THE NEW METHOD OF NORMALIZED INPUT-OUTPUT MINIMIZATION (NIOM) FOR MODELING WAVE PROPAGATION

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A new method for modeling wave propagation is described here, its application is discussed and its results are compared with those obtained using the conventional correlation and unit impulse methods. The method simplifies the input and output of a linear system by minimizing the mean square values of the input and output model when subjected to a constraint. The method is applied to simple models as well as the strong motion records of Etchujima vertical array in Japan. The travel times evaluated by the method agree with the results obtained by geophysical measurements of S-wave and P-wave velocities. The method is also effective in showing the amplification property of shallow layers at the Etchujima site.

Key Words : *wave propagation, spectral analysis, vertical array, wave amplification*

1. INTRODUCTION

The problem of spatial variation of ground motion has been considered from the statistical point of view[e.g. 1),2),3),4)] by using the surface seismic arrays. Such studies provide the basic data for seismic design of structures with large foundations (e.g., dams and nuclear power plants) and widely-spaced multiple supports (e.g., bridges and surface pipelines). On the other hand, downhole seismic arrays are designed to record the earthquake strong motion at the points below the ground surface. Such records are needed for seismic design of deeply embedded or buried facilities which are used to obtain the statistical properties of the earthquake motion at deeper points of the ground.[5),6)]

This paper presents a new method of analysis using vertical array records of earthquakes to provide a simple but clear idea about the wave propagation and to identify the dynamic properties of surface layers. The results obtained by applying the method to simple models are compared with those of conventional correlation and unit impulse response methods. The developed method is then used to analyze the earthquake strong motions recorded at the Etchujima vertical array in Japan and the results are compared with those obtained by well shooting at the sites.

2. METHODOLOGY

Consider a soil system with given components of the motion at the ground surface and at depth. For a given frequency, ω_i, the Fourier transforms of the earthquake ground motion at the surface and at depth can be related by means of the transfer function $H(\omega_i)$.

The same transfer function should satisfy the relation of input model, $X(\omega_i)$, and output model, $Y(\omega_i)$, because the transfer function depends only on physical properties of soil system. Therefore, the input and output models can be related by the following equation.

$$Y(\omega_i) = H(\omega_i)X(\omega_i) \tag{1}$$

Consider the inverse Fourier transform of the input model

$$x(m\Delta t) = \frac{1}{N\Delta t}\sum_{i=0}^{N-1} X(\omega_i)e^{j\frac{2\pi i m}{N}} \tag{2}$$

and assume that the amplitude of the input model is desired to be constant at an arbitrary time such as t=0. Therefore, Eq.(2) gives the following constraint.

$$\frac{1}{N\Delta t}\sum_{i=0}^{N-1} X(\omega_i) = 1 \tag{3}$$

Eq.(3) implies that the value of the input at the ground surface for $m = 0$ (which corresponds to $t = 0$ in the time domain) is normalized to unity.

Using the method of Lagrange multipliers, mean square values of the earthquake ground motion at the surface and depth is minimized when subjected to the constraint of Eq.(3). Therefore the Lagrange multipliers method gives

$$L = \sum_{i=0}^{N-1} \left\{ |X(\omega_i)|^2 + |Y(\omega_i)|^2 \right\} - \lambda \left\{ \frac{1}{N\Delta t} \sum_{i=0}^{N-1} X(\omega_i) - 1 \right\} \quad (4)$$

where λ is the Lagrange multiplier. Substituting Eq.(1) into Eq.(4) gives

$$L = \sum_{i=0}^{N-1} \left\{ 1 + |H(\omega_i)|^2 \right\} X(\omega_i) X^*(\omega_i) - \lambda \left\{ \frac{1}{N\Delta t} \sum_{i=0}^{N-1} X(\omega_i) - 1 \right\} \quad (5)$$

in which * denotes the complex conjugate.

