

Investigation of Solitary Wave Runup Using a New Method of Coupling Shallow Water Equation with $k-\omega$ Model

浅水流方程式と $k-\omega$ モデルを組み合わせた孤立波遡上の新しい数値計算法

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Shallow Water Equation is commonly used for solitary wave runup simulation. The Manning method is often used to assess the bed stress. In this study, a new calculation method is developed to improve the SWE model by replacing the Manning approach with turbulence $k-\omega$ model for assessing the bed stress. This method will allow the SWE equation to get a more accurate bed stress, yet efficient as compare to the full turbulent model. The new method is used to simulate a benchmark case provided from previous study. Wave profile comparison of the new method, the experiment data from previous study and calculation using manning method is given, showing that the new method has improved the SWE model accuracy. Further analysis shows that at shallow area, bed stress effects to the SWE model become higher and can not be neglected.

Key Words: Solitary wave, runup, numerical modeling, bed stress

1. Introduction

Solitary wave propagation and runup has been studied widely. Shallow Water Equation (SWE) model is commonly used for studying this phenomenon. The equation is well known for its efficiency and relatively good accuracy. SWE model typically uses Manning calculation for assessing the bed stress term in the momentum equation. This method assumes that the bed stress magnitude corresponds to the square of velocity divided by depth. However, studies regarding bed stress assessment using the boundary layer theory show that the Manning assumption is less accurate. The bed stress direction and magnitude is highly influenced by the boundary layer.

Observing boundary layer in nature is extremely difficult. Thus, numerical simulation and laboratory measurement are preferred. Tanaka and Sumer¹⁾ had conducted laboratory experiment to measure the bottom shear stress under random wave. The wave was propagated in a flume and the bottom stress was measured using a hot-film sensor.

Numerous studies on numerical model have been conducted to understand the turbulent boundary layer under the wave motion. There are several types of model for turbulence flow. The two equation models are commonly used due to its reliability. The $k-\epsilon$ model was one of the first two equation models used to study turbulence. $k-\omega$ model is a further development from the $k-\epsilon$ model. Past studies have shown that

the $k-\omega$ is superior to $k-\epsilon$ model in assessing the boundary layer properties in unsteady condition due to its ability to accommodate surface boundary condition²⁾.

Sana and Tanaka³⁾ have compared the Direct Numerical Simulation (DNS) data for sinusoidal oscillatory boundary layer over smooth bed to several 1D low Reynolds number model. However, these models were not applied to asymmetric wave profile. A detail study of the asymmetric wave profile was given by Suntoyo²⁾. They conducted laboratory experiment on oscillatory wave boundary layer using skew and sinusoidal wave. The results are compared to several 1D turbulent model. Furthermore, Suntoyo et al.⁴⁾ studied the characteristic of turbulent layer under saw tooth wave and its application to sediment transport. The study involved experimental wind tunnel and 1D numerical model. The model was further developed by Adityawan et al.⁵⁾ by advancing to 2D computation with the effect of convective acceleration term.

An experimental study regarding the solitary wave runup process had been conducted by Synolakis⁶⁾. His study covered laboratory study of the solitary wave runup over a sloping beach. He measured the water level and the runup height. His work has been accepted widely and used as benchmark for computational model.

Adityawan⁷⁾ has made computation on wave propagation and runup using the SWE and the Boussinesq equation. In his study, he has concluded that the SWE model is more practical than the Boussinesq equation. Furthermore, the study also

conducted a numerical simulation using Shallow Water Equation model and applied it to canonical problem by Synolakis⁶. The result showed good comparison to the experimental data of wave profile and deformation during the wave propagation and runup.

A full turbulent model based on k-ε equation has been used by Pengzhi et al.⁸ to simulate canonical problems from Synolakis. The results showed good accuracy to the experimental data. Nevertheless, the method is more complicated than the regular Shallow Water Equation involving grid method and free surface tracking. The SWE model seems to have the benefits in terms of efficiency, simplicity, thus allowing further development and implementation in practical application.

