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AN EFFICIENT AND SIMPLIFIED TECHNIQUE FOR NONLINEAR ANALYSIS OF STRUCTURES

Hatem TAGEL-DIN[1] and Kimiro MEGURO[2]

[1]Student member, M.Sc., Department of Civil Engineering, The University of Tokyo (7-22-1 Roppongi, Minato-Ku, Tokyo 106, Japan)
[2]Member of JSCE, Dr. Eng., Associate Professor, International Center for Disaster-Mitigation Engineering, Institute of Industrial Science, The University of Tokyo (7-22-1 Roppongi, Minato-Ku, Tokyo 106, Japan)

A new method for fracture analysis of reinforced concrete structures is proposed. The concrete is modeled as an assembly of distinct elements made by dividing the concrete virtually. These elements are connected by distributed springs in both normal and tangential directions. The reinforcement bars are modeled as continuous springs connecting elements together. Local failure of concrete is modeled by failure of springs connecting elements when reaching critical stress. The accuracy of the method is verified by comparing with experimental results. The results showed good agreement in determining the load-deformation relations, failure load and the initiation, location and propagation of cracks.

Key Words: *nonlinear analysis, fracture analysis, reinforced concrete structures*

1. INTRODUCTION

A new method for nonlinear analysis of reinforced concrete structures is proposed. The concrete is modeled as an assembly of distinct elements made by dividing the concrete virtually. These elements are connected by distributed springs in both normal and tangential directions that totally represent stresses, strains and local failure inside the elements. Reinforcement bars are modeled as continuous springs connecting elements together. Cracking of concrete is modeled by failure of springs connecting elements when reaching critical principal stress. We developed the element formulation and the computer code and verified the accuracy of the method by comparing with two experiments. In these experiments, the results showed good agreement in determining the failure load, the load-deflection relations and the initiation, location and propagation of cracks.

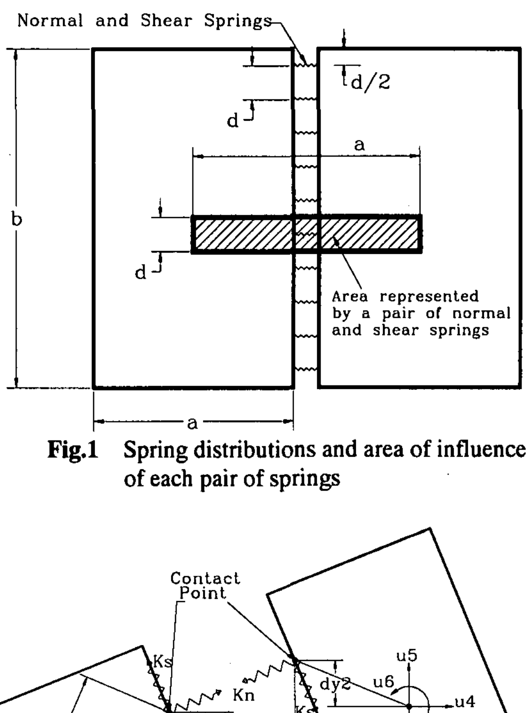

Fig.1 Spring distributions and area of influence of each pair of springs

2. ELEMENT FORMULATION

The two elements shown in **Fig.1** are assumed to be connected by pairs of normal and shear springs located at contact points which are distributed around the element edges. Referring to **Fig.1**, each pair of springs totally represent stresses and deformations of a certain area of the studied elements. The total stiffness matrix is determined by summing the stiffness matrices of individual springs around each element.

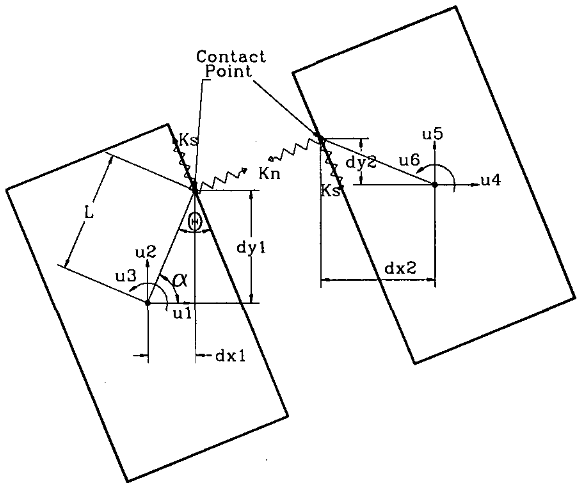

Fig.2 Element shape, contact points and degrees of freedom for two elements

Failure of springs is modeled by assuming zero stiffness for the spring being considered. Consequently, the developed stiffness matrix is an average stiffness matrix for the element according to the stress situation around the element. In the 2-dimensional model, three degrees of freedom are considered for each element and deformations are assumed to be small. This leads to a relatively small stiffness matrix which is only of size (6X6). The stiffness matrix is developed for an arbitrary contact point with one pair of normal and shear springs as shown in **Fig.2**. Two types of springs were defined. The first is concrete springs while the other is reinforcement springs. In this formulation, the element stiffness matrix depends on the contact location (distance L and the angles θ and α) and the stiffness of normal and shear springs which are determined according to the contact point type and the stress and strain at the contact point location.