One may find $X(\omega_i)$ with minimum L by requiring $\frac{\partial L}{\partial X(\omega_i)} = 0$ and $\frac{\partial L}{\partial X^*(\omega_i)} = 0$. Therefore the simplified ground motion models of the system would be determined by the following equations:

$$X(\omega_i) = N\Delta t \frac{\frac{1}{1+|H(\omega_i)|^2}}{\sum_{n=0}^{N-1} \frac{1}{1+|H(\omega_n)|^2}} \quad (6)$$

$$Y(\omega_i) = N\Delta t \frac{\frac{H(\omega_i)}{1+|H(\omega_i)|^2}}{\sum_{n=0}^{N-1} \frac{1}{1+|H(\omega_n)|^2}} \quad (7)$$

Minimizing the mean square values of the ground motion at the surface and depth when the constraint is in existence would lead to simplified models which illustrate the statistical correlation between the two motions and is named the Normalized Input-Output Minimization (NIOM) method. The inverse Fourier transform of Eq.(6) gives the input model at the ground surface in the time domain and the corresponding model at depth would be obtained by the inverse Fourier transform of Eq.(7).

It is possible to have a control on the contribution of the frequency components in the process which can be useful for determining the period of the input model and for smoothing the results. Consider $dx(t)/dt$ and $dy(t)/dt$ are also taken into account and the square values of their Fourier transforms are minimized and properly weighted. Therefore, the Lagrange multipliers method gives the following equation:

$$L = \sum_{i=0}^{N-1} \left\{ c_0 |X(\omega_i)|^2 + k_0 \omega_i^2 |X(\omega_i)|^2 + c_1 |Y(\omega_i)|^2 + k_1 \omega_i^2 |Y(\omega_i)|^2 \right\} - \lambda \left\{ \frac{1}{N\Delta t} \sum_{i=0}^{N-1} X(\omega_i) - 1 \right\} \quad (8)$$

in which the constraint of Eq.(3) is used and c_0, c_1, k_0 and k_1 are weighting constants.

One may again find $X(\omega_i)$ and $Y(\omega_i)$ with minimum L by requiring $\frac{\partial L}{\partial X(\omega_i)} = \frac{\partial L}{\partial X^*(\omega_i)} = 0$ as follows:

$$X(\omega_i) = N\Delta t \frac{\frac{1}{(1+\frac{k_0}{c_0}\omega_i^2)(c_0+c_1|H(\omega_i)|^2)}}{\sum_{n=0}^{N-1} \frac{1}{(1+\frac{k_0}{c_0}\omega_n^2)(c_0+c_1|H(\omega_n)|^2)}} \quad (9)$$

$$Y(\omega_i) = N\Delta t \frac{\frac{H(\omega_i)}{(1+\frac{k_0}{c_0}\omega_i^2)(c_0+c_1|H(\omega_i)|^2)}}{\sum_{n=0}^{N-1} \frac{1}{(1+\frac{k_0}{c_0}\omega_n^2)(c_0+c_1|H(\omega_n)|^2)}} \quad (10)$$

In Eqs.(9) and (10), the following relationship between the weighting constants is assumed:

$$\frac{k_0}{c_0} = \frac{k_1}{c_1} \quad (11)$$

which implies the same weighting relation between the output and input and the Fourier transforms of their time derivatives. As the ratio of the constants is required in Eq.(11), one of those may be fixed over the entire process. Here, c_0 is fixed to unity and k_1 is considered in terms of the other constants. Therefore, one should only specify k_0 and c_1 in the analysis. The weighting constant k_0 is mostly effective in determining the period of the input model and c_1 specifies the weight of the output with respect to the input.

