(84) Prediction of Hysteretic Moment Rotation Relation of Reinforced Concrete Elements under Variation of Axial Force

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1. Introduction

Recently, new types of very important RC structures with high axial loads, such as PC cable stayed bridges, high rise buildings etc., have been designed and constructed even in earthquake active regions. The effect of high axial force variation during the hysteretic earthquake response can not be neglected for these structures, because of the significant change of the hysteretic moment-curvature relation as well as overall structural behaviour. Since, nonlinear (hysteretic) behavior of RC structures is influenced by the various phenomena, such as material and structural resistance deteriorating process, time variation and interaction of shear and axial loads etc., which is difficult to incorporate throughout the convential analytical procedures or even hysteretic models defined on the level of discrete RC elements, the authors conducted long term, both, experimental and analytical investigation in order to develop appropriate mathematical model for hysteretic earthquake response prediction as well as to find what can be damage and failure criteria for these structures. In this study, the new analytical model for predicting hysteretic moment-rotation relation of the cross section as well as reinforced concrete members under high axial force variation is presented. The proposed mathematical model can be further extended and implemented for the complete and the most accurate reinforced concrete structures modeling and nonlinear dynamic response computation under strong earthquake ground motion.

2. Analytical Approach and Assumptions

The beam theory and fiber representation of the element materials (Fig.1) are the basis of the mathematical model proposed herein for the analysis of the reinforced concrete structures with one dimensional members. Plane cross section remains plane after the deformations, what leads to the linear strain variation with the depth. Reinforced concrete elements are composed of discrete segments with linear variation of bearing capacity between appropriately selected cross sections (Fig.2a). Stress-strain models of confined concrete, unconfined concrete and reinforcing steel material are developed based on experimental results derived from uniaxial tests of sample speciments under generalized cyclic loads. An approach with incremental numerical solution is applied for the hysteretic response computation of the reinforced concrete cross sections, elements and structural systems under high axial load time variation.

3. Steel and Concrete Stress-Strain Models

The accurate prediction of the mechanical behavior of the reinforced concrete structures, elements and cross sections during the strong earthquake ground motion strongly depends on the development of the realistic mathematical models which describe the hysteretic behavior of the critical regions or components of these structures. The experimental test results show, however, that stress-strain relation for confined concrete, unconfined concrete and steel material under arbitrary load reversals are not as simple as it is assumed in many of the previous studies. Based on the past experimental studies of the stress-strain relations of structural materials under generalized cyclic laoding, stress-strain models for concrete and steel fibers are formulated including main parameters influencing these relations. The proposed stress strain models for (confined and unconfined) concrete fibers include concrete confinement levels, tension stresses, failure in tension, crack openings, plastic strains, crush of concrete, compressive failure and stiffness degradation which is described through the complex mathematical model which consists of nine different defined paths, five of which are previous path history dependent. The proposed stress-strain model for steel fiber includes Baushinger effect, plastic strains and isotropic strain hardening and can account for arbitrary strain history.In Fig.(3),(4)are presented behaviors of proposed concrete and steel fiber models under an arbitrary strain time hystory, from which general pattern of the proposed models based on experimental data can be recognized.

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4. Multi Section Fiber Element Model

The instentaneous tangent element stiffness matrix is defined by inversion of the element flexibility matrix which can be estimated by integration of the flexibility along the element taking into account current state of cross section flexibilities. The cross section flexibility matrices are obtained by inversion of the cross section stiffness matrices which are defined by sumation of the curent tangent fiber stiffnesses. The shape functions relating cross section deformations to element displacement are derived from the current section and element flexibilities.

4.1. Cross Section Stiffness and Flexibility Matrix

Using the assumption of plane section, the strain of a fiber can be expressed

$$\varepsilon_i = \varepsilon_a + \Phi y_i \quad \text{....Eq.(1)}$$

(where ε_a :strain at reference point, Φ : curvature and y_i : distance from the reference point). The tangent stiffness of the fiber is the slope of the stress-strain curve for the given strain, which is derived taking into consideration of the previous strain history. By using section equilibrium equations, the incremental section forces can be related with incremental section deformations. Eq.(2) or Eq.(3), (where $[K]_s$: current cross section tangent stiffness matrix). Corresponding cross-section tangent flexibility matrix is given by inversion of section stiffness matrix, Eq. (4)

