(76) FREQUENCY AND DAMPING OF TOWER-SOIL SYSTEM WITH PARTIAL UPLIFT OF FOOTING

by

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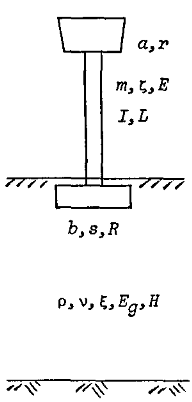
Introduction

This study aims to quantify the change in fundamental frequency and modal damping ratio of an inverted-pendulum-type tower, when the foundation soil is a viscoelastic medium rather than an extremely hard rock, and when partial uplift of the base mat is allowed. Nondimensionalized parameters are introduced, even as different tower proportions and soil profiles are considered (Fig.1).

A rigid circular shallow footing is assumed, and frequency-dependent foundation impedance functions <1,2> are adopted. The same functions are used in an approximate way when the possibility of partial uplifting of the foundation mat is included. Other recent studies of partial uplift have usually neglected this frequency-dependence.

Mathematical Model

The tower is considered a Bernoulli-Euler beam whose axial deformations are negligible while axial displacements are mainly due to lateral beam deflections. For the coupled rocking-swaying of the tower, four generalized displacements are defined, and corresponding constraint shapes are used as generalized coordinates (Fig.2-a). The footing mat being shallow, uncoupled are the three mass-dashpot-spring sets (now as if anchored to some rigid rock) which are used in place of the viscoelastic soil. Unlike the dashpots and virtual masses, the spring constants may be considered wholly independent of vibration frequency (Fig 2-b).



TOP MASS = aLm
TOP ROTATORY INERTIA = rL^3m
COLUMN MASS PER UNIT LENGTH = m
COLUMN YOUNG'S MODULUS = E
COLUMN SECTIONAL MOMENT OF INERTIA = I
COLUMN LENGTH = L
* COLUMN STOCKINESS = I/L^4
TOWER DAMPING FACTOR = ζ
BASE MASS = bLm
BASE ROTATORY INERTIA = sL^3m
BASE RADIUS = R
SOIL MASS DENSITY = ρ
SOIL POISSON'S RATIO = ν
SOIL HYSTERETIC DAMPING CONSTANT = ξ
SOIL YOUNG'S MODULUS = E_g
SOIL LAYER THICKNESS = H
* SOIL RELATIVE SHALLOWNESS = R/H
* ASPECT RATIO = L/R
* MASS RATIO = $m/\pi R^2\rho$
* ELASTICITY RATIO = E/E_g

Fig.1 : Major nondimensionalized parameters of the system marked with asterisk (*)

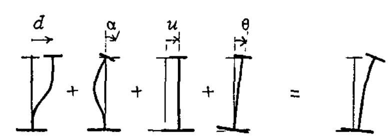


Fig.2-a : Generalized coordinates

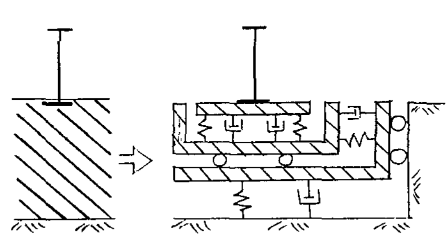


Fig.2-b : Structural model

Time is nondimensionalized into $t^* = t\sqrt{EI/mL^4}$, and, correspondingly, each nondimensionalized circular frequency is denoted as ω^*. Another nondimensionalized frequency is relevant to the foundation impedance functions, namely, a_o. The two frequencies are related thus:

$$a_o^2 = \omega^{*2} \cdot (E/E_g) \cdot (I/L^4) \cdot (2\{1+\nu\}/\pi)/(m/\pi R^2 \rho).$$

The real-valued dashpot coefficients and virtual soil masses shown schematically in Fig.2-b depend on a_o, as well as on R/H, ν, and ξ. Table 1 summarizes the general trend with respect to a_o.

a_o	virtual mass	dashpot coefficient
low	large	small if R/H >> 0.0
high	small	large for any R/H

Table 1 : General trend of frequency-dependence

Denoting a frequency-dependent quantity Q as $Q(\omega^*)$, the time-independent matrix equation of free oscillation (coupled swaying and rocking) of the base-fully-bonded-to-the-soil case may be written as:

$$[M(\omega^*)]\{\ddot{q}\} + [C(\omega^*)]\{\dot{q}\} + [K]\{q\} = \{0\}$$

where

$$\{q\} = (d \quad \alpha L \quad u \quad \theta L)^t \quad , \quad \{\dot{\ }\} = d\{\ \}/dt^* \quad .$$

The third and fourth diagonal elements of $[M]$ and $[C]$ contain the virtual soil masses and the soil damping. $[K]$ can include axial-force effect, here due to gravity. The frequency-independent part of $[C]$, representing the structural damping, is selected such that it is proportional to $[K]$ and contributes ζ % to the first natural mode.

