(72) AN ENERGY-BASED THREE-DIMENSIONAL MODEL TO PREDICT PERMANENT GROUND DISPLACEMENTS CAUSED BY LIQUEFACTION

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INTRODUCTION

Permanent displacements of liquefied grounds ranging between a few centimeters to several meters have been observed during past earthquakes and have caused severe damage to roads, embankments, building foundations and lifeline facilities. Therefore, engineering measures to mitigate liquefaction and the associated permanent ground deformations must include an assessment of the magnitude and pattern of ground movement and soil failure resulting from liquefaction.

Towhata et al. (1990) developed a simplified two-dimensional analytical model to predict permanent ground displacements induced by liquefaction. In this model, the distribution of ground displacements was derived so that the potential energy of the ground would take minimum value, and using variational principle, a closed-form solution was obtained.

This paper deals with the three-dimensional extension of the above model for evaluating the spatial distribution of ground displacements caused by liquefaction. In the proposed model, the area is divided into two-dimensional finite elements and the total energy of the ground is expressed in terms of the unknown nodal displacements. Variational principle is again employed to obtain the solution. To illustrate its validity, the model is used to analyze the ground displacements observed in laboratory shaking table tests as well as those measured in Noshiro City during the 1983 Nihonkai-Chubu earthquake.

FORMULATION OF THE MODEL

The model employs the finite element shown in Fig. 1. Each ground element consists of an unliquefiable base (B), a liquefiable layer (H), and a surface unsaturated layer (T). The surcharge load, P, includes the weight of the surface layer and any additional loading. These parameters vary linearly with the coordinates x and y, i.e.,

$$B = B_o + a_1 x + b_1 y \quad H = H_o + a_2 x + b_2 y$$
$$T = T_o + a_3 x + b_3 y \quad P = P_o + a_4 x + b_4 y \qquad (1)$$

Based on experimental results, the lateral displacement in a vertical cross section of a liquefied ground is maximum at the top and zero at the bottom. Thus, at any point (x, y, z) in the liquefied layer, the lateral displacements along x and y-axes, denoted by u and v, respectively, are approximated by sinusoidal distributions in z direction:

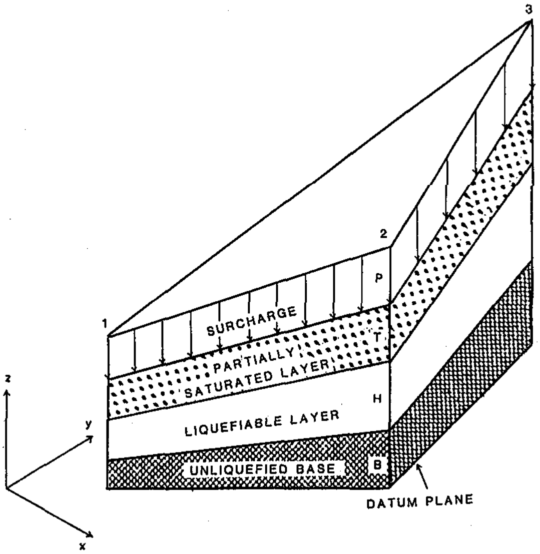

Figure 1: A finite element of the model ground

$$u(x,y,z) = F(x,y)\sin\frac{\pi[z-(B_o+a_1x+b_1y)]}{2(H_o+a_2x+b_2y)}$$
$$v(x,y,z) = J(x,y)\sin\frac{\pi[z-(B_o+a_1x+b_1y)]}{2(H_o+a_2x+b_2y)} \qquad (2)$$

With the assumption that there is no slip between the surface unsaturated layer and the liquefiable layer, the functions $F(x,y)$ and $J(x,y)$ actually represent the displacements at the ground surface ($z = B+H$).

The surface unsaturated layer behaves like an elastic plate subjected to in-plane stresses, with an elastic modulus E and Poisson's ratio ν. On the other hand, the stress-strain relation of the liquefiable portion is modeled by

$$\tau = G\gamma + \tau_r \qquad (3)$$

where G is the shear modulus and τ_r is the residual strength.

The settlement w at any point is related to the lateral displacements u and v by the equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (4)$$

Since constant-volume condition is assumed, the settlement resulting from consolidation should be considered separately.

