ANALYSIS OF SOLID-LIQUID SEPARATION CURVE BASED ON BOTH QUANTITY AND RATIO IN SEPARATED LIQUID

分離水の量と比率に基づく固液分離曲線の解析

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ABSTRACT: This paper deals with an analysis of solid-liquid separation curve based on both quantity and ratio (relativity quantity) in separated liquid. It uses a statistical model based on the analysis of the separation curve (i.e., the speed of separation) through a multivariate analysis of variance. By using this technique, effectiveness of separation factors affecting the solid-liquid separation in sludge can be evaluated, and quality of the separated liquid resulting from a filter can be also examined. Moreover, this analysis can be extended to the quality of separated liquid as related to the quantity, viz., the speed of separation.

KEYWORDS; sludge, solid-liquid separation, quantity of water, water quality, MANOVA

1 INTRODUCTION

Roughly reviewing the past studies relating to the solid-liquid separation of sludge, solid-liquid separation in sludge has been mainly evaluated with the following methods[1]: 1) Measurement of the final moisture content of sludge cake, 2) Measurement of the ratio of decrease in sludge moisture content, 3) Calculation of specific resistance and 4)Dynamic analysis. These methods can be classified into two categories; one consists of the first two methods while the other, the second two methods. The basic difference between the two categories is that the first is directly based on data obtained, whereas the second, on theoretical analysis. In practical application, it appears that the first category, and especially the first method, is more suitable for evaluating the effectiveness of solid-liquid separation since it employs the moisture content directly.

Considering a total environmental conservation system, information about the sludge generated from a sewage treatment plant and its dewatering effectiveness must be established. The dewatering should be evaluated not only from the quantity of the liquid separated, but also from the quality of the separated liquid.

In the present paper, as a preliminary work to the solution of the problem mentioned above, a technique for evaluating solid-liquid separation in sludge for a filter is proposed based on the concept of the first category, while a multivariate analysis of factors affecting the separation is employed. In particular, we introduce an analysis of solid-liquid separation curve based on both quantity and ratio in separated liquid.

The evaluation of the separation effectiveness is made by comparing the rates of sludge moisture as it decreases within given times or, in other words, to analyze the "separation curve" which describes the quantity of separated liquid as a function of separation time.

The most important parameters of this new technique are α' and β. The former estimates the volume of the initial

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quantity of the separated liquid whereas the latter estimates the time-tendency of the increase of volume of separated liquid. From these two values, it is possible to predict the quantity of separated liquid at an arbitrary separation time. The model reveals that when the value of α' is maximum, the value of β falls to a minimum. A rapid separation is possible at the initial stage of the separation. First, the maximum value of parameter α' is identified and then the minimum value of parameter β. If several points are available, the optimum condition is determined by giving priority to the point where α' has its maximum value.

2. ANALYSIS OF SEPARATION CURVE

2.1 Analysis based on Absolute Quantity in Separated Liquid[2)]

When putting the sludge in the separation machine, in view of the operational condition, the separation factors on the sludge affecting the sludge dewaterability are selected, and these factors are called separation factors. These factors have the levels each {(e.g., in three-way layout method, $A_i B_j C_k$ ($i = 1\sim\ell$, $j = 1\sim m$, $k = 1\sim n$, for example, $\ell = m = n = 3$)}. In the design of the experiment, three types of filter cloths (e.g., A_1 = PF-8044, A_2 = PF-401, and A_3 = P-2088), sludge layer thickness (e.g., B_1 = 10 mm, B_2 = 20mm, and B_3 = 30mm), and filtration pressure / suction (e.g., C_1 = 0.03MPa, C_2 = 0.06MPa, and C_3 = 0.09MP) are used as main dewatering factors affecting the quantity of filtration both experiments. The quantity of filtrate is measured at 5 min., 10 min., 15 min., 30 min., 45 min., and 60 min. after the experiment starts. In each case of the combinations of the factor evels $A_i B_j C_k$, the separation curve as shown in **Fig. 1** can be obtained. Assuming the quantity of separated liquid, V, to be a function of the separation time t (The Separation time t should be here changed according to the sludge.), then from the form of the separation curve, the following model is adopted:

$$V(t) = \alpha t^{\beta} \qquad (1)$$

where $\alpha > 0$, $1 > \beta > 0$, $t > 0$

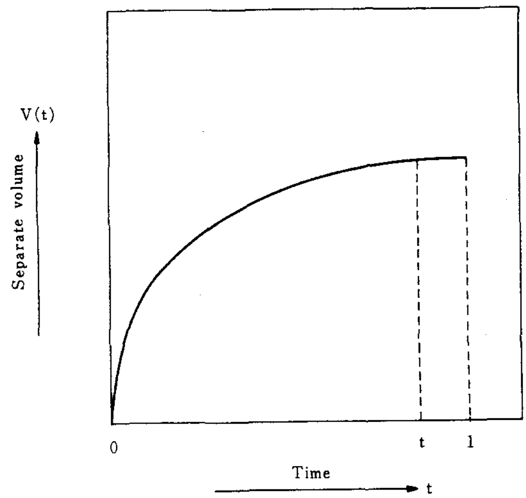

Fig.-1 Typical separation curve.

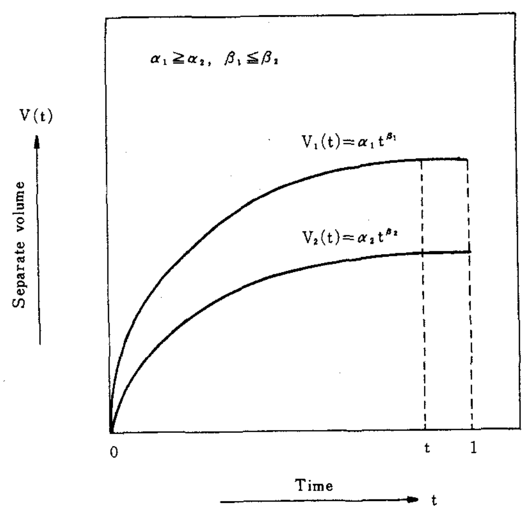

Fig.-2 One formula for faster separation and the other for slower separation.

The speed of separation at the initial stage shall be next examined and the curve retained which has the fastest separation by estimating the parameters, α and β. The procedure is based on the following fact.

Considering two separation curves as shown in **Fig. 2**, $V_1(t) = \alpha_1 t^{\beta_1}$ and $V_2(t) = \alpha_2 t^{\beta_2}$ at the initial stage $0 < t < 1$.

The ratio of the two curves is

$$\frac{V_1(t)}{V_2(t)} = \frac{\alpha_1}{\alpha_2} \cdot (t)^{\beta_1 - \beta_2}$$

If the parameters $\alpha_1 \geqq \alpha_2$ and $\beta_1 \leqq \beta_2$ are taken, then the ratio is always greater than one, and so $V_1(t) \geqq V_2(t)$ for $0 < t \leqq 1$. This means that, when the fastest separation curve at the initial stage ($0 < t \leqq 1$) is selected, the largest possible α and the smallest possible β should be chosen.

On the other hand, the condition with small α and large β means that the speed of separation at the initial stage is very slow.

From the model (1), it is known that the parameter α indicates the quantity of separated liquid when the unit time ($t = 1$) has passed. The parameter β indicates the quantity of filtrate when the time ($t = 1$) has passed. The parameter β ($0 < \beta < 1$) determines the shape of the separation curves and gives us some information about the relative rate of change to the quantity of the separated liquid V(t), that is, the ratio of the derivative V'(t) to V(t),

$$\frac{V'(t)}{V(t)} = \frac{\beta}{t} .$$

In order to estimate parameters and β in the model (1) through a linear regression analysis, natural logarithms of both sides of Eq.(1) shall be taken: The model (1) is defined for comparing the filtration curves, and also designed to describe $\ln t$ and $\ln V(t)$. Moreover, the model is not introduced with the physical implications other than time. Therefore, based on the model, the differential at $t = 0$ is not defined, and the model is considered only in $t > 0$. In case of fitting the statistical model ($V(t) = \alpha t^{\beta}$) to the various kinds of filtration curves, the interpretation of the estimated values ($\hat{\alpha}$, $\hat{\beta}$) of population parameter is the subject. Fortunately, the comparison of filtration curves at the initial stage ($0 < t \leqq 1$) gives some light on the interpretation of population parameter (α, β) including physical information, except for time. At the initial stage ($0 < t \leqq 1$), we can arbitrarily set up the unit time ($t = 1$).