3. APPLICATION OF THE METHOD TO SIMPLE MODELS

To clarify the effectiveness of the NIOM method, the results of the method are compared here with the results obtained when using conventional autocorrelation, cross-correlation, and impulse response methods by using simple models. The models are made by means of a time history and applying successive shifting and adding. The time history $f(t)$ is considered as the input and the following combination is assumed as the output of the linear system when white noise $n(t)$ is also accompanied.

$$g(t) = 4f(t) + 3f(t-\tau) + 2f(t-2\tau) + f(t-3\tau) + n(t) \quad (12)$$

The simplified input and output model of the system for τ=0.1 are obtained by transforming the results of Eqs.(9) and (10) into the time domain. The results of the NIOM method are compared with the results obtained by the conventional autocorrelation and cross-correlation methods and with the results of conventional impulse response function in Fig.1. As Fig.1 shows, the results of the NIOM method clearly present the relationship of the input and the output as one expects from Eq.(12), whereas the results of correlation and impulse response methods do not.

The weighting constants of $c_0 = 1$, $c_1 = 0.001$ and $k_0 = 0.001$ are used for analyzing the input and output.

4. ANALYSIS OF THE ETCHUJIMA STRONG MOTION RECORDS

The acceleration time histories recorded at the Etchujima vertical array during the earthquakes M6.0 of February 27, 1983, and M6.7 of December 17, 1987 are used in this analysis. The observation points of the array are located at elevations GL-1.0 m, GL-40.0 m, and GL-100.0 m.

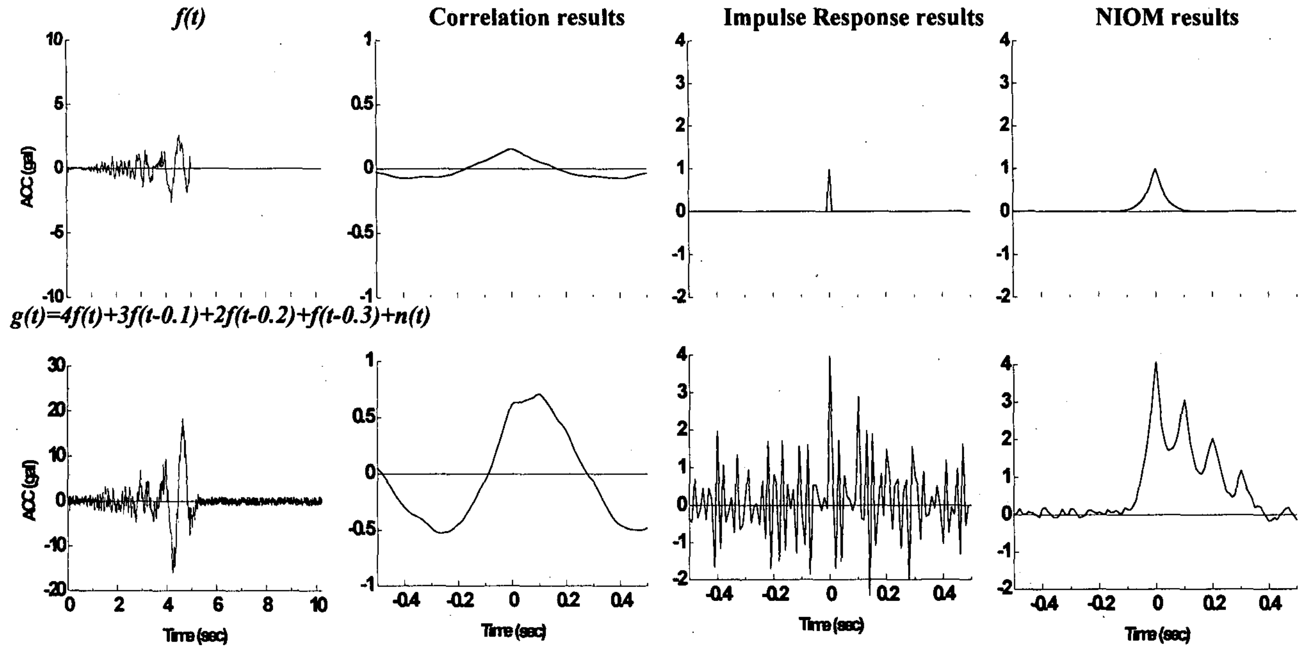


Fig.1 Application of the NIOM method to simple models and comparison with the results obtained by the correlation and impulse response methods. The first column shows the strong motion time history considered as the input *f(t)* and some combinations of that plus white noise *n(t)* as the output *g(t)*. The second column shows the autocorrelation of *f(t)* at the top and cross-correlation of *f(t)* and *g(t)* below that. The third column shows the unit impulse function at the top and the impulse response below that. The fourth column shows the simplified input by the NIOM at the top and the response to the simplified input below that.

