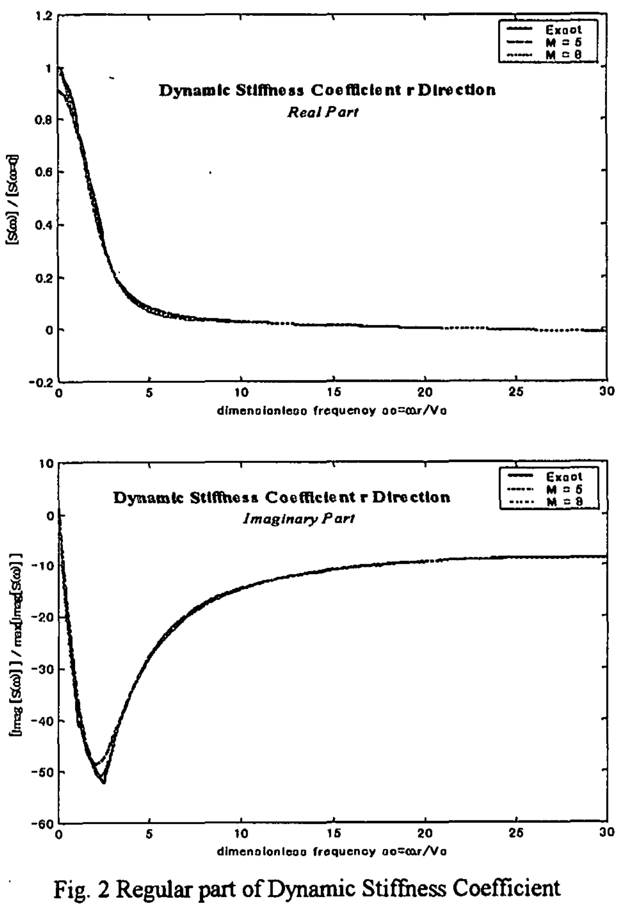


Fig. 2 Regular part of Dynamic Stiffness Coefficient

To integrate a rational function as Eq. (9) it is usually necessary to write it as a sum of partial fraction of the form:

$$\frac{P(i\omega)}{Q(i\omega)}=\sum_{i=o}^{N}\frac{A_i}{i\omega-s_i} \qquad (10)$$

where s_i are the roots of $Q(i\omega)$ and A_i's are the residues at the poles. Applying Discrete Fourier Transformation:

$$S(t)=\sum_{i=1}^{N}A_i e^{s_i t} \qquad (11)$$

Eq. (11) represents the regular part of the dynamic stiffness coefficient in time domain derived from frequency domain using ratio of polynomials (Fig. 3). The singular part is added later in the total recursive equation.

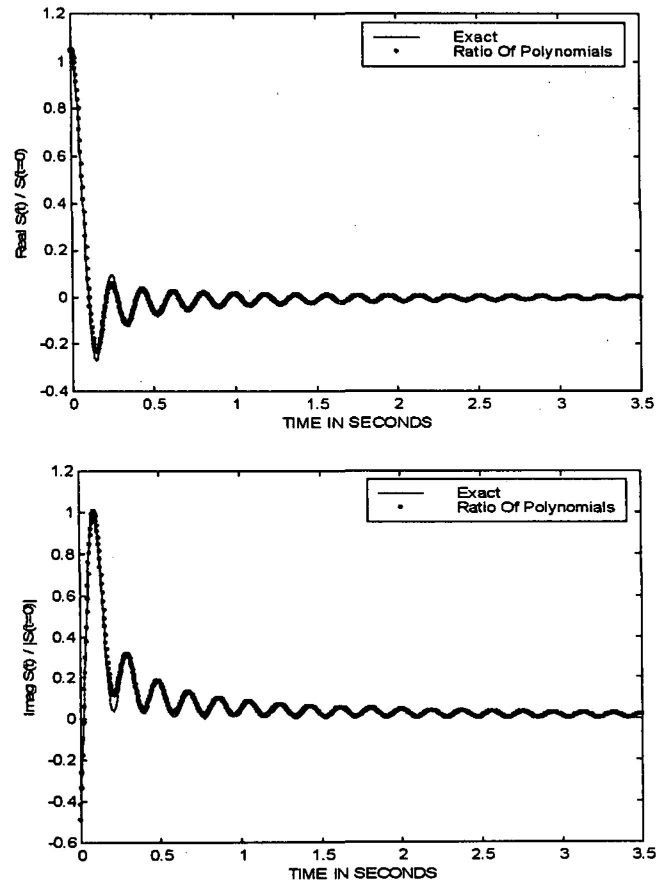


Fig. 3 Dynamic Stiffness Coefficient in Time Domain

(b) Recursive equation

Discretizing Eq. (11) and applying a Z-Transformation at time corresponding to the Nyquist frequency Nf_i:

$$S(z)=\frac{b_o+b_1z^{-1}+\cdots+b_{N-1}z^{-(N-1)}}{1-a_1z^{-1}-\cdots-a_Nz^{-N}} \qquad (12)$$

Identifying the coefficients of the terms involving z^{-1} the recursive equation of interaction forces acting from the far-field are thus obtained.

$$R_r^l=\sum_{i=1}^{m}R_r^{li} \qquad (13)$$

$$[R_{r,n}^{lm}]=\sum_{i=1}^{N}a_{ik}^{lm}R_{r,n-i}^{lm}+\Delta t\sum_{i=o}^{N-1}b_{ik}^{lm}.U_{n-i}^{m} \qquad (14)$$

where m is equal to the dimension of the dynamic stiffness matrix [S(ω)], U_{n-i}^{m} being the displacement vector of time $(n-i)\Delta t$.