$$\ln V(t) = \alpha' + \beta \cdot \ln t \qquad \text{where} \qquad \alpha' = \ln \alpha \qquad (2)$$

By carrying out the regression analysis with respect to Eq.(2), α' and β as the estimated values of α' and β are obtained.

The value of α' estimates the volume of the initial quantity of the separated liquid when a unit time passes, and the value of β estimates the time-tendency of the increase of volume of the separated liquid. From these two values, it is possible to predict the quantity of the separated liquid at an arbitrary separation time. If there are a few points at the initial stage, the maximal condition of quantity of filtrate is determined by giving priority to the point which has the maximum value of α', because the filtration curve with respect to the point dominates the remaining portion of the curve as time passes.

2.2 Analysis based on Relativity Quantity (Ratio) in Separated Liquid

In order to consider a balance among various factors such as quantity and quality of water and sludge, as well as energy for evaluating solid-liquid separation process in a sewage system, we here propose a cumulative proportion curve shown in Eq. (3).

$$P(t) = \frac{V(t)}{T} \qquad (3)$$

where V(t): quantity of separated liquid at the arbitrary separation time

T : total quantity of separated liquid

If we can define a cumulative proportion curve as mentioned above, the following two model are considered.

Logit transformation :

$$\ln \left(\frac{P(t)}{1 - P(t)} \right) = \alpha + \beta \cdot \ln t \qquad (4)$$

Complementary - log - log Transformation :

$$\ln [- \ln (1 - P(t))] = \alpha + \beta \cdot \ln t \qquad (5)$$

By drawing the scatter diagram with respect to a relationship between ln t and V(t), we select a model which shows the stronger linear tendency.

3. EVALUATION OF SOLID-LIQUID SEPARATION IN SLUDGE USING MULTIVARIATE ANALYSIS OF VARIANCE

Here we consider the two methods using a multivariate analysis of variance (hereafter called "MANOVA") for evaluating the optimum condition of solid-liquid separation as viewed from a balance among various factors such as quantity and quality of water, sludge and energy. One is a three-way layout method, and the other is five-way one.

3.1 MANOVA of the Three-Way Layout Method

If the fitting of the regression equation is good enough to summarize the data, the analysis can be continued further. There are two estimates α' and β for each separation curve of each level $A_iB_jC_k$ ($i = 1 \sim \ell$, $j = 1 \sim m$, $k = 1 \sim n$). They shall be put together in a vector notation as

$$\chi_{ijk} = (\chi^1_{ijk}, \chi^2_{ijk}) = (\alpha', \beta)' \qquad (6)$$

for $A_iB_jC_k$. As there is a 2-dimensional observational vector for each separation curve, a MANOVA to the data set shall be applied.

$$\{(\chi^1_{ijk}, \chi^2_{ijk})', i = 1 \sim \ell, \ j = 1 \sim m, \ k = 1 \sim n \}$$

MANOVA is a technique for testing the main effect and interaction of various kinds of factors taking up the p-variate at the same time. This method corresponds to the direct extension of the analysis of variance with a single variate. After obtaining each mean vector, each variation is calculated. Using these values, a MANOVA of the three-way layout method is conducted in the form as shown in **Table 1**. Regarding the application of the technique, we are concerned for referring the other paper[1].

