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LINEAR STRUCTURAL SYSTEM IDENTIFICATION USING THE H_∞ FILTER

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Identification algorithms are proposed using the H_∞ filter to identify linear structural systems. Characteristics of the H_∞ filter for structural system identification was studied in detail by checking digital simulation results obtained by using the H_∞ and the Kalman filters. Application of the proposed identification algorithms to SDOF and MDOF structural systems shows that the H_∞ filter is more robust than the Kalman filter for identifying linear structural systems.

***Key Words:** identification, the H_∞ filter, the Kalman filter, structural system, convergence, robust*

1. INTRODUCTION

The H_∞ filtering problem is a state estimation problem of minimizing the maximum energy in the estimation error over all the disturbance trajectories. The state estimation based on the H_∞ criterion is valid when a significant uncertainty in the disturbance statistics exists [1)].

A robust filter, the H_∞ filter, has been used to identify the parameters of linear structural systems in our research. Identification algorithms are proposed for linear structural systems. Digital simulation results show that the characteristics of the H_∞ filter are better than those of the Kalman filter for structural system identification.

2. BACKGROUND OF THE H_∞ FILTER

Consider a system described by

$$x_{t+1} = A_t x_t + B_t \omega_t \tag{1}$$

$$y_t = C_t x_t + D_t v_t \tag{2}$$

$$u_t = L_t x_t \tag{3}$$

in which x_t is the state vector, y_t the measurement and u_t the vector to be estimated. The exogenous signals ω_t and v_t respectively are the process and measurement noises. Moreover, we assume that $R_t := D_t D_t^T > 0$ holds for any t.

The finite-horizon H_∞ filtering problem is to find estimates of u_t and x_t based on the measurement set $\{y_0, \cdots, y_t\}$ such that

$$\sup_{\omega,v,x_0} \frac{\sum_{k=0}^{N} \left\| u_t - \hat{u}_t \right\|^2}{\sum_{t=0}^{N}\left(\left\|\omega_t\right\|^2 + \left\|v_t\right\|^2\right) + (x_0 - \bar{x}_0)^T \Pi^{-1} (x_0 - \bar{x}_0)} < \gamma^2 \tag{4}$$

where $\hat{u}_t$ is the estimate of u_t, and $\bar{x}_0$ is the a priori estimate of the initial state x_0. Π is a positive definite weighting matrix which represents the uncertainty of the initial state. γ is a positive constant which represents the magnitude of the penalty. The central H_∞ filter which satisfies the above H_∞ bound is given by [1)]

$$\hat{x}_t = \bar{x}_t + K_t (y_t - C_t \bar{x}_t) \tag{5}$$

$$\bar{x}_{t+1} = A_t \hat{x}_t, \ \ \hat{x}_0 = \bar{x}_0 \tag{6}$$

$$\hat{u}_t = L_t \hat{x}_t \tag{7}$$

$$K_t = \bar{P}_t C_t^T R_t^{-1} \tag{8}$$

$$\overline{P}_t = (P_t^{-1} + C_t^T R_t^{-1} C_t)^{-1} \quad (9)$$

in which $\hat{x}_t$ is the estimate of system state vector, $\hat{u}_t$ the estimate to be obtained and K_t the gain of the H_∞ filter at time *t*. $\overline{x}_{t+1}$ is the predicted value of the system state vector at time *t*+1. The covariance matrix P_t satisfies the Riccati difference equation

$$P_{t+1} = A_t P_t \left\{ I + (C_t^T R_t^{-1} C_t - \gamma^{-2} L_t^T L_t) P_t \right\}^{-1} A_t^T + B_t B_t^T, \quad P_0 = \Pi \quad (10)$$

and

$$V_t := \gamma^2 I - L_t P_t (I + C_t^T R_t^{-1} C_t P_t)^{-1} L_t^T > 0 \quad (11)$$

in which I is identical matrices.

3. ALGORITHM FOR STRUCTURAL SYSTEM IDENTIFICATION

(1) Algorithm for the case of linear structural system for which the acceleration, velocity and displacement of each floor is available

For a n DOF structural system, the measurement equation of the identification algorithm can be derived from the motion equation of the structural system. The motion equation is given by

$$\ddot{z}_t = \Theta_t H_t - \ddot{Z}_t \quad (12)$$

where $\Theta_t = \left[-M^{-1}C \quad -M^{-1}K \right]$, $H_t = \left[\dot{z}_t \quad z_t \right]^T$.

