(150) EFFECT OF POST-ELASTIC STIFFNESS ON THE RESPONSE OF SINGLE DEGREE OF FREEDOM STRUCTURES

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INTRODUCTION: An investigation has been initiated at the Public Works Research Institute, Tsukuba, Japan in order to understand the likely seismic behaviour of bridges by studying the seismic response of single-degree-of-freedom (SDOF) oscillators. This investigation is being carried out in order to assess existing methods to predict maximum inelastic displacements of structures, to understand the P-delta effect and the shape of the hysteresis loops and to obtain appropriate loading regimes for the testing of structures. This paper describes some of the preliminary results from this study and shows that the acceleration, energy absorbed and residual displacements of structures with different ductilities are significantly influenced by the value of post-elastic stiffness.

PROGRAM AND ANALYSES: A computer program was developed to obtain the yield acceleration, yield displacement and residual ductility of single-degree-of-freedom oscillators (SDOFO's) with different post-elastic factors, r, under earthquake ground motions for specified values of target ductility, μ_t. This sort of bilinear curve is shown in Figure 1. Iteration of yield displacement, d_y, was used until the target ductility was obtained. Fifty one oscillators with periods from 0.1 to 3.0 seconds were analysed with the transverse component of the Kaihokubashi record during the Miyagikenoki earthquake (12.6.1978) [1] assuming a damping ratio of 2%. Period steps of 0.025 secs, 0.05 secs and 0.1 secs were used for oscillators with periods between 0.1 and 0.45 secs, 0.45 and 1.5 secs, and 1.5 and 3.0 seconds respectively.

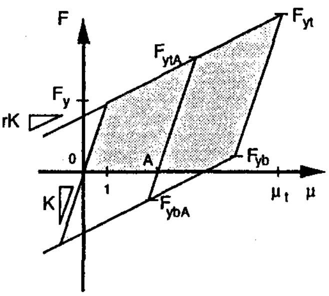

Figure 1. Bilinear Loop

FORCE: The response spectrum for this earthquake, which is shown in Figure 2, is typical of that expected on a stiff soil site. The average yield acceleration ratio, R, over the range of periods analysed for each value of post-elastic stiffness is given in Figure 3, where the value of R for each of the oscillators was defined as the yield acceleration for a particular ductility, a_μ, divided by the elastic acceleration, a_e. It may be seen that

1) the average yield acceleration ratio, R, was less than $1/\mu$ when the post-elastic stiffness was greater than unity. This showed that the yield acceleration ratio from the equal energy assumption, calculated as $1/\mu$, was on average conservative in this range.

2) the average yield acceleration ratio, R, increased with greater negative post-elastic factors. That is, the loops with negative post-elastic stiffnesses had to have a higher yield strength than loops with positive post-elastic stiffness in order to obtain the same ductility.

3) a post-elastic factor of between 0.1 and 0.5 gives the lowest yield acceleration ratio. Conventional methods for assessing structural displacement will be most conservative in this range.

The average maximum acceleration ratio resisted, defined as the maximum acceleration for a particular ductility, a_{max}, divided by the elastic acceleration, a_e, with each post-elastic factor is given in Figure 4. It may be seen that loops with a post-elastic stiffness factor, r equal to zero, resist the least amount of force.

RESIDUAL DISPLACEMENTS: The residual displacement of each oscillator at zero force, d_r, was obtained by subtracting any elastic displacement present at the end of the earthquake record. The residual ductility, μ_r, was defined as the residual displacement, d_r, divided by the yield displacement, d_y. The maximum possible residual ductility, μ_{mr}, of an oscillator depends on whether the negative yield strength at the maximum displacement, F_{yb}, is less or greater than zero as shown in Figure 1. The value of μ_{mr} is given in Equations 1a and 1b.

When $r(\mu_t - 1) < 2$ (ie. $F_{yb} < 0$) then $\mu_{mr} = (\mu_t - 1)(1 - r)$ (1a)

When $r(\mu_t - 1) > 2$ (ie. $F_{yb} > 0$) then $\mu_{mr} = (1 - r)/r$ (1b)

The residual ductility ratio, μ_{rr}, is the ratio of the residual displacement to the maximum possible residual displacement. This ratio, μ_{rr}, which is calculated as μ_r/μ_{mr} was averaged over all of the periods and is shown in Figure 5. It may be seen that:

1. The residual ductility ratio decreases with increasing values of post-elastic stiffness.
2. The residual ductility ratio was very close to the maximum possible displacement when the post-elastic stiffness was less than about 0.10 and was very low for loops with a post-elastic stiffness greater than about 0.25. This average ratio was seen to be very sensitive for post-elastic factors between -0.10 and 0.10.

A large scatter of residual ductility was seen to occur with period when the post-elastic factor was 0.0 as shown in Figure 6. However no trends were observed to occur with period.