The objective of this study is to increase the accuracy of SWE model for simulating wave runup by assessing the bed stress with boundary layer approach, without reducing its efficiency. The SWE model will be upgraded by coupling with turbulent model (k-ω). The coupling is conducted by replacing the conventional Manning method used for bed stress term calculation in the SWE model with the calculated bed stress from the turbulent model. The k-ω model is used to simulate the flow within the boundary layer to obtain a more accurate bed stress. In this study, a 1D Depth Averaged Shallow Water Equation model is coupled with a 2D Vertical k-ω model. The coupled model is applied for solitary wave runup on a sloping beach, Synolakis⁶.

2. Model Development

2.1 Shallow water equation model

The SWE consists of the continuity equation and the momentum equation as follows.

$$\frac{\partial h}{\partial t} + \frac{\partial(Uh)}{\partial x} = 0 \quad (1)$$

$$\underbrace{\frac{\partial U}{\partial t}}_A + U \underbrace{\frac{\partial U}{\partial x}}_B + g \underbrace{\frac{\partial(h+z_b)}{\partial x}}_C + \underbrace{\frac{\tau_o}{\rho h}}_D = 0 \quad (2)$$

where h is the water depth, U is depth averaged velocity, t is time, g is gravity, z_b is the bed elevation, τ_o is bed stress. Manning equation is commonly used to assess this parameter. The bed stress relation in the conventional manning method is assumed linear to the square of upstream velocity as shown below.

$$\frac{\tau_o}{\rho} = \frac{gn^2}{R^{1/3}} \times U \times |U| \quad (3)$$

where R is the hydraulic radius or can be considered as water depth for a very wide channel, and n is the Manning roughness.

The governing equation above is solved using Mc.Cormack predictor corrector finite difference scheme. Forward difference scheme is used in the predictor step and backward difference scheme is used in the corrector step. The new value of h , U the next time step is obtained from the initial

time step and the corrector time step. This scheme has been known for its good performance in obtaining numerical solution for the St. Venant equation⁷.

Wet dry moving boundary condition is applied in the model to allow runup simulation. A threshold depth is selected. If the calculated water depth is lower than the threshold, then the water depth and velocity in the corresponding grid is given zero value (dry cell). The model also adapts numerical filter for better stability in calculation. The filter acts as an artificial dissipation. A numerical filter which acted as an artificial dissipation^{9,10} is used for each time step at each node. The value of depth and velocities are updated with the following equation.

$$F(i) = C.F(i) + 0.5(1-C).(F(i-1) + F(i+1)) \quad (4)$$

with F corresponds to the filtered parameter (velocity and depth) and i corresponds to the grid number location. The C value in Eq. (4) ranges between to 0.9-1. The value of C must be taken carefully with respect to the time step by trial and error. The result should be kept stable with no significant effect to the accuracy.

2.2 k-ω model

The governing equation for the k-ω model is based on the Reynolds-averaged equations of continuity and momentum, which can be written as follows:

$$\frac{\partial u_i}{\partial x_i} = 0 \quad (5)$$

$$\rho \frac{\partial u_i}{\partial t} + \rho u_j \frac{\partial u_i}{\partial x_j} = -\frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j} (2\mu S_{ij} - \overline{\rho u_i u_j}) \quad (6)$$

where u_i and x_i denotes the mean velocity and location in the grid, u_i' is the fluctuating velocity in the x ($i = 1$) and y ($i = 2$) directions, P is the static pressure, ν is the kinematics viscosity, $\overline{\rho u_i u_j}$ is the Reynolds stress tensor, and S_{ij} is the strain-rate tensor from the following equation:

$$S_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \quad (7)$$

The Reynolds stress tensor is given through eddy viscosity by Boussinesq approximation:

$$-\overline{u_i u_j} = \nu_t \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \frac{2}{3} k \delta_{ij} \quad (8)$$

with k is the turbulent kinetic energy and δ_{ij} is the Kronecker delta. The k-ω model equation is given as follows:

$$\frac{\partial k}{\partial t} + u_j \frac{\partial k}{\partial x_j} = \tau_{ij} \frac{\partial u_i}{\partial x_j} - \beta^* k \omega + \frac{\partial}{\partial x_j} \left[(\nu + \sigma^* \nu_t) \frac{\partial k}{\partial x_j} \right] \quad (9)$$

3. Simulation

Each model is verified using appropriate case from previous study. The $k-\omega$ model is verified using case study of bed stress measurement from random waves. The St. Venant model using Manning approach along with the new developed method are verified using runup case on sloping beach (canonical problem). Comparison of the new method performance to the Manning approach is conducted in this scenario.