3. EFFECTS OF ELEMENT SIZE AND THE NUMBER OF SPRINGS

To illustrate the effects of element size, a series of analyses were made for the laterally loaded cantilever shown in **Fig.3**. Elastic analyses were performed using our proposed method for the different cases shown in the figure. The results were compared with those obtained from elastic theory of structures. The percentage of error in maximum displacement and the CPU time (CPU: DEC ALPHA 300 MHz) are also shown in the figure. To study the effects of the number of connecting springs, two different analyses were performed using 20 and 10 springs connecting each pair of adjacent element faces. From the figure, it is evident that increasing the number of base elements leads to decreasing the error but increasing the CPU time. The error reduces to less than 1% when the number of elements at the base increase to 5 or more. Although the CPU time in case of 10 springs is almost half of that in case of 20 springs, its results congruent with those of 20 springs.

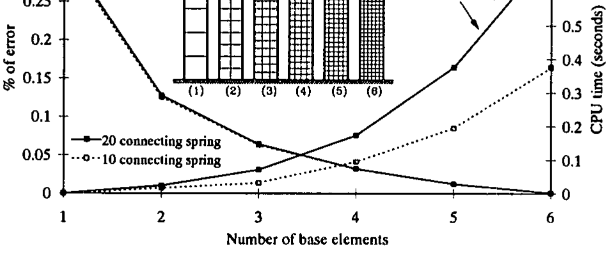

Fig.3 Relation between the number of base elements, percentage of error and CPU time

It should be also noted that in the previous analysis using rigid elements, like RBSM[1)], the results obtained were of poor accuracy. This may be due to:

- The spring stiffness not being determined in a proper way to simulate the element deformation,
- The use of relatively large sized elements, and
- The use of relatively small number of springs between edges which leads to an inaccurate failure mechanism.

4. SIMULATION OF TWO-STORIED RC WALL STRUCTURE SUBJECTED TO MONOTONIC LOADING

To verify the accuracy of the model, the simulation results were compared with the experimental results of a two-storied RC wall. The shape of the wall, reinforcement and loading location are shown in **Fig.4**. Reference (2) gives more details on the columns, beams and wall reinforcement, or the material properties. The wall is modeled using 1,845 square elements. The number of springs between each two adjacent faces is 10. Reinforcement locations are defined by their nearest spring coordinates. For vertical reinforcement, x-coordinate is defined at the steel bar location while for horizontal reinforcement, y-coordinate is defined.

Fig.4 shows a comparison between measured and calculated load-rotation relations. First, to discuss the effects of load increment in failure analysis, three models of different load increments, calculated by dividing the estimated failure load by 50, 250 and 500, with the constant number (10) of springs were used. Next, to study the effects of the number of connecting springs between faces, additional simulations were carried out using the case of 250 load increments with 5 and 2 springs between faces and the results were compared with that with 10 springs. The failure load calculated in all cases were within the range from 64 to 70 tf while the measured one was 67 tf. The calculated failure load using the FEM was 64 tf[3)]. In general, the calculated failure loads are very close to the measured ones. The results of 50, 250 and 500 increments are almost congruent till at least 95% of failure load. It should be emphasized that the CPU time of analysis of 500 increments is 10 times that of 50 increments. To avoid long CPU time, load increments can be reduced after about 90% of expected failure load. Moreover, it can be noticed easily from **Fig.4** that the agreement between experimental and numerical results is fairly good for 250 increments with 10 or 5 connecting springs.