The results of the analysis by using the NIOM method are shown in Fig.2. Based on the geophysical and geological information[7], the arrival times of S-wave and P-wave are computed and shown in the figure. The figure shows the simplified input and its responses at different observation points. The simplified input is obtained at elevation GL-1.0 m and the responses are computed and shown at GL-40.0 m and GL-100.0 m. The weighting constants of $c_0=c_1=c_2=1$ and $k_0=0.0001$ are used in this analysis.

The simplified outputs computed by using the horizontal strong motion records (Fig.2, EW and NS components) show two clear peaks corresponding to the incident S-wave and the reflected S-wave from the ground surface at GL-40.0 m and GL-100.0 m. The results are compared with the arrival times obtained by downhole well shooting and show good agreement. The mentioned peaks are stable and are observed at both the horizontal component responses of the analyzed events. Some other stable peaks are also observed at GL-40.0 m and GL-100.0 m of horizontal components. There is not enough evidence to relate them to propagation of P or S waves and reflection from the known interfaces.

The simplified outputs computed by applying the NIOM method to the vertical strong motion records of the earthquakes (Fig.2, UD component) also show clear peaks which are in agreement with the elastic P-wave arrival times obtained by downhole well shooting. The remarkable achievement of analyzing the vertical strong motion records is that the vertical simplified outputs at GL-40.0 m and GL-100.0 m of both the earthquakes do not show any peak corresponding to the S-wave propagation. This can also be confirmed by analyzing the S-portions of the vertical component strong motions.

The method also shows the effect of soil amplification. Significant differences are observed between the amplitude of the peaks at deeper layers and the relating peaks at shallower layers, and imply that the waves are amplified mostly in the layers from elevation GL-40.0 m to the ground surface.