The energy of each ground element is formulated by considering the strain energy and gravity components of the liquefied and surface unsaturated layers. In the calculation of the strain energy, the present analysis makes use of only the predominant components of the strain tensor. Hence, for the liquefied layer, only the shear strains associated with $\partial u/\partial z$ and $\partial v/\partial z$ are employed as the contributions of the other strain components are negligible. Similarly, the strain energy of the surface unsaturated layer is calculated assuming plane stress condition.

For elements located at free boundaries of the study area, additional terms are included to account for the movement of such boundaries. The change in energy consists of the potential energy of the displaced volume of soil and the work done by hydrostatic pressure, which is assumed to exist in cracked boundaries.

Therefore, the total energy in each element, represented by the functional Π_p, is expressed in terms of the unknown surface displacements $F(x,y)$ and $J(x,y)$, their first-order derivatives, and the coordinates x and y. The unknown displacements can be obtained by applying variational principle on the the functional Π_p. Due to the complexity of the expressions involved, a closed-form solution is not attempted; instead, a finite element-based formulation is employed.

In the finite element formulation, the surface displacements within an element are represented by the linear functions

$$\begin{aligned} F(x,y) &= \alpha_1 + \alpha_2 x + \alpha_3 y \\ J(x,y) &= \beta_1 + \beta_2 x + \beta_3 y \end{aligned} \quad (5)$$

or, in terms of the unknown nodal displacements F_i and J_i (i=1 to 3),

$$\begin{aligned} F(x,y) &= N_1 F_1 + N_2 F_2 + N_3 F_3 \\ J(x,y) &= N_1 J_1 + N_2 J_2 + N_3 J_3 \end{aligned} \quad (6)$$

where N_1, N_2 and N_3 are referred to as shape functions and are expressed in terms of the coordinates x and y.

Upon substitution of Eqtn. (6) into the functional Π_p, the total energy of the whole system is minimized by taking the variation of Π_p with respect to F_i and J_i and setting it to zero, i.e.,

$$\delta \Pi_p = \frac{\partial \Pi_p}{\partial F_i} \delta F_i + \frac{\partial \Pi_p}{\partial J_i} \delta J_i = 0 \quad (7)$$

The above equation can be solved by setting

$$\frac{\partial \Pi_p}{\partial F_i} = 0 \quad \text{and} \quad \frac{\partial \Pi_p}{\partial J_i} = 0 \quad (8)$$

Eqtns. (8) represent $2n$ equations where n is the number of nodal points. By rearranging these set of equations, they can be written in a more familiar form

$$\{\mathbf{P}\} = [\mathbf{K}]\{\mathbf{U}\} \quad (9)$$

where $[\mathbf{K}]$ is the stiffness matrix and $\{\mathbf{P}\}$ is the equivalent load vector. Thus, the equation becomes a typical finite element problem where the displacement vector $\{\mathbf{U}\}$ is required.

In the application of the above model, the total stress in the surface layer of each element is computed by combining the stress components induced when subsoil liquefaction occurred and the initial static stresses in the surface layer given on the average by

$$S_o = 1/2\, K_o \gamma_s t \quad (10)$$

where K_o is the coefficient of earth pressure at rest (assumed to be equal to 0.5 in this study), γ_s is the unit weight of the soil and t is the average thickness of the surface layer. Since sandy soil has no tensile resistance, elements which show tensile principal stresses are picked out and the corresponding E are reduced to only five percent of their normal values.

EXAMPLE CALCULATIONS

The proposed method was used to simulate the ground displacements observed in shaking table tests conducted at the Public Works Research Institute (PWRI) and those measured in Noshiro City. Due to the complexity involved in determining the soil parameters for case history studies, the present study makes use of a common set of soil properties for each analysis. The values employed are shown in Table 1. In addition, the liquefied soil was assumed to behave like liquid, with $G = 0$ and $\tau_r = 0$. For the surface unsaturated layer, $\nu = 0.30$ was used.

Shaking Table Tests

A series of large scale shaking table tests on model soil deposits were conducted at PWRI (Sasaki et al., 1991) to study the mechanism of ground flow induced by soil liquefaction. The proposed method is applied to two of the tests in the series.