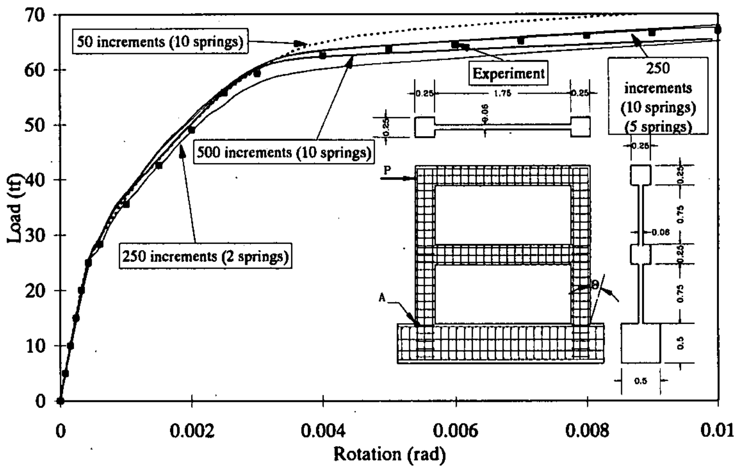

Fig.4 Relation between load and base rotation for 2-storied RC wall

Surprisingly, for the case of 250 increments with only 2 springs connecting each two adjacent faces, the results are also reliable till reaching failure of the structure. It is noted also that using few number of load increments leads to the results in slightly higher failure load (70 tf) while using a few number of connecting springs gives slightly lower one (64 tf). This means that our model gives reliable results even when using a few number of connecting springs or few number of load increments.

Although increasing the number of springs leads to increasing the calculation time required for assembling the global stiffness matrix, the time required for the solution of equations, which is dominant of CPU time when the number of elements is large, does not change. Because the number of degrees of freedom is independent of the number of springs used. This means that we can use larger number of springs between edges without significant change of the CPU time of analysis.

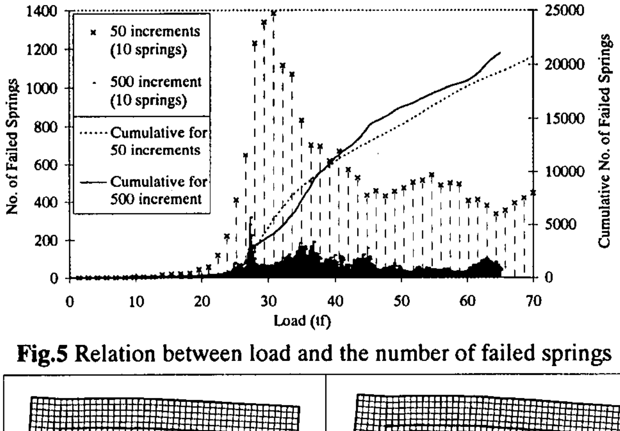

Fig.5 Relation between load and the number of failed springs

Fig.5 shows the relation between load and the number of failed springs for each increment. Cumulative curves also show the total number of failed springs till that increment. It should be noted that although the number of increments in both cases are different, both cumulative curves close each other. This gives good indication that the solution is generally stable. Excessive cracking begins to appear when the applied load is about 28 tf. At the same load, behavior of the structure begins to be highly nonlinear.

Fig.6 shows the deformed shape during the application of load in case of 500 load increments with 10 springs. The location of cracks and crack propagation can be easily observed. The location of cracks and crack propagation are very similar to those obtained from the experiment. This means that the proposed model can be applied for fracture behavior of RC structures, such as, failure load and deformation, crack generation, crack location and crack propagation, etc.

It should be emphasized that although the shape of elements used in the analysis are squares, it does not affect the crack generation or crack propagation in the material. Diagonal cracks, as shown in **Fig.6**, coincide well with those obtained from the experiment. In the analysis using rigid elements, like RBSM[1)], shapes and distributions of elements were decided based on the assumption that cracks were generated and propagated in previously expected locations and directions.

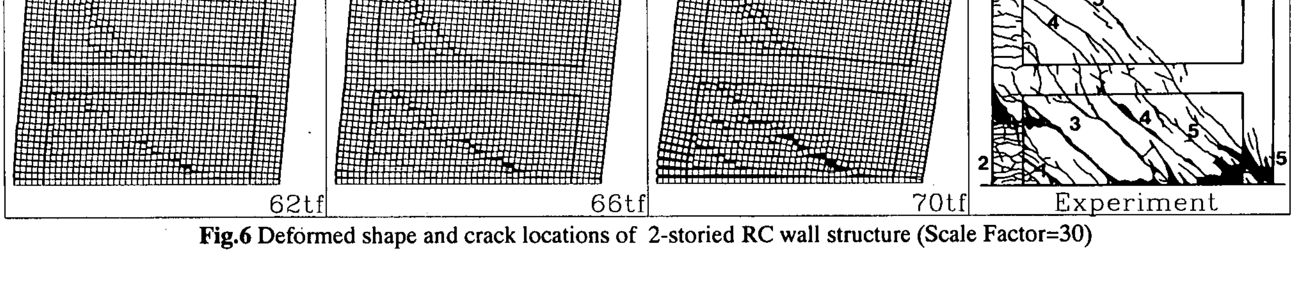

Fig.6 Deformed shape and crack locations of 2-storied RC wall structure (Scale Factor=30)

5. SIMULATION OF RC FRAME STRUCTURE

The third verification example is an RC frame structure. Dimensions, loading conditions and reinforcement details[4)] are shown in **Fig.7**. This frame is modeled using 1,880 elements with 10 connecting springs. The maximum load is applied in 200 increments at the shown location. All reinforcement details, including location and diameters of stirrups, were taken into account. **Fig.7** shows also the relation between load and deformation calculated from our simulation model and measured from the experiment. It can be noticed that excellent agreement between the two results has been achieved. **Fig.8** shows the deformed shape and crack location at the final stage of our result and experiment. Good agreement between the measured and calculated crack locations, crack inclination and crack length can also be obtained.

