(74) IMPEDANCE FUNCTIONS FOR EMBEDDED FOUNDATIONS ---B.E.M. APPLICATION

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SUMMARY

The problem of the soil-structure interaction as characterized by the impedance functions is studied by applying the boundary element method. The boundary element method is based on representing the boundary conditions as resulting from a set of sources. The shortcoming of this numerical practice is the requirement of a large number of sources, which leads to a long calculation time. A modification is made, in the case of a rigid foundation, by introducing an equivalent rigid-body displacement. By this modification, relatively good results are obtained with only a small number of sources.

INTRODUCTION

The seismic response of a structure embedded in a soil may be calculated by analysing the structure and the soil-foundation imdependently. The soil-foundation interaction problem involves the evaluation of the dynamic response of the foundation when excited by both external forces and incoming seismic waves. The evaluation of the response of the foundation to external forces and moments reduces to the problem of determining the impedance matrix. The evaluation of the response of the foundation to seismic waves is associated with the problem of determining the driving force vectors (the forces and moments for keeping the foundation fixed under seismic excitation). These two problems are related. Once the impedance matrix and the response of the virgin free field to the seismic excitation are known, the driving force vectors can be decided.

The problem of determining the impedance matrix for three-dimensional foundation is considerably complex, for it must satisfy the traction-free condition on the surface of the soil, the radiation condition at the infinity and the condition on the foundation-soil intersurface. Plus, the soil generally is non-homogeneous semi-infinity. It is less likely to be solved by pure analytical means. The finite element methord cannot be successful unless a fictitious boundary is introduced and when the size of the elements is sufficiently small and the element grid is sufficiently large.

An integral technique known as the boundary element method has recently been developed. It is based on representing the boundary conditions as resulting from a set of sources. The fact that the energy transmitting to the infinity can be automatically taken into account, makes the boundary element method especially atractive in the problem of calculating the impedance functions. However, as a numerical method, it is still questionable : can the boundary conditions be efficiently represented by a relatively small number of sources?

Numerical experimentation reveals that a large number of sources is needed, especially when the frequency is high. To overcome this difficulty in the numerical practice, a modification is made by introducing an equivalent rigid-body motion. The validation of this method is presented by comparison of the results obtained for cylindrical foundations with previous results obtained by other methods.

FORMULATION

The soil-foundation model is illustrated in Fig.(1). The foundation, *assumed to be rigid* and massless, is perfectly bonded to the soil along the interface S. The external generalized force Fo

$=(Fox, Foy, Foz, Mox, Moy, Moz)^T$, acting on the foundation, has a harmonic time dependence of the type e^{iwt}, in which w is the frequency. The displacement U(x) over the interface S must satisfy the condition

$$U(x) = T(x)\ Uo \tag{1}$$

where $Uo=(Uox, Uoy, Uoz, Qox, Qoy, Qoz)^T$ represents the generalized rigid-body displacement. The matrix

$$T(x)= \begin{vmatrix} 1 & 0 & 0 & 0 & (z-z) & -(y-y) \\ 0 & 1 & 0 & -(z-z) & 0 & (x-x) \\ 0 & 0 & 1 & (y-y) & -(x-x) & 0 \end{vmatrix} \tag{2}$$

is a rigid-body motion influence matrix. The traction vector $V(x)=(Vx, Vy, Vz)^T$ on the interface S must lead to the resultant force

$$Fo = \int_S T(x)^T V(x)\ dx \tag{3}$$

Now, we represent U(x) by the response of a set of external loads Q acting in the free field within the interface S, as illustrated in Fig.(2). The displacement Up(x) and the surface traction Tp(x) produced by Q can be formulated as

$$Up(x) = G(x)\ Q \tag{4}$$

$$Tp(x) = H(x)\ Q \tag{5}$$

where the matrices G(x) and H(x) contain the Green's functions for the displacement and the stress respectively.

For a finite number of sources the displacement-boundary condition U(x) can be satisfied only in an average sense as

$$\int_S W(x)^T (\ Up(x)-U(x)\ ) dx = 0 \tag{6}$$

where W(x) denotes the matrix containing the weighting functions.

Substituting Eq.(1) and Eq.(4) to Eq.(6), it is possible to write

$$Q = A\ Uo \tag{7}$$

Once the source parameters have been obtained from Eq.(7), the Up(x) and Tp(x) can be abtained from Eq.(4) and Eq.(5) respectively.

Finally, from Eq.(5), Eq(3) and Eq.(7), the desired force-displacement relation can be written in the form

$$Fo = K\ Uo \tag{8}$$

where

$$K = \int_S T(x)^T H(x)\ A\ dx \tag{9}$$

Taking H(x) as W(x), Eq.(6) corresponds to the Euler's equation for a function which has an extreme value which is the work of the unknown tractions on the given displacements. Imposing the rigid-body displacement at the interface S in a least square sense, W(x) becomes G(x).

EQUIVALENT RIGID-BODY MOTION

Considering the fact that the rigid-body displacement boundary condition U(x) can hardly be well represented by the response to a set of concentrate loads, especielly at high frequencis, the boundary element method can not be successful unless the number of the sources is sufficiently large. This is the major difficulty in applying the boundary element method. As an example, the displacements on the surface of a cylinder are plotted in Fig.(3), in the case of offset=0.0 and 1.0 meter. It is understood that in both cases, the displacements produced by the external loads are quite different from the rigid body displacements.

It is possible to divide the displacement Up(x) into two parts, as illustrated in Fig.(3), U1(x) which corresponds to a rigid-body displacement, and the resultant displacement U2(x) to which the corresponding general force is desirable to be 0.