The residual displacement results agree well with those observed from the shaking table testing of actual piers. Large residual displacements and yielding in predominantly one direction occurred for steel piers with slightly negative post-elastic stiffnesses [2] while almost no residual displacement occurred in tests of base-isolated bridge models with positive post-elastic stiffnesses [3]. Residual displacements are undesirable in real structures because

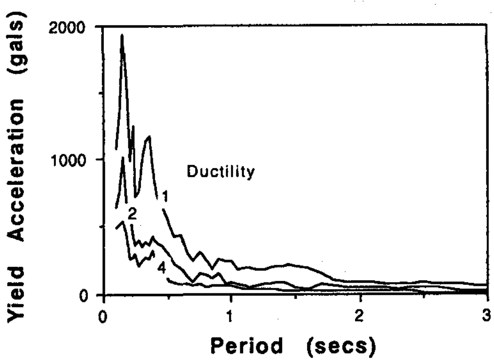

Figure 2. Kaihoku Bridge Transverse Acceleration Response Spectrum

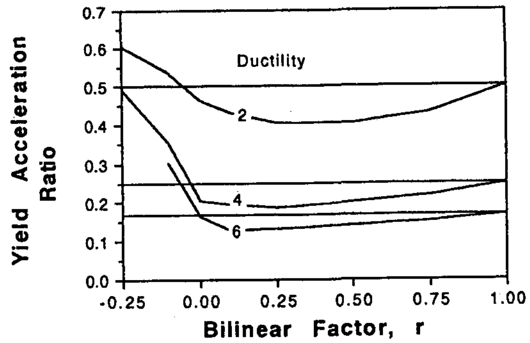

Figure 3. Average Yield Acceleration Ratio versus Post-elastic Factor

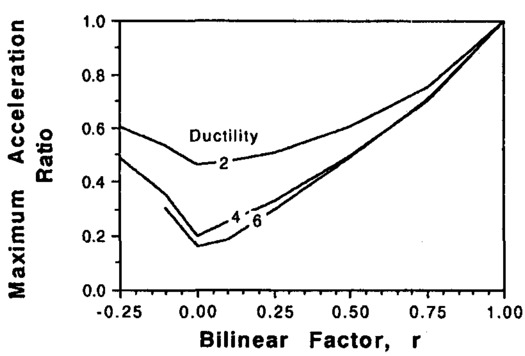

Figure 4. Average Yield Acceleration Ratio versus Post-Elastic Factor

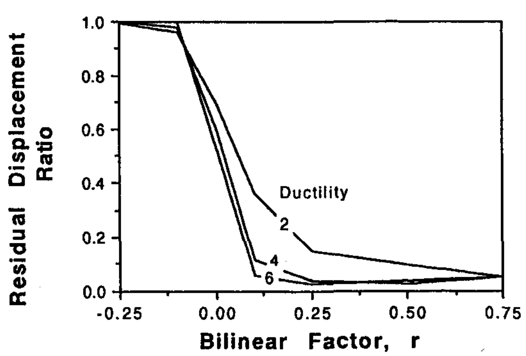

Figure 5. Average Residual Ductility Ratio versus Post-Elastic Factor

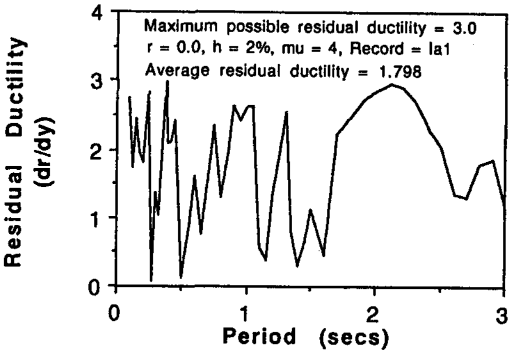

Figure 6. Residual Ductility for an Elastically-Perfectly Plastic Oscillator

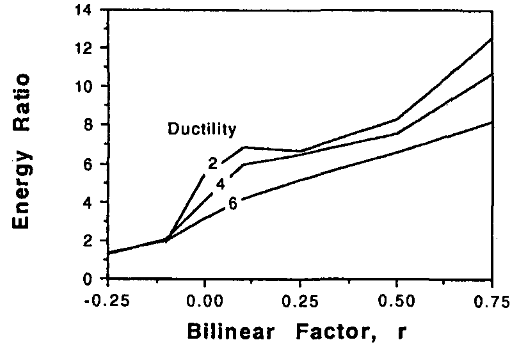

Figure 7. Average Energy Ratio versus Post-Elastic Factor

of difficulties with straightening them after an earthquake and the possible greater susceptibility to damage in further earthquakes or aftershocks.

ENERGY ABSORBED: The average energy absorbed is expressed as an energy ratio in Figure 7. This energy ratio was calculated as the total hysteretic energy absorbed, $\Sigma F\Delta x$, divided by the energy absorbed in a monotonic hysteretic curve to the same maximum displacement as shown in the shaded area of Figure 1. This ratio is never less than unity but is equal to unity for a loop in which there is no reversal of yielding. The energy ratio equals 4 for an elastic perfectly-plastic hysteresis curve ($r = 0.0$) subject to one full cycle of loading but it tends toward 2 for loops with post-elastic stiffnesses approaching unity. It may be seen that very little energy was absorbed in loops with post-elastic stiffnesses less than -0.1. The behaviour of oscillators with these hysteresis loops was almost monotonic which also accounted for the large residual displacements observed. The energy ratio increased with increasing post-elastic stiffness and with decreasing ductility.