In the first case (Case 6), a gently sloped gravel surcharge was placed over one half the length of the loose saturated deposit, as shown in Fig. 2(a). The width of the box was 0.8 m. Shaking was then applied in four stages with the acceleration increasing in each subsequent stage.

The resulting cumulative lateral deformation of the ground surface after each step is shown in

SOIL PARAMETER	CASE 6	CASE 8	NOSHIRO CITY
Modulus of Elasticity E (kN/m^2)	98	294	10780
Unit Weight of Surface Layer γ_s (kN/m^3)	14.5	13.3	15.7
Unit Weight of Liquefied Layer γ_l (kN/m^3)	19.6	18.0	17.6

Table 1: Soil parameters used in the analyses

Fig. 2(b). Also plotted in the same figure are the calculated permanent surface displacements using the proposed method. In the calculation, the ends of the test box are assumed as fixed boundaries. It can be seen that the calculated displacements agree well with the observed values.

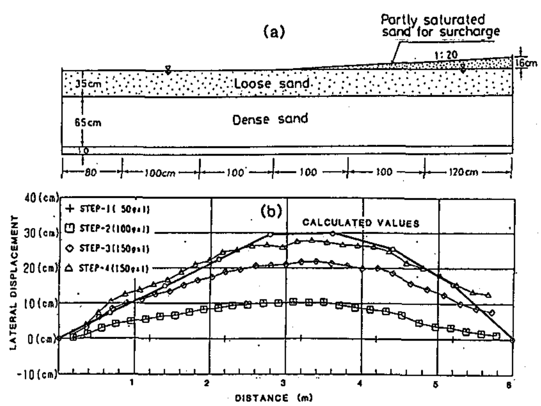

Figure 2: Case 6 Test of PWRI: (a) Ground model; (b) Observed and calculated permanent ground displacements

In the second case (Case 8), a semi-circular liquefiable deposit of 2 m in radius and 0.25 m in height was overlain by a cone-shaped gravel embankment with radius of 1 m and a height of 0.15 m at the center. This model ground was shaken in the longitudinal direction in three stages to determine the effect of the direction of excitation on the lateral movement. Fig. 3(a) shows the direction of ground flow after the first two stages. From this figure, it is noted that the surface of the embankment as well as the neighboring horizontal ground moved almost radially, i.e., in the direction of slope.

Fig. 3(b), on the other hand, illustrates the spatial distribution of the ground displacement obtained by the proposed model. Although the calculated maximum displacement of 3.2 cm exceeds the maximum observed value of 2.7 cm, comparison of the two figures reveals that the distribution calculated conforms with those obtained from the shaking table test and the ground displacements occurred in the radial directions.

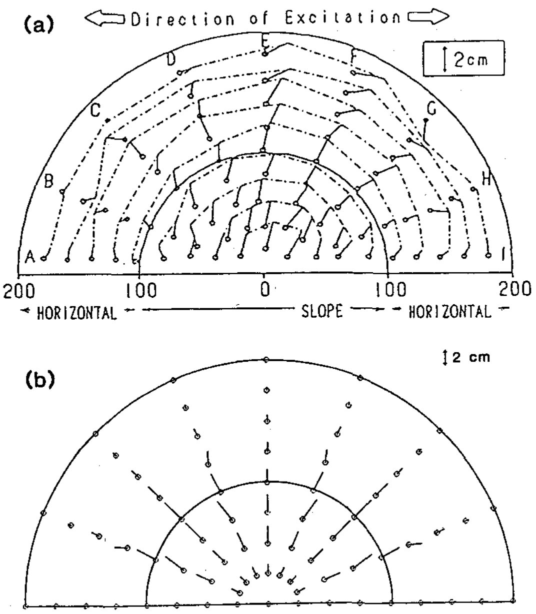

Figure 3: Case 8 Test Results of PWRI: (a) Observed direction of ground flow; (b) Calculated permanent ground displacements

Noshiro City

Analyses were also made on the northern and southern portions of Noshiro City where large ground displacements were observed. In the analyses, fixed boundaries were assumed at the lowland areas where non-occurrence of liquefaction was observed. Similarly, cracks in the upper slope were modeled by free boundaries.