3.1 $k-\omega$ model verification

The $k-\omega$ model is calibrated using measurement data from Tanaka et al. experiment. The data sets consist of upstream velocity and bed stress over time as shown in the Fig. 3. The measured velocity data is used to calculate the gradient pressure which is applied as the model input. The simulation is conducted to obtain the bed stress. The vertical grid is divided with the near bottom spacing starts from 0.0005 meter. The simulation is conducted with the time step of 0.001 second for 25 seconds.

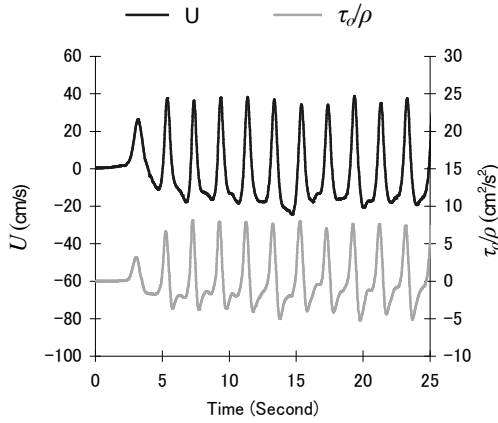


Fig.3 Measured velocity and bed stress over time (Tanaka et al. ¹⁾)

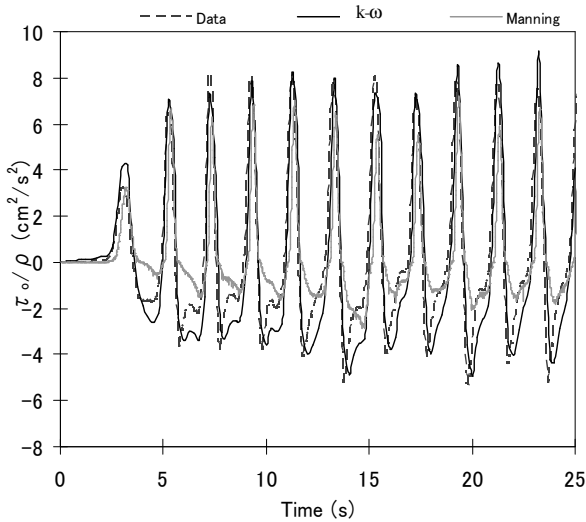


Fig. 4 Bed stress comparison

Simulation result is shown in Fig. 4. The bed stress obtained from simulation shows good comparison with measurement data. Similar behaviour is also observed between the data and simulation result.. It can be seen that the model is able to assess the bed stress under wave motion.

3.2 Wave runup

The wave runup simulation is verified with the case of runup from previous study by Synolakis⁶⁾. The runup occurs due to a solitary wave on a sloping beach or commonly known as canonical problem. Two type of model are simulated for this scenario, the SWE model using the conventional Manning method, and the upgraded SWE model using the new method proposed in this study. The model setup is shown in Fig. 5 below.

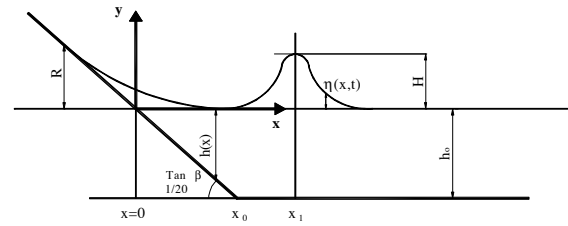


Fig.5 Model sketch setup for benchmark (Synolakis⁶⁾)

Non-dimensional variables are introduced as shown in Eq. (15) to Eq. (18).