By use of the least square method, the "best-fitted" approximate result can be obtained

$$U1(x)= T(x)H \int_S T(x)^T Up(x)\ dx \tag{10}$$

where H is a 6×6 matrix defined by

$$H = \int_S T(x)^T \, T(x) \, dx \qquad (11)$$

It is worth noting that by this process one may obtain an 'exact' result in the discretized field. In fact, the general force Fo in a discretized field can be written as

$$Fo = T^T K \, Up \qquad (12)$$

where K is the symmetric impedance matrix, the force-displacement relation between the concentrated nodal force vectors and the displacement at each nodal.

Assuming $U2 = Up - T(T^T T)^{-1} T^T Up$, the corresponding force

$$\begin{aligned} Fo' &= T^T K \, (\, Up - U1 \,) \\ &= T^T K \, (\, 1 - T \, (T^T T)^{-1} T^T) \, Up \\ &= T^T (\, 1 - T \, (T^T T)^{-1} T^T) \, K \, Up \\ &= 0 \end{aligned} \qquad (13)$$

This means the displacement U1 calculated by Eq.(10) may be taken as an equivalented rigid-body displacement corresponding to the general force Fo.

NUMERICAL INVESTIGATION

In order to conduct the numerical calculation effectively, we will discuss two variables: the sources (location and number), and the observations. Refer to Fig.(9). Theoretically, an exact result can always be obtained when taking the number of sources and observations to infinity. So the problem of how to determine the source and the observation is rather a numerical one than an analyticale one.

For avoiding the unneccessary singular integration, the source should be located within the interface S. Increasing the offset may smooth the displacement on the interface S, but in the three-dimensional problem in order to dominate the displacement on the interface S, the offset should be taken as small as possible. The only way to obtain a boundary displacement without distortion is to apply a large number of sources and the offset should be small enough.

For calculating the response to the sources accurately, there should be a sufficient number of observation points. In practice, the mesh near the source should be small enough, say, less than 0.5 of the offset. For the sake of economy, the mesh far from the source can be relatively bigger.

RESULTS AND COMPARISONS

Refer to Fig.(4). Numerical calculations are conducted in the frequency domain for a cylinder rigid foundation, embedded in a layered viscoelastic medium with a rock at a certain depth. The Green's functions for ring loads, obtained by E. Kausel, are used. These solutions are based on a discretization of the medium in the direction of layering, which results in a formulation yielding algebraic expressions, whose integral transformation can readily be evaluated (no numerical integration necessary).

The five complex impedance functions calculated by different methods discussed in this paper for different modeling of the sources are plotted in Fig.(5), with the frequency from 0.0 to 10.0. Results are compared to those obtained by the finite element method with use of the so-called transmitting boundary method.

First, calculations are conducted for the case shown in Fig.(5)a, by both the direct boundary element method and the indirect boundary element method. A good agreement between the boundary element method and the finite element method can be seen in Fig.(6), The only significant differences appear at high frequencies where the finite element method results are lower.

The calculations for only 2 sources shown in Fig.(5)b are conducted by the modification method suggested in this paper and by the indirect boundary element method. It can be seen that after the modification, the results improved extremely, especially at high frequencies. From the numerical calculation and comparisons, we may conclude that by the method developed in this paper the impedance functions can be calculated efficiently. This method overcomes the major difficulty in the application of the boundary element method, in which a large number of sources are generally required.

REFERENCE

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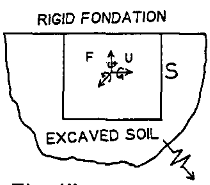


Fig. (1)

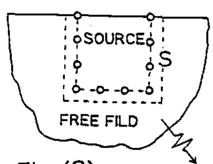
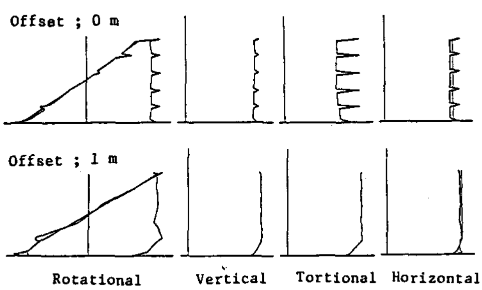


Fig. (2)



Displacement distribution on the surface of the cylinder

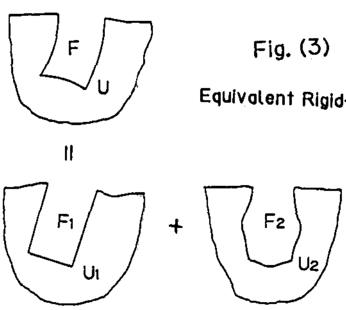


Fig. (3)

Equivalent Rigid-Motion

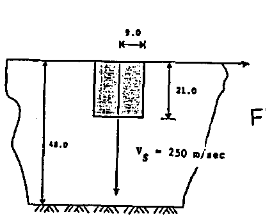


Fig. (4)

Axisymmetric foundation-soil system

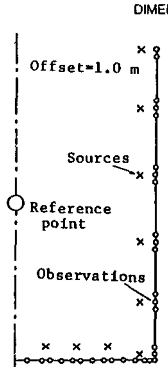


Fig. (5)a

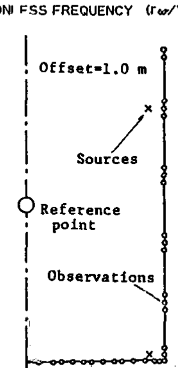


Fig. (5)b

Fig. (6)