$$\begin{Bmatrix} \Delta M \\ \Delta N \end{Bmatrix}_s = \begin{bmatrix} \Sigma A_i E_i Y_i^2 & \Sigma A_i E_i Y_i \\ \Sigma A_i E_i Y_i & \Sigma A_i E_i \end{bmatrix} \begin{Bmatrix} \Delta\Phi \\ \Delta\varepsilon_a \end{Bmatrix}_s \ldots(2) \qquad [F]_s = [K]_s^{-1} \ldots(4) \qquad \begin{Bmatrix} M \\ N \end{Bmatrix}_s = \begin{bmatrix} -1+x/L & x/L & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} M_i \\ M_j \\ N \end{Bmatrix} \ldots(6)$$

$$\{\Delta F\}_s = [K]_s \{\Delta r\}_s \ldots(3) \qquad \{\Delta r\}_s = [F]_s \{\Delta F\}_s \ldots.(5) \qquad [F]_e = \int_0^{\ell} [b(x)]^T [F(x)]_s [b(x)] dx \ldots(7)$$

4.2. Element Flexibility and Stiffness Matrix

For the element, Fig.2.(a), the two end rotations along with axial displacement constitute the three local degree of freedom (θ_i, θ_j, and δ_l) associated with two end moments (M_i, M_j) and axial load (N). Assuming linear variation of flexibility between appropriate selected cross sections the element flexibility can be calculated by closed form of integration along the length of the element. The first step is to relate the cross section forces $[S]_s$ to the member end forces $[S]_m$, Eq(6), by implementing equilibrium equations through $[b]$ matrix. If flexibility matrix (3x3) is obtained by intergration, Eq.(7). the element stiffness matrix can be obtained by inversion of the element flexibility matrix and relation between incremental element end forces and element end deformations is defined, Eq.(8)

$$\begin{Bmatrix} \Delta M_i \\ \Delta M_j \\ \Delta N \end{Bmatrix} = \begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{bmatrix} \begin{Bmatrix} \Delta\Phi_i \\ \Delta\Phi_j \\ \Delta\delta_\ell \end{Bmatrix} \ldots.(8)$$

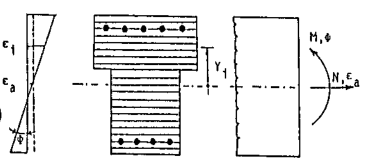

Fig.1.Typical cross section fiber model (CSFM)

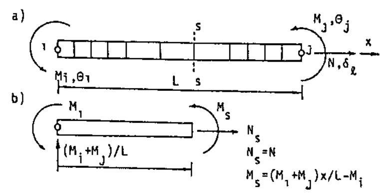

Fig.2.Beam element in local coordinate system. Local degrees of freedom and element forces (a),relating section forces to element forces (b)

4.3. Hysteretic Moment-Curvature Relation of RC Cross Section

Following the incremental solution approach, the hysteretic moment-curvature relation for a single cross section of the RC member can be derived by implementation of Eq.(2) under time varying moment and axial force.

4.4. Hysteretic Moment-Rotation Relation of RC Members

Similarly, the nonlinear relation between member end forces and member end deformations can be calculated by incremental solution of Eq(8) for the given time varying member end moments and axial force.

5. Verification of The Proposed Mathematical Model Based on Experimental Results

5.1. Short Description of HYLSER Experimental Tests and Results

In order to obtain experimental results for the hysteretic behavior of RC elements under simultaneous bending and time varying axial loads, a number of On-line Hybrid Loading System (HYLSER) tests have been conducted,in which case earthquake response is calculated by a digital computer adopting real element hysteretic restoring force, directly measured from a loading actuator. For the experimental tests different levels of recorded acceleration time histories are used as well as different axial force time histories.

In the experiment, reinforced concrete speciments with cross section of 150x150mm and total length of 2090mm where used. The longitudinal reinforcement of the speciments exists of 4 Ø 12.7mm steel bars, while the lateral reinforcement of Ø 5mm tie loops was changed by assuming different pitch of 60mm and 90mm for different speciments (Fig.5). The compressive strength of concrete was also varied, so the experimental data for hysteretic behavior of the speciments with different design parameters have been obtained. For all tested speciments, large number of data have been stored in the computer in a digital form such as restoring force time history, displacement time history, axial force time history, curvature time history of the critical cross section, strain time histories of longitudinal and lateral reinforcement etc. Using the collected experimental data actual force-displacement and moment-curvature hysteretic loops have been plotted for each tested speciment separately. For example, for the tested speciment No.4 hysteretic moment-curvature relation of the critical cross section is plotted in Fig.6 for different time intervals t= 0-10sec, t= 10-20 sec and t= 20-30 sec. In this case, N-S component of El-Centro record obtain during the Imperial Valley earthquake (5-18-1940) in the U.S. have been used as input acceleration time history. The complete hysteretic moment-curvature loop for the critical cross section obtained from the experimental test of speciment No.4 is plotted separately in Fig.10.a. The actual hysteretic moment-curvature loop in this case is obtained for the case of simultaneously applied time varying compressive axial force which time history is plotted in Fig.9.