$$x^* = x/h_o \quad (15)$$

$$h^* = h/h_o \quad (16)$$

$$\eta^* = \eta/h_o \quad (17)$$

$$t^* = t(g/h_o)^{0.5} \quad (18)$$

where h_o is the initial water depth (normal depth), η is the water elevation x is the coordinate according to the model sketch. In the experiment, the parameter are given as follows:

$$H/h_o = 0.019$$

$$\text{beach slope} = 1:20$$

The solitary wave initial profile and velocity is applied for the model initial condition according to the following equation:

$$\eta(x,0) = \frac{H}{h_o} \text{sech}^2 \left[\sqrt{\frac{3H}{4h_o}} (x - X_1) \right] \quad (19)$$

$$U(x,0) = \frac{c\eta}{1+\eta} \quad (20)$$

$$c = \sqrt{g(H+h_o)} \quad (21)$$

The wave profile is given by Eq. (19) with the initial

velocity as given from Eq.(20) and Eq. (21). The location of this initial wave peak is at X_l as shown in Fig.5. X_l is situated at half of the initial wave length ($L/2$) from the initial slope (X_0). The wave length (L) can be calculated according Eq. (22).

$$L = \frac{2}{\sqrt{3H/4ho}} \left[\operatorname{arccosh} \left(\sqrt{\frac{1}{0.05}} \right) \right] \quad (22)$$

The initial solitary wave profile is given in the Fig. 6.

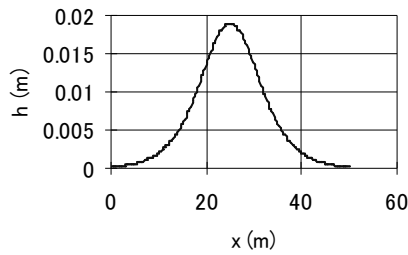
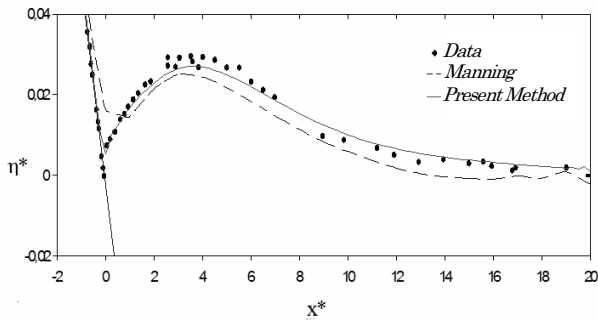
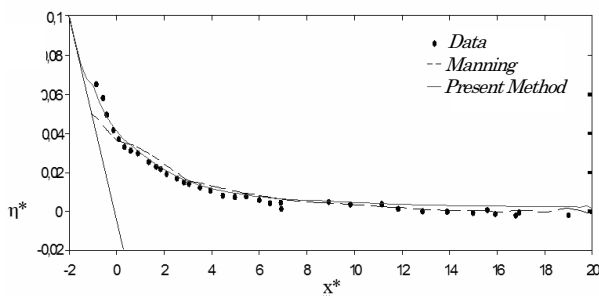


Fig. 6 Solitary wave profile



(a) $t^*=45$



(b) $t^*=55$

Fig. 7 Water level profile

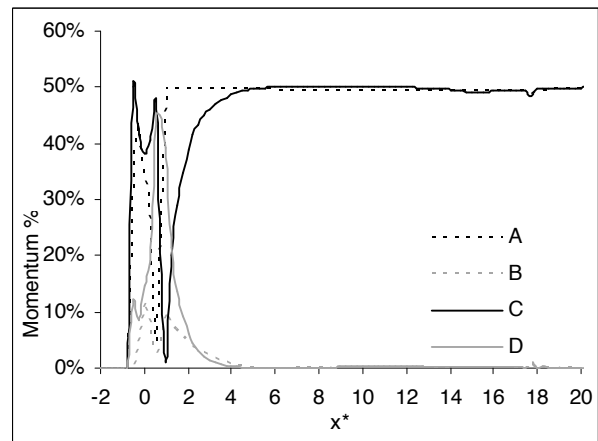
Table 1. Computation error and calculation time comparison

method		Error Quantification		Calc. Time (minute)
		Mean	Correlation	
n	0.05	0.00278	0.93154	Approx. 10
n	0.04	0.00279	0.93150	
n	0.03	0.00280	0.93148	
n	0.025	0.00280	0.93144	
n	0.02	0.00281	0.93140	
n	0.01	0.00281	0.93138	
Present Method		0.00181	0.95144	