Figs. 4 and 5 show the calculated ground displacements in the northern and southern parts of the city, where the maximum permanent displacements obtained were 3.6 m and 6.6 m, respectively. Hamada et al. (1986) observed that the maximum displacements in these areas were approximately 3 m and 5 m for the northern and southern portions, respectively. Although the analyses slightly overestimate, the method shows good agreement with the measured data. Moreover, the spatial distributions are also consistent with those observed, i.e., displacements are large at high elevations and small at low elevations.

It is noted that the proposed method, being static in nature, does not require complicated soil

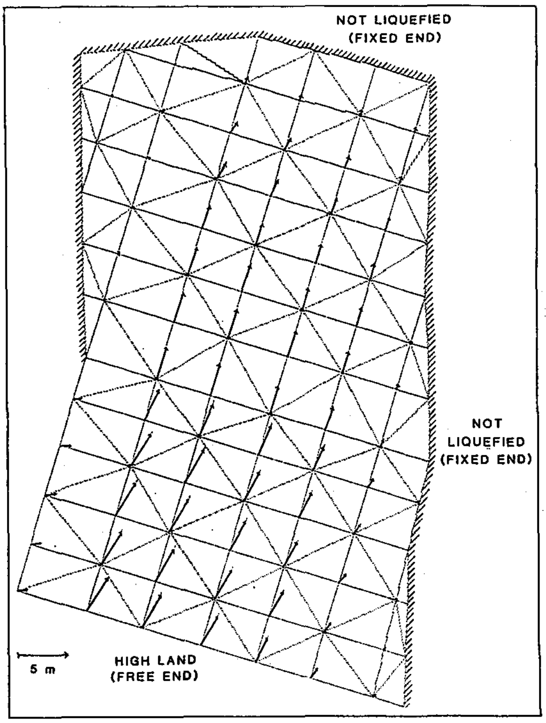

Figure 4: Calculated permanent ground displacements in Noshiro City - northern portion

properties as input parameters. The heterogeneity of the ground can be considered by allowing different values of soil properties to various elements, if necessary. Furthermore, since the liquefiable layer and surface unsaturated layer were assumed as elastic material and liquid, respectively, the computation is fairly straight forward and is not time consuming. For example, the analysis of the southern slope of Noshiro City, wherein the area was divided into 314 finite elements, took only 6 minutes of CPU time in an ordinary personal computer.

CONCLUSION

A simplified numerical procedure based on the principle of minimum potential energy is developed to predict the magnitude and spatial distribution of ground displacements induced by liquefaction. Based on the example calculations made on both experimental and field conditions, it is noted that the method can reasonably predict the resulting displacements.

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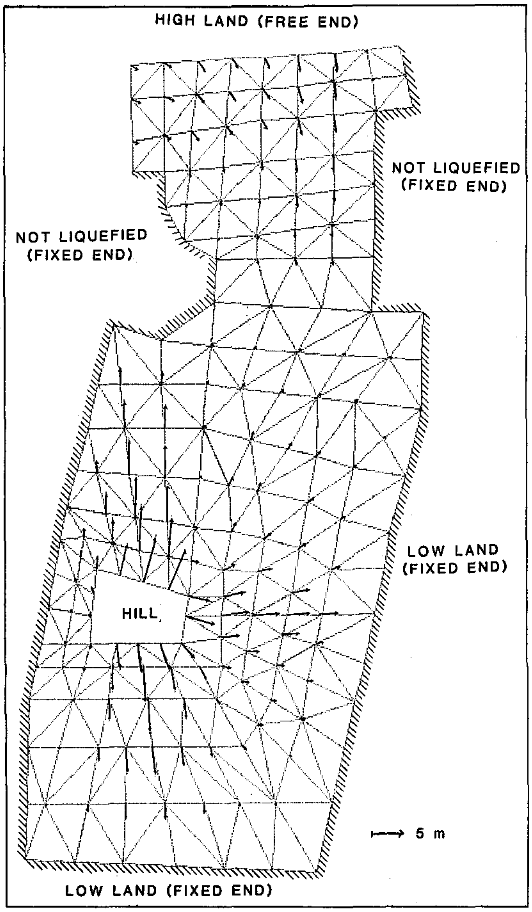

Figure 5: Calculated permanent ground displacements in Noshiro City - southern portion

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