3.2 MANOVA of the Five-Way Layout Method

The five-way layout method is as follows. For convenience, the values of α' and β are shown as a vector notation.

$$(\alpha', \beta) = \chi_n \qquad n = 1, 2, \cdot\cdot\cdot, 32$$

The variation due to each separation factor and error can be calculated through the following formula.

$$S_f = \frac{1}{32} (\mathbf{T}_1 - \mathbf{T}_2)(\mathbf{T}_1 - \mathbf{T}_2)' \qquad (7)$$

where S_f : variation due to each dewatering factor A, B, C, D, G vand E. (f = A, B, ..., G, A×B, A×C, ..., D× G, A×B×C, A×B×D, ..., C×D×G and E)

$\mathbf{T}_1, \mathbf{T}_2$: total vector of data corresponding to the level 1 and 2 of the orthogonal array, respectively.

Using these values of variation obtained from Eq. (7), MANOVA of the Five-way layout method is conducted in the form as shown in **Table2** [1]. We will examine the application of the technique in the near future.

By using these methods, we can compare the solid-liquid separation process considering various factors in each sewage treatment system. Therefore, through a compareson of the separation process, we might be able to show an optimum path of sludge for its management.

4. CONCLUSIONS

For the estimation of solid-liquid separation effectiveness in sludge, traditionally attention has been paid mainly to the cake moisture content. In the present paper, an emphasis has been put on the quantity and quality

Table -1 Multivariate analysis of variance of the three-way layout method.

(1) Design of experiments

Factor A : ℓ level Factor B : m level Eactor C : n level

Number of repetitions : 1 Data : p-dimensions

(2) Multivariate analysis of variance

Factor	Variation(S_f)	Degree of freedom(ν_f)	Chi-square	Test standard
A	S_A	$\ell-1$	χ^2_A	$\chi^2_{p(\ell-1)}(\alpha)$
B	S_B	$m-1$	χ^2_B	$\chi^2_{p(m-1)}(\alpha)$
C	S_C	$n-1$	χ^2_C	$\chi^2_{p(n-1)}(\alpha)$
A × B	$S_{A\times B}$	$(\ell-1)(m-1)$	$\chi^2_{A\times B}$	$\chi^2_{p(\ell-1)(m-1)}(\alpha)$
B × C	$S_{B\times C}$	$(m-1)(n-1)$	$\chi^2_{B\times C}$	$\chi^2_{p(m-1)(n-1)}(\alpha)$
A × C	$S_{A\times C}$	$(\ell-1)(n-1)$	$\chi^2_{A\times C}$	$\chi^2_{p(\ell-1)(n-1)}(\alpha)$
E	S_e	$(\ell-1)(m-1)(n-1)$		
T	S_T	$\ell mn-1$		

$$\chi^2_f = -\{(\ell-1)(m-1)(n-1) - \frac{1}{2}(p - \nu_f + 1)\} \log_e \nu_f$$

$$\nu_f = \frac{|S_e|}{|S_f + S_e|} \qquad (f = A,\ B,\ C,\ A\times B,\ B\times C,\ A\times C)$$

Table -2 Multivariate analysis of variance of the five-way layout method.

(1) Design of experiments

Factor A: 2 level Factor B: 2 level Factor C: 2 level Factor D: 2 level
Factor G: 2 level
Number of repetition: 1 Data: 2-dimensions