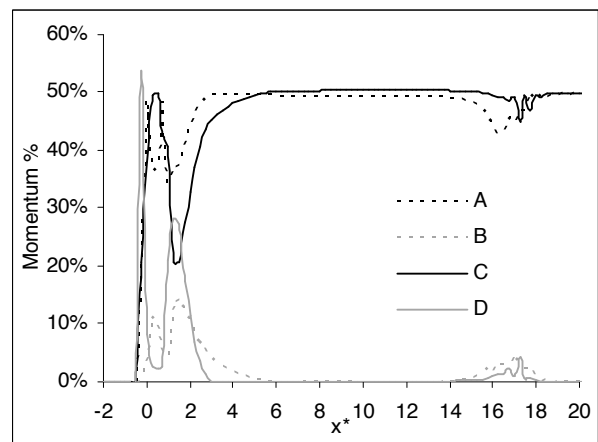
For computation, grids are constructed to resemble the experimental channel. The grid was divided with the interval of 0.075 meter. Total number of grid is 300. The vertical grid for the new method is divided with the near bottom spacing starts from 0.0005 meter. The threshold value for the wet/dry boundary is given as 1×10^{-6} . The time step for the computation is set to 0.0031944 second which corresponds to $0.1 t^*$, allowing output recording to be consistent with the non dimensional variables.

Simulation results for the new method and the Manning method are shown in Fig. 7 to Fig. 11. The water level profile comparisons are shown in Fig. 7 (a) and (b) for $t^*=45$ and $t^*=55$ respectively. Time step $t^*=45$ corresponds to the condition where the wave starts to reach the shore line. Time step $t^*=55$ corresponds to the condition where the wave just reach its runup peak and starting to drawback. It easily observed from the figures that the new method provides a better agreement to the experimental data than the Manning approach.

Overall, the present method provides a better estimation of wave profile as compare to the manning method. Error comparison and correlation coefficient between the two methods to the experiment data is shown in Table 1. It is also shown in the table that by increasing the roughness value, a more accurate result can be achieved. However, increasing the Manning value might lead to instability of the computation.



(a) New method

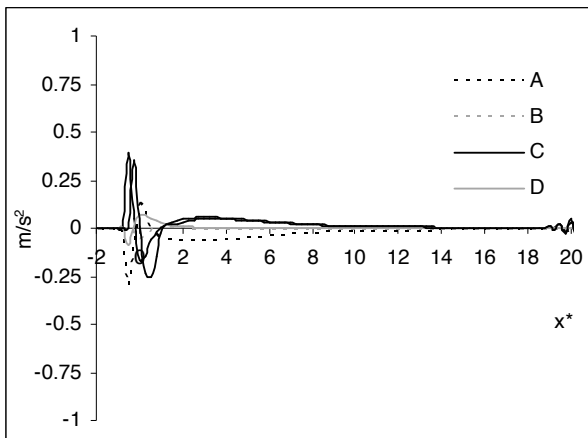


(b) Manning method

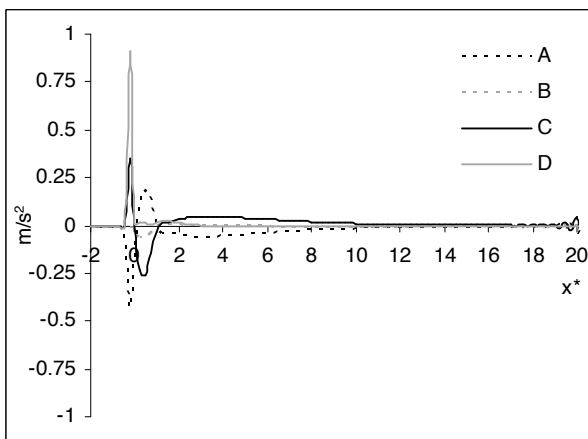
Fig. 8 Percentage of momentum terms ($t^*=45$)

The momentum balance and percentage are checked in order to investigate the effect of bed stress during the runup. The calculation is based from the SWE momentum equation (Eq. (2)). The A term corresponds to the local acceleration, the B term corresponds to convective acceleration, the C term corresponds to the pressure gradient and the D term corresponds to the bed stress. Percentage is calculated as the absolute value of a term divided by the total sum of absolute value of each term.

Percentage of momentum terms at $t^*=45$ calculated using the new method and the conventional Manning method is shown in Fig. 8 (a) and (b) respectively, and the same results for times step $t^*=55$ is shown in Fig. 10 respectively. The magnitude of each terms are given in Fig. 9 and Fig. 11 for $t^*=45$ and $t^*=55$ for both new method (a) and Manning method (b). Several things can be observed from the figures. Overall, the bed stress value is comparable to the convective acceleration, and the acceleration is comparable to water level gradient.