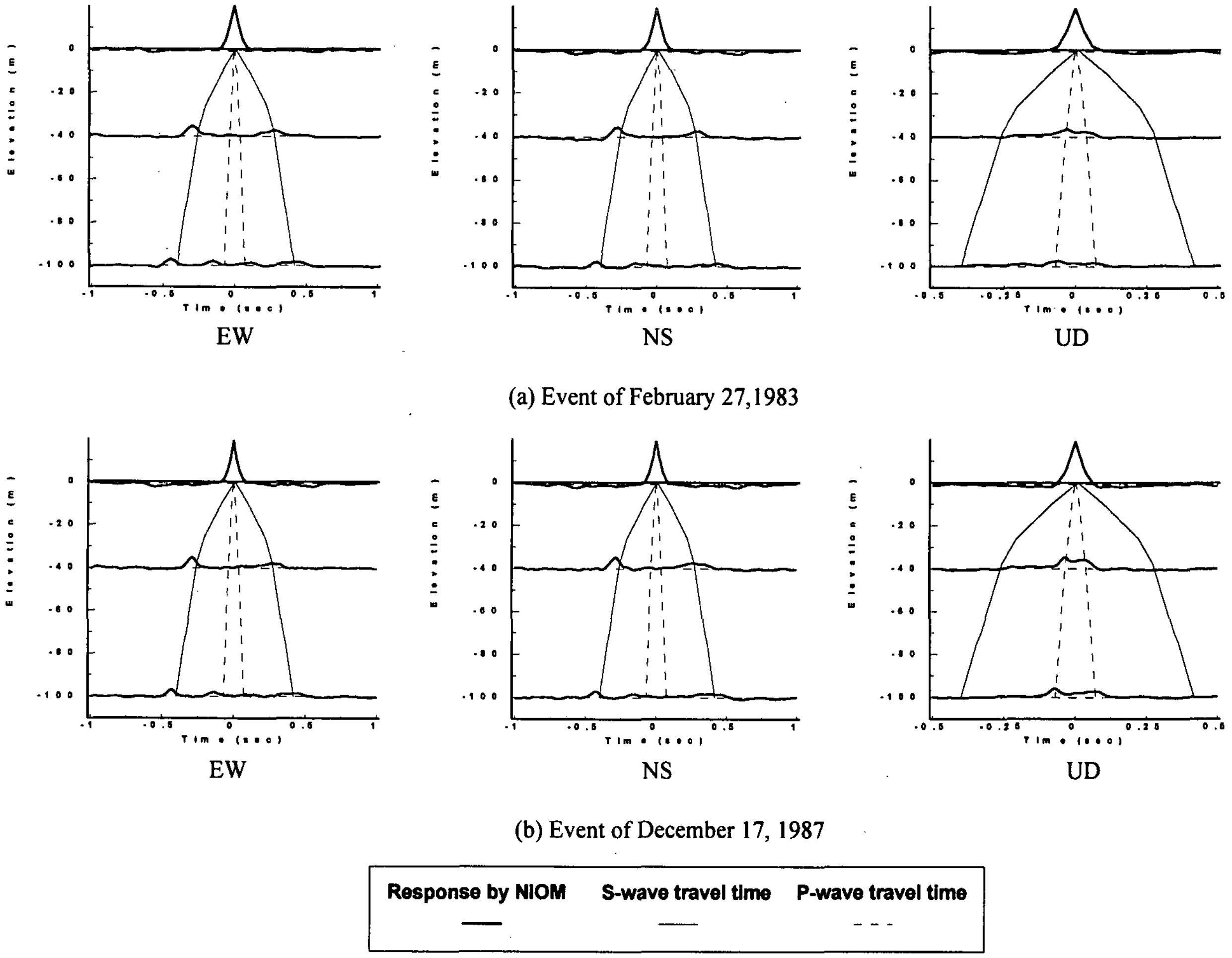
The incident and reflected peaks revealed by the NIOM method also show a reasonable relationship between the amplitudes of the incident wave and the reflected wave. The reflected wave amplitude is smaller than the incident amplitude which is in accordance with the multiple reflection theory.

5. CONCLUSIONS

The Normalized Input-Output Minimization (NIOM) method is capable of revealing a simplified relationship between the input and output of linear systems. The results obtained by applying the method to simple models including one time history as the input and some different combinations of that as the outputs show a distinct correlation between the input and the output, and the method is more effective than the conventional cross-correlation and impulse response methods.

Application of the method to the records of the Etchujima vertical array also yields clear arrival times for the incident and reflected S-waves and P-waves. The results are in agreement with the downhole well shooting measurements at the site.

There is also a reasonable relationship between the amplitudes of the incident and reflected waves in the



(a) Event of February 27,1983

(b) Event of December 17, 1987

Fig.2 The results obtained by the NIOM method in comparison with the P-wave and S-wave elastic velocities measured by downhole well shooting at the Etchujima site during two earthquakes. Three components (EW, NS, and UD) of strong ground motion are considered in the analysis.

shallow layers. The amplitude of the reflected wave from the ground surface is smaller than the amplitude of the incident wave which is consistent with the multiple reflection theory.

The method also shows the effect of shallow layers on the wave amplification at the Etchujima site. The layers from GL-40.0 m up to the ground surface have significantly larger effect on amplification of the wave than do the deeper layers at the site.

The NIOM method clearly show the simplified correlation of input and outputs of linear systems and is useful for studying wave propagation in shallow layers.

The NIOM method is also potential for processing the horizontal and/or vertical components of ground motion records at the same or different observation points which is useful for studying the propagation of surface waves. This application is also considered by the authors and the results will be published in a separate paper.

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