M is the $n \times n$ mass matrix, *C* damping matrix and *K* stiffness matrix; z_t is the n element vector of displacement responses relative to the ground, and $\ddot{Z}_t$ is the ground motion acceleration. The measurement equation in the identification algorithm can be given as [2)]

$$y_t = C_t x_t + D_t v_t \quad (13)$$

in which $y_t = \ddot{z}_t + \ddot{Z}_t$. C_t is the measurement matrix with the dimensions of $n \times 2n^2$; x_t, the system state vector with $2n^2$ elements to be identified, is defined by the elements of matrix Θ as

$$x_t = \{\theta(1, 1), \cdots, \theta(1, 2n), \theta(2, 1), \cdots, \theta(2, 2n), \cdots, \cdots, \theta(n, 1), \cdots, \theta(n, 2n)\}^T \quad (14)$$

The system transfer equation is given by

$$x_{t+1} = x_t + B_t \omega_t \quad (15)$$

(2) Algorithm for the case of a linear structural system for which only the velocity and displacement of each floor is available

Assume that only the structural responses of velocity and displacement are available for this structural system identification. The mass matrix is assumed to be given, we therefore identify the natural frequency and damping constant of each story of the n DOF structural system instead of the damping coefficient and stiffness matrices. The state vector to be identified is defined by

$$x_t = \left\{ \cdots \; z_i \; \dot{z}_i \; h_i \; \omega_i \; \cdots \right\}^T, \quad i = 1, \cdots, n. \quad (16)$$

The state transfer equation is expressed as a non-linear equation of x_t, this equation must be linearized by a proper linearization scheme [3)]. Then we can get the system equation

$$x_t = A_{t-1} x_{t-1} + d_{t-1} + B_t \omega_t \quad (17)$$

The measurement equation is given as

$$y_t = C_t x_t + D_t v_t \quad (18)$$

in which y_t is the *2n* observation vector defined by

$$y_t = \left\{ \cdots \quad z_i \quad \dot{z}_i \quad \cdots \right\}^T, \quad i = 1, \cdots, n \quad (19)$$

and C_t is the $2n \times 4n$ measurement matrix.

4. STRUCTURAL SYSTEM IDENTIFICATION

The proposed identification algorithms were applied to different structural systems. The seismic responses of these systems are simulated as the observed data in the identification. The El Centro NS (1940) earthquake record with a scaled peak value of 50.0 gal is used as the input excitation. Pink noise with a standard deviation set at 5% of the standard deviation of the structural response is used as the measurement noise. Identification results obtained using by the H_∞ filter are compared with those obtained by using the Kalman filter to show the performance of the H_∞ filter.

(1) Identification for the case of a single degree of freedom (SDOF) structural system

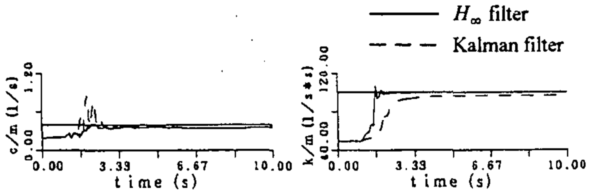

Fig.1 Identified para. of the SDOF system

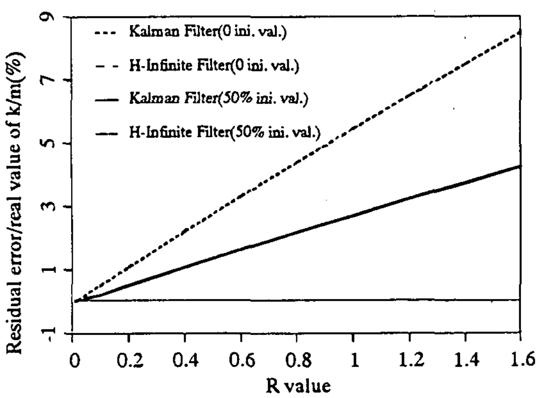

Fig.2 Residual error of the identified para. k/m when $\bar{x}_0 = 0$

Assume that the responses of acceleration, velocity, and displacement of the SDOF linear structural system are available for the identification. A model of the SDOF system with parameters m=1.0, c=0.4 and k=100.0 was used to generate observation time history. As shown in **Fig.1**, the parameters identified using the H_∞ filter converge faster and closer to the real values than those identified using the Kalman filter.

The initial value of the state vector, $\bar{x}_0$, and value R are set different value to check their effect on the identified results. **Fig.2** show the ratio of the residual error of the identified value k/m to the real value obtained with the algorithms using the H_∞ and Kalman filters when the initial state vector $\bar{x}_0$ is given by 50% of the real value and is set at zero. If the initial value of the system state vector $\bar{x}_0$ is set far from the real value, the performance of the Kalman filter would deteriorate (bold solid and dashed), whereas the residual error of the identified parameter k/m for the H_∞ filter is very small (thin solid and dashed which are almost coincident). Compared with the Kalman filter, the H_∞ filter performs better for the identification of parameter k/m when the initial value of the state vector can not

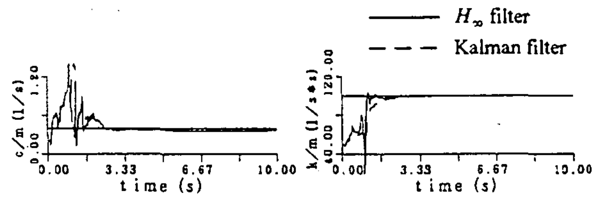

Fig.3 Identified para. of the SDOF system

be set near the real value. When the R is set to be small, the identified parameters converge quickly, but oscillation is large before the identified parameters to be converged, as shown in **Fig.3** (R=0.01). In the case of the Kalman filter, when the value of R becomes large, a large residual error in the identified value is expected. For the H_∞ filter, the residual error of the identified parameter k/m is very small and is not affected by the value of R.