Generally, the experimental results for the tested speciments show significant change of the hysteretic loops caused by the time varying amplitude of applied axial force, which practically leads to the significant change in overall structural response.

5.2. Mathematical Model and Theoretical Results

Mathematical fiber model of the critical cross section of the speciment No.4 is presented in Fig.5. It consists of 28 confined concrete fibers, 34 unconfined concrete fibers and 4 steel fibers or in total 66. The parameters for the stress-strain models of concrete and steel fibers are obtained on the basis of the uniaxialy testets of material sample speciments (Fig.8.a and 8.b)

In order to obtain theoretically hysteretic moment-curvature relation which can be compared with the one plotted by using experimental results the measured axial force and curvature time histories have been used as a input parameters. Theoretically calculated hysteretic moment-curvature relation is plotted similarly in fig.7 for time intervals t= 0-10sec, t= 10-20sec and t= 20-30sec. In addition, the complete theoretical hysteretic moment-curvature loop for the analyzed critical cross section of speciment No.4 is plotted in Fig.10.b. In Fig.8. typical fiber stress-strain histories are presented for the specified confined concrete fiber No.3 and steel fiber No.1. From the plotted experimental and theoretical hysteretic moment-curvature relations (fig.6, Fig.7 and Fig.10) it is evident that the complete pattern as well as corresponding moment and curvature levels are in good agreement even for this specific case where input curvature and axial force histories are given in quite general (irregular)form. It can be noted that generally good agreement between experimental and theoretical results is obtained for all 21 experimentally tested speciments with different strength of concrete and confinement levels as well as different loading conditions.

6. Further Research Prospects and Conclusions

In this paper, a new mathematical model for hysteretic moment-curvature prediction of the RC cross sections and Moment-Rotation prediction of the RC elements under simultaneously applied time varying axial force, have been presented which is based on formulated material stress-strain models. The applicability of the model is demonstrated by comparing the theoretical and experimental reuslts obtained by newly developed Hybrid Loading System for Earthquake Response computation (HYLSER). From the agreement obtained between experimental and theoretical results it can be concluded that mathematical model proposed herein can be further implemented for the refined RC complete structure modeling as well as nonlinear dynamic earthquake response computation on the basis of direct integration of the dynamic equation of motion.

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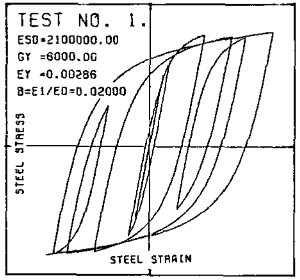

Fig.3.Proposed steel stress-strain model under arbitrary strain time history

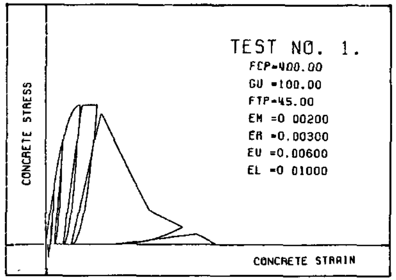

Fig. 4.Proposed concrete stress-strain model under arbitrary strain time history

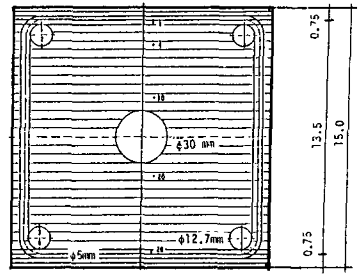

Fig.5.Cross section fiber model of speciment No.4.

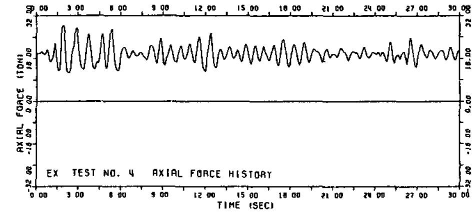

Fig.9.Applied time varying axial force for experi-*mental test and analytical solution in case* of the speciment No.4.