(a) New method



(b) Manning method

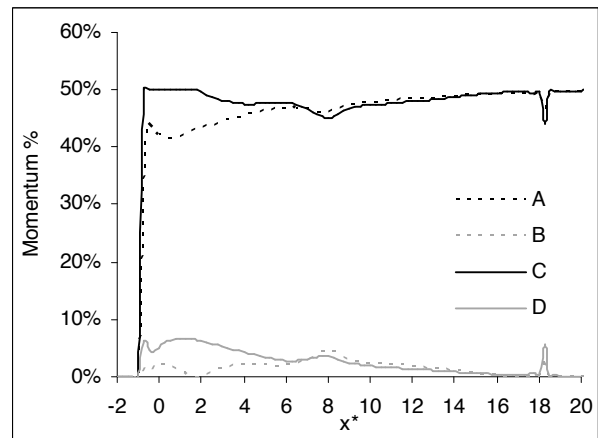
Fig. 9 Momentum balance ($t^*=45$)

At the deep area, the Manning method provides a near zero value of bed stress and the effect to the flow is almost zero percent, except at the transition point from flat bed to sloping beach ($x/h_o=20$). The near zero value corresponds to the assumption of bed stress relation to square of velocity divided

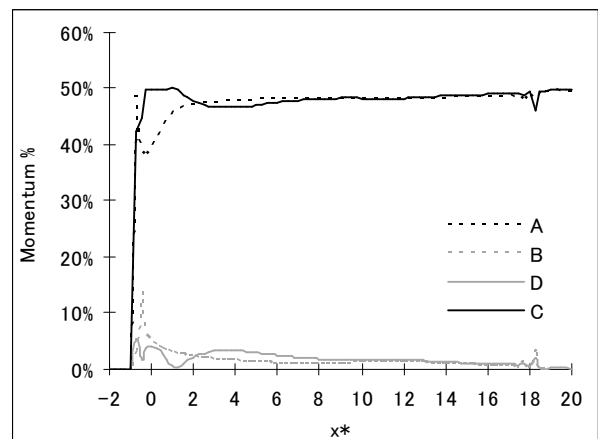
by depth (Eq. (3)). The velocity magnitude would be small and the water depth would be high, thus the bed stress value will be very small. However, the new method assesses the bed stress from the pressure gradient which is approached by acceleration. Therefore, a higher value of bed stress with slightly larger effect to the wave motion can be observed in the deep area.

It can be seen that the acceleration and the water level gradient terms will have more influenced to the wave motion in the deeper area until it reaches to a point where the convective acceleration and bed stress suddenly increase their effect to the wave motion.

At the initial slope, it is also observed a sudden fluctuation in the momentum percentage. Nevertheless, this phenomenon is small as compare to the fluctuation occurs in the wave front at the shallower area, especially around the wave peak. The location of maximum percentage of bed stress term from the new method appears slightly in front of the wave peak. As for the Manning method, the maximum percentage location is following the wave peak.



(a) New method



(b) Manning method

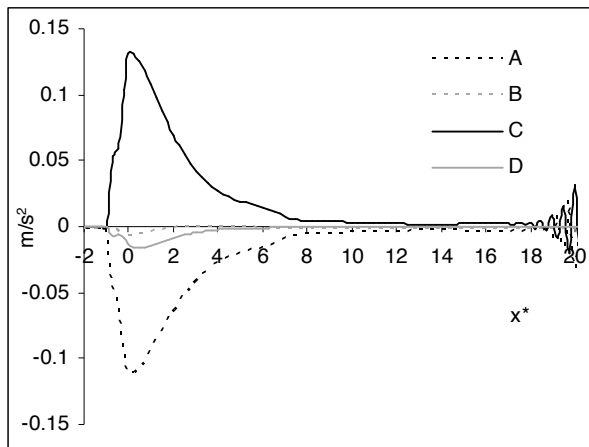
Fig. 10 Percentage of momentum terms ($t^*=55$)