When partial uplift occurs, the equation of nonlinear free oscillation becomes

$$[M(\omega^*, t^*)]\{\ddot{q}\} + [C(\omega^*, t^*)]\{\dot{q}\} + [K(t^*)]\{q\} = \{0\} ,$$

subject here to an initial rocking velocity. The present approximate method to accomodate dependence on both frequency and time is presented here later.

Full-Bond Case

To deal with non-proportional damping, a complex-eigenvalue analysis is made. Virtual masses and dashpot constants are used that correspond to the first (lowest) subcritically damped mode. (Therefore note that the natural frequencies and, particularly, the damping ratios of the other modes so obtained may be grossly misrepresented.) The fundamental mode (nonclassical) consists basically of tower flexure, base rocking, and small base swaying (see right part, Fig.2-a).

The typical values of the parameters defining different tower proportions are summarized in Table 2. Numerical results for each case are shown and labeled VSL, SL,or ST in Figs.3 to 6.

	$I/L^4, 10^{-6}$	$m/\pi R^2\rho, 10^{-3}$	$E/E_g, 10^3$	L/R
Stocky (ST)	25.0	30.0	0.1	3.0
Slender (SL)	1.0	3.0	1.0	3.0
Very Slender (VSL)	0.1	0.3	10.0	3.0

Table 2 : Typical values of nondimensionalized parameters

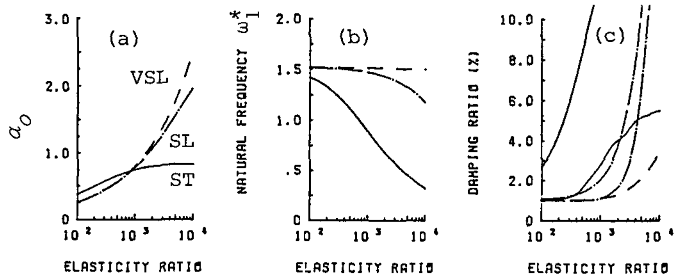


Fig.3 : Fundamental mode of full-bond case

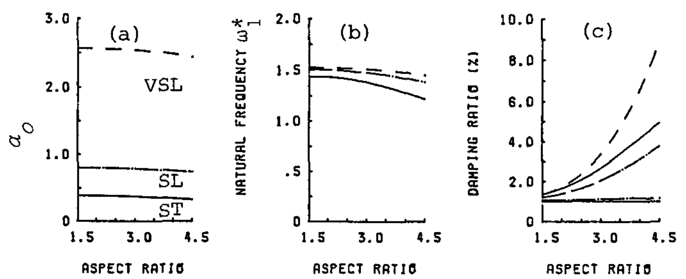


Fig.4 : Fundamental mode of full-bond case

The other parameters in the examples have these values: a=1.0 ; r=0.020 ; b=8.0 ; s=0.222 ; ζ =1.0% ; ν =0.333; ξ =10.0% ; R/H=0.0 (or 0.5). When a tower has two damping curves, the lower one means R/H=0.5 (shallow layer).

Fig.3 shows the influence of elasticity ratio. Next, with the respective elasticity ratios fixed, the slenderness ratio (and the base rotatory inertia) can be varied, with results shown in Fig.4. Meanwhile, other numerical results (not shown here) indicate that the mass ratio has little effect even on the modal damping. Likewise the axial-force effect due to gravity is slight.

Case with Partial Uplift

A certain pseudostatic criterion is defined here to indicate uplift. Given that the rigid-body vertical oscillations of the tower-soil system is so highly damped as to be pseudostatic, the critical angle of base rotation (in radians) may be simply related to the vertical deformation of the soil (here due to the structural weight) and the base radius:

$$\theta_{cr} = (\pi/2)\cdot(1-\nu)\cdot(1+\nu)\cdot(1 + a + b)\cdot(m/\pi R^2\rho)\cdot(g\rho R/E_g)\cdot(L/R)/(1 + 1.28\ R/H).$$

A value of 8.0×10^{-3} is taken as typical of the nondimensionalized vertical acceleration, $g\rho R/E_g$. Uplift is considered to be initiated when the base mat rotation exceeds θ_{cr}. Accordingly R is replaced by an effective radius for the next time step in the direct-integration scheme. New values of the soil virtual masses and dashpot coefficients are obtained (again in the frequency domain) for the instantaneous pseudolinear system. Further uplift thru the next time step is ascertained after replacing R even in the expression for θ_{cr}. Thus θ_{cr} increases, i.e., further uplift becomes progressively difficult, as the effective radius decreases while all other parameters are fixed. This simple approach goes well with the physical observation that superstructures do not normally proceed to overturn on soil which is compliant unless, of course, the soil liquefies.