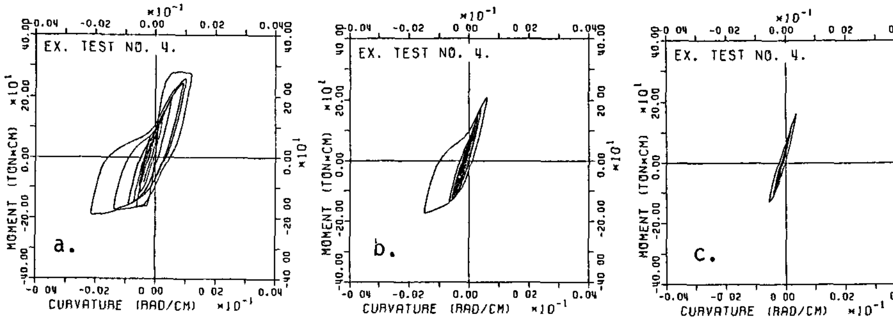

Fig.6.Experimental moment-curvature relation obtained for critical cross section of speciment No.4. tested by HYLSER system.Earthquake:El-Centro, component N-S, Time intervals:t=0-10sec.(a), t=10-20sec,(b), t=20-30sec,(c).

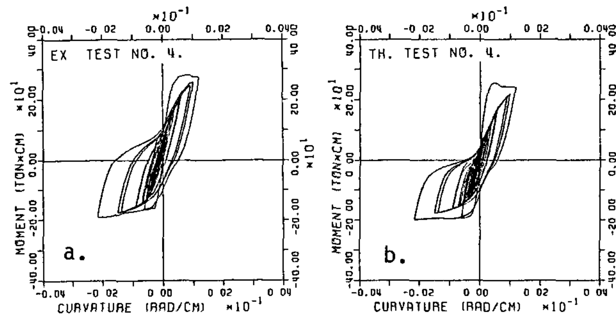

Fig.10.Comparision of the experimental (a) and theoretical (b) moment-curvature relations for the total time t=30 sec.

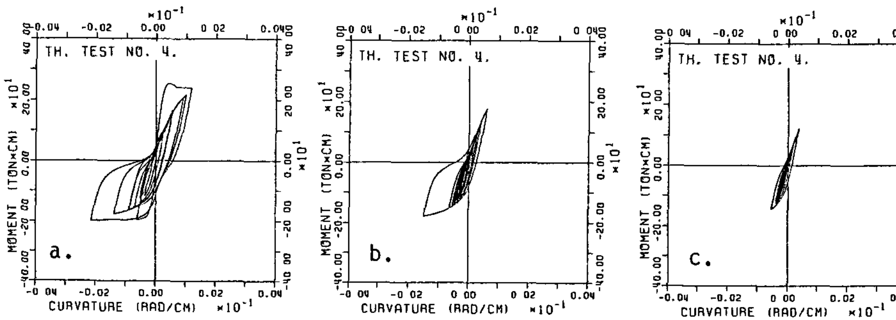

Fig.7.Theoretical moment-curvature relation obtained for critical cross section of speciment No.4. Time intervals : t=0-10sec,(a), t=10-20sec,(b), t=20-30sec,(c).

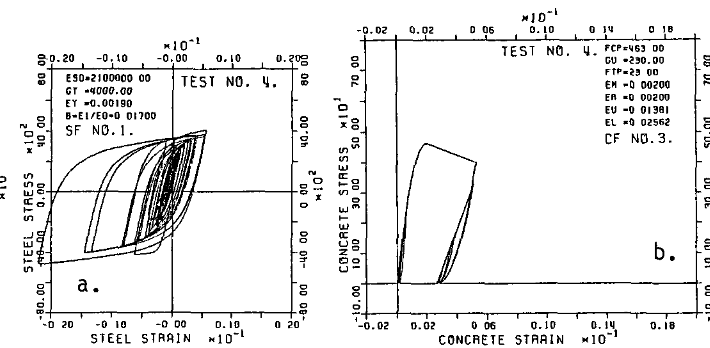

Fig.8.Typical stress-strain histories for specified cross section steel fiber No.1,(a),and concrete fiber No.3,(b).