(2) Identification for the case of a 5 DOF linear structural system for which the acceleration, velocity and displacement responses of each floor are available

In this case, the respective dimensions of state vector x_t, given by Eq. (14), and measurement matrix C_t in Eq. (13), are $2n^2 \times 1$ and $n \times 2n^2$. If the number of degrees of freedom increases, the dimensions of the variables in the identification program increase rapidly. A large amount of computer memory is needed, leading to difficulties in calculation. To cope with this, we divided the system defined by Eqs. (15) and (13) into n sub-systems

$$x_{t+1}^i = x_t^i + B_t^i \omega_t^i \quad (i=1,\ n) \tag{20}$$

$$y_t^i = C_t^i x_t^i + D_t^i v_t^i \quad (i=1,n) \tag{21}$$

where $x_t^i = \{\theta(i,\ 1),\ \cdots,\ \theta(i,\ 2n)\}^T$ and $C_t^i = H_t^T$. To identify the parameters defined by the respective n sub-systems, we can identify the parameters of the structural system without defining the large dimensions of the variables.

The 5 DOF structural system with parameters $m_i = 0.12553$, $c_i = 0.07$, $k_i = 24.5$ $(i = 1,\ \cdots,\ 5)$ was used to generate the observation time history. The simulation results show that the H_∞ filter performs better for MDOF structural system

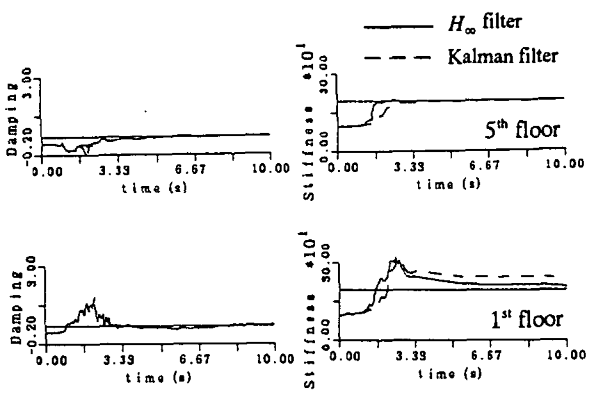

Fig.4 Identified para. *k/m* of the 5 DOF system

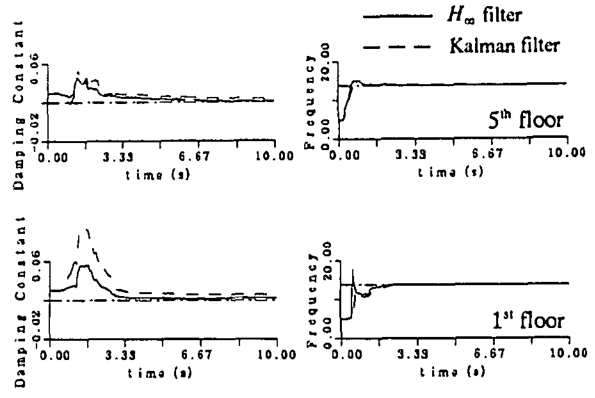

Fig.5 Identified para. of the 5 DOF system

identification. The identified parameters obtained with the H_∞ filter converge faster, and residual error is smaller than the values obtained with the Kalman filter. **Fig.4** shows the time history of the identified parameter *k/m* for the 1st and 5th floors of the 5 DOF structural system.

(3) Identification for a 5 DOF linear structural system for which all the floor responses of velocity and displacement are available

The effect of P_0 on the identification is checked by setting different values. If P_0 is set properly, good identified results can be obtained with algorithms using the H_∞ and Kalman filters. When the initial value, P_0, can not be set properly, the residual error of the identified values obtained with the Kalman filter is very large. **Fig.5** shows the identified parameters of the damping coefficients and frequencies of the 1st and 5th floors of the structure when the initial covariance matrix can not be set properly. The figure also shows that the H_∞ filter gives very good identification results even if the initial value, P_0, is not guaranteed the accuracy of identification results when we use the algorithm with the Kalman filter. The identification algorithm with the H_∞ filter is more robust than that with the Kalman filter for structural identification.

5. CONCLUSION

The algorithms proposed for linear structural identification were applied to different structural systems. Results of the digital simulations show that the performance of the H_∞ filter in the structural system identification is better than that of the Kalman filter. The conclusions of this study are as follows:

(1) The identified parameters of the structural system obtained with the H_∞ filter converge faster and closer to the real values of the structural systems than do those obtained with the Kalman filter.

(2) The initial value of the system state vector has no effect on the identified parameter *k/m* obtained with the H_∞ filter

(3) The residual error in the identified parameter *k/m* is very small and is not affected by value *R* when the H_∞ filter is used.

(4) The algorithm using the H_∞ filter can obtain good identification results when the value P_0 is not guaranteed the accuracy of identification for the algorithm using the Kalman filter.

(5) The H_∞ filter is more robust than the Kalman filter for the identification of structural systems.

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