(2) Multivariate analysis of variance

Factor	Sum of squares	Degree of freedom(ϕ_f)	Statistic	Test standard
A	S_A	$1(=\phi_A)$	χ^2_A	$\chi^2_p(\alpha)$
B	S_B	$1(=\phi_B)$	χ^2_B	$\chi^2_p(\alpha)$
C	S_C	$1(=\phi_C)$	χ^2_C	$\chi^2_p(\alpha)$
D	S_D	$1(=\phi_D)$	χ^2_D	$\chi^2_p(\alpha)$
G	S_G	$1(=\phi_G)$	χ^2_G	$\chi^2_p(\alpha)$
A×B	$S_{A\times B}$	$1(=\phi_{A\times B})$	$\chi^2_{A\times B}$	$\chi^2_p(\alpha)$
A×C	$S_{A\times C}$	$1(=\phi_{A\times C})$	$\chi^2_{A\times C}$	$\chi^2_p(\alpha)$
A×D	$S_{A\times D}$	$1(=\phi_{A\times D})$	$\chi^2_{A\times D}$	$\chi^2_p(\alpha)$
A×G	$S_{A\times G}$	$1(=\phi_{A\times G})$	$\chi^2_{A\times G}$	$\chi^2_p(\alpha)$
B×C	$S_{B\times C}$	$1(=\phi_{B\times C})$	$\chi^2_{B\times C}$	$\chi^2_p(\alpha)$
B×D	$S_{B\times D}$	$1(=\phi_{B\times D})$	$\chi^2_{B\times D}$	$\chi^2_p(\alpha)$
B×G	$S_{B\times G}$	$1(=\phi_{B\times G})$	$\chi^2_{B\times G}$	$\chi^2_p(\alpha)$
C×D	$S_{C\times D}$	$1(=\phi_{C\times D})$	$\chi^2_{C\times D}$	$\chi^2_p(\alpha)$
C×G	$S_{C\times G}$	$1(=\phi_{C\times G})$	$\chi^2_{C\times G}$	$\chi^2_p(\alpha)$
D×G	$S_{D\times G}$	$1(=\phi_{D\times G})$	$\chi^2_{D\times G}$	$\chi^2_p(\alpha)$
A×B×C	$S_{A\times B\times C}$	$1(=\phi_{A\times B\times C})$	$\chi^2_{A\times B\times C}$	$\chi^2_p(\alpha)$
A×B×D	$S_{A\times B\times D}$	$1(=\phi_{A\times B\times D})$	$\chi^2_{A\times B\times D}$	$\chi^2_p(\alpha)$
A×B×G	$S_{A\times B\times G}$	$1(=\phi_{A\times B\times G})$	$\chi^2_{A\times B\times G}$	$\chi^2_p(\alpha)$
A×C×D	$S_{A\times C\times D}$	$1(=\phi_{A\times C\times D})$	$\chi^2_{A\times C\times D}$	$\chi^2_p(\alpha)$
A×C×G	$S_{A\times C\times G}$	$1(=\phi_{A\times C\times G})$	$\chi^2_{A\times C\times G}$	$\chi^2_p(\alpha)$
A×D×G	$S_{A\times D\times G}$	$1(=\phi_{A\times D\times G})$	$\chi^2_{A\times D\times G}$	$\chi^2_p(\alpha)$
B×C×D	$S_{B\times C\times D}$	$1(=\phi_{B\times C\times D})$	$\chi^2_{B\times C\times D}$	$\chi^2_p(\alpha)$
B×C×G	$S_{B\times C\times G}$	$1(=\phi_{B\times C\times G})$	$\chi^2_{B\times C\times G}$	$\chi^2_p(\alpha)$
B×D×G	$S_{B\times D\times G}$	$1(=\phi_{B\times D\times G})$	$\chi^2_{B\times D\times G}$	$\chi^2_p(\alpha)$
C×D×G	$S_{C\times D\times G}$	$1(=\phi_{C\times D\times G})$	$\chi^2_{C\times D\times G}$	$\chi^2_p(\alpha)$
Error	S_e	$6(=\phi_e)$		
T	S_T	$31(=\phi_T)$		

$$\chi^2_f = -m \cdot \log_e \nu_f$$

(f = A, B, C, D, G, A × B, ······, D×G , A × B × C, ······, C × D × G)

$$m = \phi_e - \frac{1}{2}(p - \phi_f + 1)$$

$$\nu_f = \frac{|S_e|}{|S_f + S_e|}$$

of the separated liquid obtained. Considering the problem caused by the separation factors and the optimum combination among them, an examination on a technique for evaluating solid-liquid separation effectiveness in sludge for a filter is shown using a statistical model. Quality of the separated liquid for the filter is also analyzed. The research is further extended to the examination of the relationship between the speed of separation and the quality of separated liquid.

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