4. FORMULATION OF THE INNER ZONE

(1) Linear Finite Element Formulation

(a) Stiffness and mass matrices

The inner irregular zone can be modeled for the linear finite element analysis by a quadrilateral solid elements interconnected at a total of N points. The locations of nodal points are defined in the 2-D plane by the coordinates (r,z). Expansion to the 3-D space is done using Fourier expansion in the tangential direction of Eq. (3) and the corresponding shape functions. The element Static Stiffness Matrix and the Mass Matrix are expressed as follows:

$$[K]^e=\int_{-1}^{1}\int_{-1}^{1}[B]^T[S]_e[B]r\det J d\xi d\eta \qquad (15)$$

$$[M]^e=\int_{-1}^{1}\int_{-1}^{1}\rho[N]^T[N]r\det J d\xi d\eta \qquad (16)$$

where $[B]$ is the strain-displacement transformation matrix and $[S]_e$ is the linear material property matrix and $[J]$ is the 2-D Jacobian matrix. The interpolation matrix $[N]$ is defined as:

$$[N]=([N]_1,\cdots[N]_4) \text{ And } [N]_i=diag(n_i,n_i,n_i) \qquad (17)$$

where $$\left.\begin{array}{l} n_i(\xi,\eta)=\frac{1}{4}(1+\xi_i\xi)(1+\eta_i\eta) \\ \xi_i=[1,1,-1,-1];\eta_i=[-1,1,1,-1] \end{array}\right\} \qquad (18)$$

Total stiffness and mass matrices are symmetric and banded.

(b) Equations of equilibrium of dynamic system

The equation of motion governing the linear dynamic response of a system of finite element at time $(n+1)\Delta t$ can be written as:

$$[K]u_{n+1}+[C]\dot{u}_{n+1}+[M]\ddot{u}_{n+1}=\{R_{n+1}\} \qquad (19)$$

A direct integration method is used for the solution of the above equation. In the numerical implementation, the Newmark and θ-Wilson integration schemes are used for a step-by-step solution of Eq. (19).

(2) Materially Non-Linear Finite Element Formulation

(a) Introduction and model concept

For the Materially Non-Linear only analysis, the non-linear effect lies in the nonlinear stress-strain relation. The displacements and strains are assumed to be infinitesimally small, as in the linear analysis. The finite irregular region is constructed from an eight-node 3-D brick element. To minimize the computational load, all initial analysis is carried out in local Cartesian coordinates. A transformation to global cylindrical coordinates is performed only on the elements joined to the outer half-space region.

(c) Stiffness matrix and mass matrix

In the finite element formulation of elastoplastic problems, the incremental elastoplastic stiffness matrix for one typical element is given by:

$$K_T^e=\iint\int_{-1}^{1}[B]^T[S]_{ep}[B]r\det J d\xi d\eta d\zeta \qquad (20)$$

where $[B]$ is the strain-displacement transformation matrix composed of derivatives of shape functions and $[S]_{ep}$ is the constitutive elastoplastic matrix, which is constructed using the flow approach to describe the elastoplastic material [1], and $[J]$ is the 3-D Jacobian matrix.

The element mass matrix is similarly written for the 3-D formulation as:

$$[M]^e=\iint\int_{-1}^{1}\rho[N]^T[N]r\det J d\xi d\eta d\zeta \qquad (21)$$

$$[N]=([N]_1,\cdots,[N]_8) \; ; \; [N]_i=diag(n_i,n_i,n_i)$$

and $$n_i(\xi,\eta,\zeta)=\frac{1}{8}(1+\xi_i\xi)(1+\eta_i\eta)(1+\zeta_i\zeta)$$

The stiffness and the mass matrices are symmetric and banded.

(d) Incremental equations of equilibrium of dynamic system and solution schemes

The equilibrium of the finite element system at time $(n+1)\Delta t$ in nonlinear analysis requires that an iteration be performed. Using the modified Newton-Raphson iteration scheme the equation of motion can be written as:

$$[M]\ddot{u}_{n+1}^{(i)} + [C]\dot{u}_{n+1}^{(i)} + [K_T]_n \Delta u^{(i)} = \{R\}_{n+1} - \{F\}_{n+1}^{(i-1)} \qquad (22)$$

where *{F}* is the internal forces for the regular finite element zone due to the external and recursive forces of interaction of regular and irregular zones.