In the present case when geometric nonlinearity at the base-soil interface is allowed, one can speak only of apparent natural frequency and modal damping ratio. A Wilson-theta method is used here to generate a time history of $\{q\}$ spanning the first few oscillations; then the apparent frequency and damping are evaluated from the displacement record of θ . Each structure is imparted a certain initial energy in rocking that is enough to obtain a peak base rotation θ_p that is 10 times θ^0_{cr} , the critical rotation at the initial, full-contact condition. The results are shown as points in Fig.5 and Fig.6 (compared with curves from Fig.3 and Fig.4, respectively).

It must be noted, however, that in real (time domain) terms the imparted energies implied must exceed approximately

$$50.0 \cdot (a + r + s + 1/3) \cdot L^3 \cdot m \cdot \theta_{cr}^2 \cdot \omega_1^2 .$$

Hence greater energy is actually required for the taller and heavier tower, and for the stiffer tower-soil system.

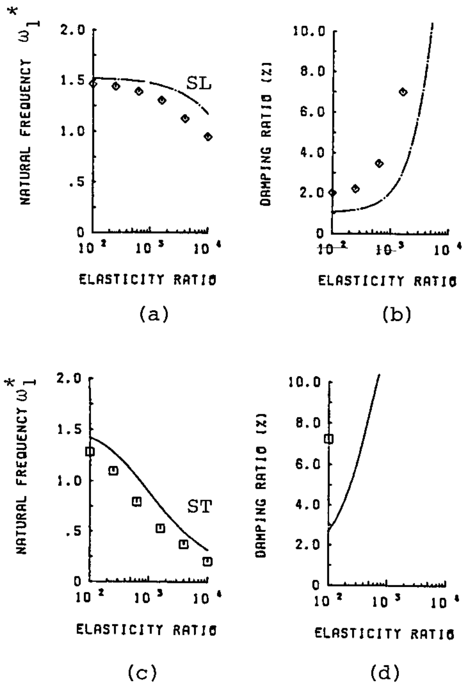


Fig.5 : Apparent fundamental mode with moderate uplifting

Conclusions

Full-bond case compared with fixed-base case:

The stockier and relatively heavier tower is most susceptible to a reduction of natural frequency (at least 10%). The very slender tower is almost not affected. (Figs.3-b,4-b)

When the soil layer is deep (R/H << 0.5) the modal damping may become double or triple. Otherwise the stockier tower gets no (little) additional damping.(Figs.3-c,4-c)

With increasing elasticity ratio between tower and soil, the stockier tower gets much more damping from the soil. (Fig.3-c)

With increasing aspect ratio, the very slender tower obtains much more damping from the soil. This is regardless of R/H , because the associated a_o is always high.(Figs.4-a,c)

With uplift, compared with full-bond case:

The apparent natural frequency may become about 10-15 % lower, given a certain moderate extent of uplifting of the base. (Figs.5-a,c, 6-a,c)

When the soil is rather deep, the apparent modal damping may become double.(Figs.5-b,d,6-b,d)

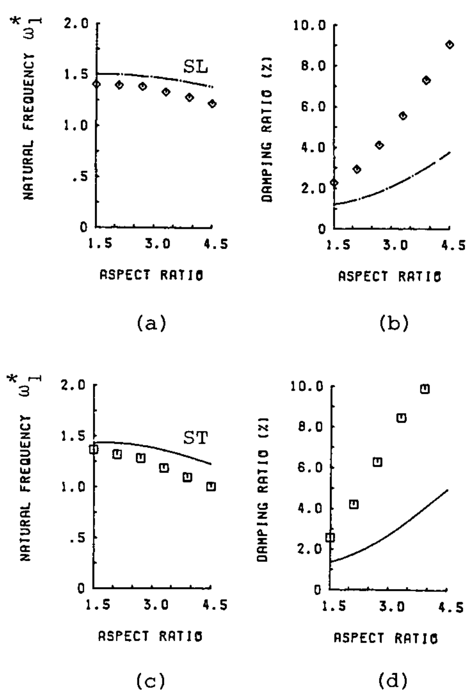


Fig.6 : Apparent fundamental mode with moderate uplifting

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