LOOP STABILITY: In Figure 1 it may be seen that a SDOFO oscillating about point "A" will tend to yield at the lower absolute yield force, F_{ybA}, before point F_{ytA} causing yielding in the direction toward the zero displacement position. This restoring tendency increases at higher displacements. Conversely, loops with negative post-elastic stiffnesses have a destabilizing tendency and larger displacements in one direction are likely to result. The maximum displacement of oscillators with negative post-elastic stiffnesses is strongly dependent on the magnitude and number of cycles of yielding which is affected by the length of earthquake record.

The maximum velocity of an oscillator subjected to non-forced loading tends to be greatest at zero force. Therefore, after yielding has occurred in one direction the momentum may cause a significant force and possible yielding in the opposite direction depending on the earthquake record and the shape of hysteresis loop. Loops with positive post-elastic stiffnesses are "stable" because there is always a tendency for yielding to occur in the direction toward the position of zero displacement. Conversely, loops with negative post-elastic stiffnesses are inherently "unstable".

The stability of a point on a general hysteresis loop, λ, may be calculated as $\lambda = (F_{ybA} + F_{ytA})/2.\mathrm{Sgn}(A)$ where A is the displacement from the origin at zero applied load as shown in Figure 1. If $\lambda > 1$ at point "A" then yielding will tend to occur first in the direction toward the position of zero displacement and this point will be "stable" with the degree of stability reflected in the value of λ. Conversely if $\lambda < 1$ at point "A" then yielding will always tend to occur first in the direction away from the origin causing instability.

An unconditionally stable loop is one in which all the points on the loop are stable. In these loops the centre curve, defined as $(F_{ybA} + F_{ytA})/2$, only crosses the line of zero force once with a positive slope at the origin as shown in Figure 8. The origin is defined as the point at which this centre curve crosses the line of zero force. An unconditionally unstable loop is one in which the centre curve of the loop goes through the origin with a negative slope and a partially stable loop is one in which the centre curve of the loop crosses the line of zero force more than once creating several origins. The stability value over a certain range of the hysteresis loop is given as the slope of the centre curve.

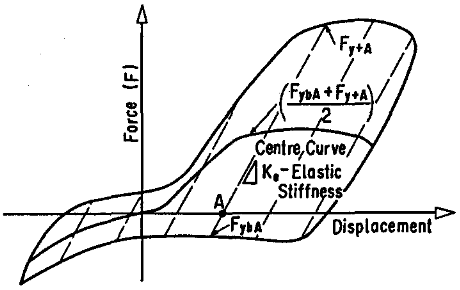

Figure 8. Unconditionally Stable Hysteresis Loop

It is possible to know whether or not the residual displacements will be large or not by assessing the stability of the hysteresis loops. For example, reinforced concrete piers have loops with pinched hysteresis loops but which are "stable" so they would therefore be expected to have low residual displacements. This assumption is consistent with the results of shaking table tests of actual reinforced concrete piers which have been tested at the PWRI [4] where the residual displacements were significantly less those of the steel piers tested. Most of the residual displacement which did occur in these RC pier tests was because the origin of the loop moved away from the initial displacement position.

SUMMARY: In this paper some analyses carried out with SDOF oscillators with both positive and negative post-elastic stiffnesses were described. It was found that:

1. Both the equal energy assumption and the equal displacement assumption, which are used for predicting response, were on average conservative for oscillators with post-elastic stiffnesses, r, between 0.0 and 1.0. However these assumptions became inconservative as the post-elastic stiffness became more negative.
2. The minimum yield acceleration occurred in SDOFOs with post-elastic stiffnesses between about 0.1 and 0.5, however the minimum acceleration occurred in SDOFOs with no post-elastic stiffness.
3. The residual displacements were very sensitive to the post-elastic factor. Oscillators with very large and very small post-elastic factors had very small and very large residual displacements respectively.
4. The ratio of the average energy absorbed divided by the static energy absorbed increased with increasing displacement ductility but decreased to almost the same as the monotonic response for loops with more negative values of post-elastic stiffness.
5. Yielding tended to occur in only one direction and very little hysteretic energy was absorbed in the loops with negative post-elastic stiffnesses. This caused large residual displacements. The maximum displacement in this sort of loop is dependent on the number of yielding cycles which is dependent on the length of the earthquake record. An inelastic response spectrum design approach, which considers only the maximum acceleration, is therefore not appropriate to obtain the inelastic response of a structure with negative post-elastic hysteresis loops to a general earthquake.
6. The commonly held assumption that structures with "fat loops are good" does not hold when the post-elastic stiffnesses of oscillators are negative.
7. The behaviour of oscillators was able to be understood by considering the *stability* of the hysteretic loops. A simple method for assessing the stability of general hysteresis loops was proposed.
8. The results obtained were consistent with those obtained from the shaking table testing tests of actual model bridges.

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