{R} is the external loading .

$[K]_T$ is the tangent stiffness matrix.

[C] is the constant damping matrix.

[M] is the mass matrix of the finite irregular region.

5. ANALYSIS OF TOTAL SYSTEM OF PILE-SOIL INTERACTION.

(1) Final Equation Of Motion

The final equation of motion for the pile-soil system is written directly by adding the interaction forces of the layered far-field to the irregular zone. It can be written as follows:

$$[\widetilde{K}]\{u\}_I = P(t)_I + R_r^l(t) - F_I(t) - M_I\ddot{u} - C_I\dot{u} \qquad (23)$$

where

$[\widetilde{K}]$ is the effective stiffness matrix

$P(t)_I$ is the external load force vector

$F_I(t)$ is the vector of internal forces of the finite element zone

$R_r^l(t)$ is the vector of nodal recursive forces acting on the vertical boundary between layered zone and irregular zone

(2) Numerical Implementation

Three models are considered (Fig. 4), the first describes a system with a core irregular zone , two cases are studied when the core region is homogenous and when it's non homogenous , the second model describes the layered zone connected directly to the pile . A harmonic force is applied on the head of the pile in r direction.

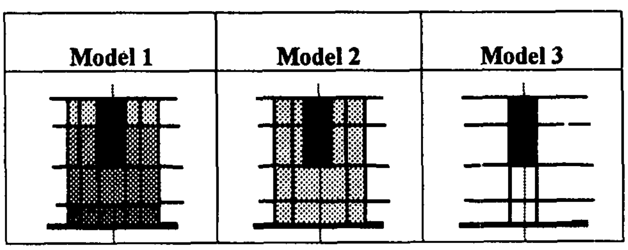
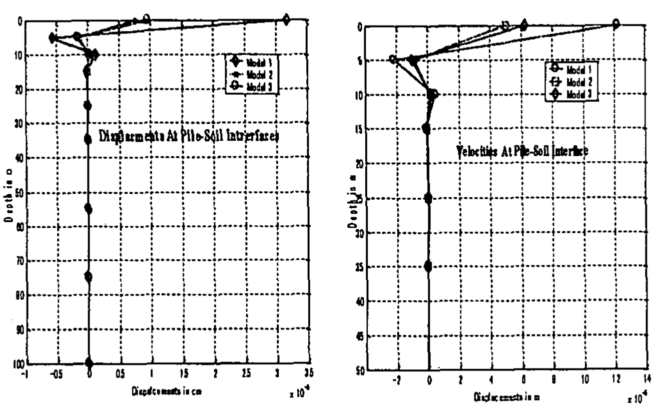
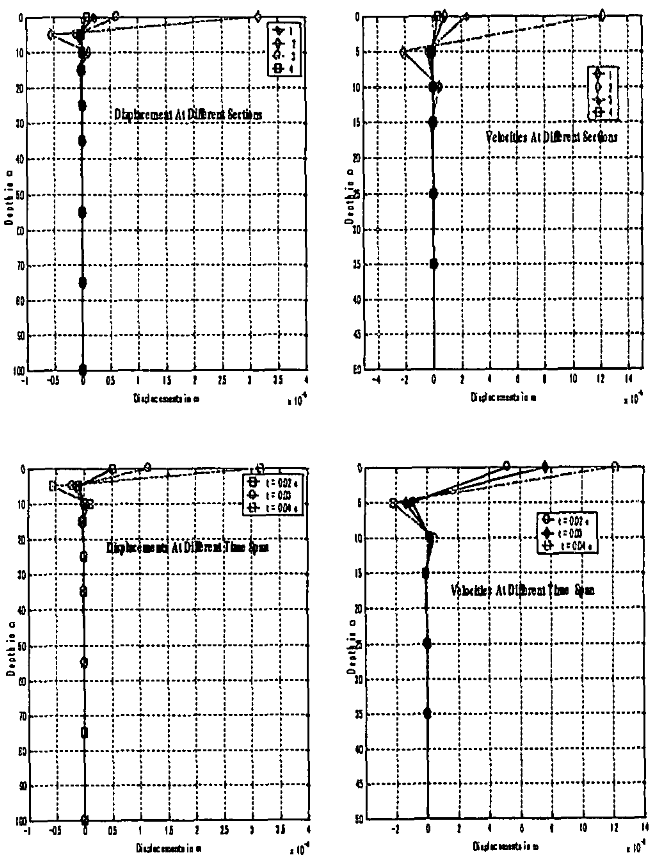


Fig. 4 Model used for the numerical computations





6. CONCLUSION:

The efficient of the combined method of finite element and thin layer element in three-dimension formulation of a half space with irregularities, can be successfully applied to soil-structure problems. Since the thin layer formulation is only performed in frequency domain, an accurate method is needed to transform the dynamic stiffness matrix to Time domain. The recursive procedure requires lesser computer storage and solution time and there is a drastic reduction of the degrees of freedom compared to using direct methods.

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