6. CONCLUSIONS

Through numerical simulations of reinforced concrete structures, it is confirmed that the new proposed model is capable of simulating the fracture behavior of concrete structures. The results of load and displacements in monotonic loading almost coincide with the experimental results by at least 95% accuracy. The expected failure mode is also very close to the measured one. Through this new model, stresses, strains, load-deformation relations, initiation and propagation of cracks can be calculated with high accuracy and relatively simple techniques. In addition, all reinforcement details, such as bar location and shear reinforcement details, can be taken into account without any additional complications, like in case of FEM, to the analysis. Also, it is very easy to follow the mechanical behavior of steel and concrete at any point.

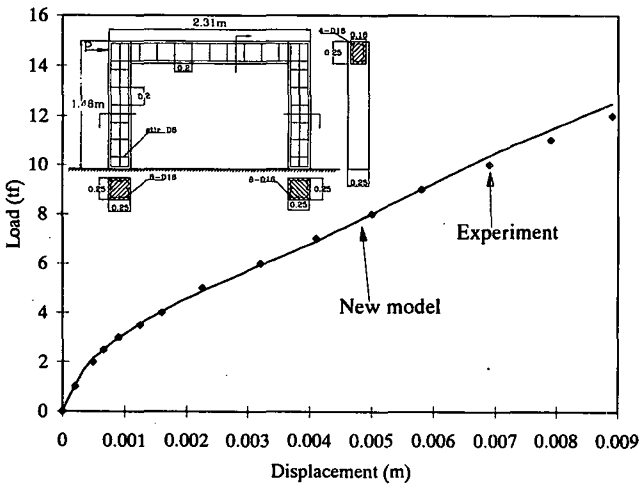

Fig.7 Relation between load and maximum displacement of RC frame

Although the shape of the element used was square, it did not affect the crack propagation in the material. This method does not require complicated techniques for the representation of cracking or special elements, such as joint elements, to follow the crack propagation. Although the effects of Poisson's ratio is not taken into account in the model, the results obtained by the proposed method agree well with experimental results. It means that the model can be applied to fracture behavior cause of which is not strongly related to the Poisson's ratio. This model can be easily combined with the EDEM to simulate the total behavior of structure till complete failure. The proposed model is expected to give high accuracy to wide applications where FEM can not give reliable accuracy.

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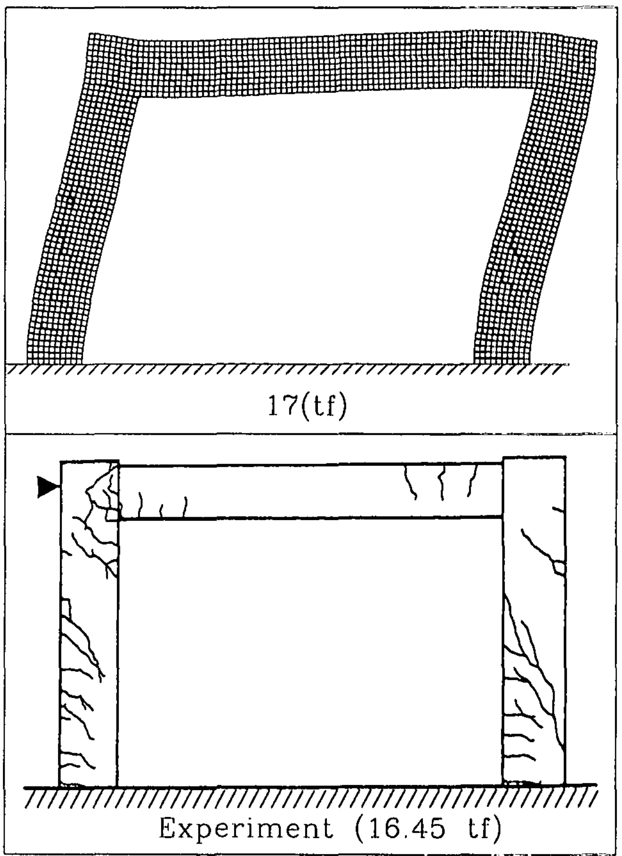

Fig.8 Crack pattern of RC frame