Overall, the bed stress influence is higher as the wave is nearer to the shoreline. Further analysis is conducted regarding the location of the separation point when the bed stress starts to influence the wave motion. The location is determined by stating a percentage threshold for the bed stress term. The

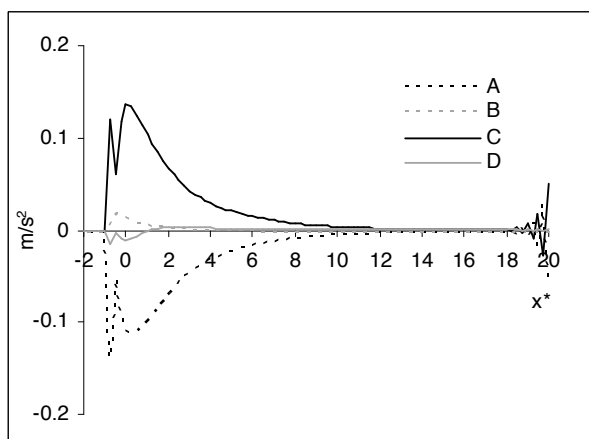
threshold value is taken as 5%, considering that less than 5% value can be considered as error in computation, thus insignificant. A value of 5% or higher is considered to have significant impact to the calculation, thus can not be neglected.

This separation points start to occur as the wave approaching the sloping beach or approximately the around time step $t^*=30$. Most of them appear in the shallow water region, on the sloping bottom. However, as the wave travels, these points eventually move to the shoreline.

The occurrence of this threshold in both models is located at the shoreline ($x/h_o=0$) for the wave profile at $t^*=55$. At this point, the water depth will be equal to the wave height ($\eta^*/h^*=1$). However, for the wave profile at $t^*=45$, the separation point is located in the shallow area. The separation point location was estimated at $x/h_o=2.5$ with the value of η^*/h^* (wave height per total depth) equals to 0.175 for the new method η^*/h^* equals to 0.200. For each time step, both models provide slightly different location of the separation point. Results for various time steps can be seen in Table. 2 (a) and (b).



(a) New method



(b) Manning method

Fig. 11 Momentum balance ($t^*=55$)

Table 2. Separation point location

(a) New method

t^*	x^*	$z^*(x)$	$\eta^*(x)$	$h^*(x)$	$\eta^*(x)/h^*(x)$
30	19	-0.95	0.018	0.968	0.019
35	13.5	-0.675	0.017	0.692	0.025
40	9	-0.45	0.022	0.472	0.047
45	2.5	-0.125	0.027	0.152	0.175
50	1	-0.05	0.030	0.080	0.377
55	0	0	0.056	0.056	1.000

(b) Manning method

t^*	x^*	$z^*(x)$	$\eta^*(x)$	$h^*(x)$	$\eta^*(x)/h^*(x)$
30	18.5	-0.925	0.019	0.944	0.020
35	12.5	-0.625	0.021	0.646	0.032
40	8.25	-0.4125	0.023	0.435	0.053
45	2.5	-0.125	0.031	0.156	0.200
50	1.75	-0.0875	0.034	0.121	0.280
55	0	0	0.054	0.054	1.000

Non-dimensional parameter, which is the ratio of wave height and total depth, has been proposed in Table 2 to explain the phenomenon. Nevertheless, there is no similarity to the separation point. Each time step has different value of this parameter. Thus, the phenomenon does not correspond to the wave height per depth ratio. However, the location of the separation point seems to correspond to the location of the wave peak.

4. Conclusion

A new method has been developed for simulating solitary wave runup on sloping beach (canonical problems). The conventional Manning approach in the Shallow Water Equation model has been replaced with bed stress assessment from the boundary layer using $k-\omega$ model.

The new method was developed with concern to efficiency of the calculation. Therefore, a new type of grid system has been developed. The grid system covers horizontal and vertical calculation. However, the vertical grid is only required in near bed area to assess the boundary layer for obtaining the bed stress. This is more efficient than the full turbulent model which requires the construction of grid system up to the surface. The acquired bed stress is used in the SWE model, replacing the conventional Manning method. Both models are coupled and solved simultaneously, allowing an efficient and accurate calculation.

The performance of each model has been verified using appropriate case with good results. $k-\omega$ model was verified using experimental study of bed stress measurement under random wave. The results showed the good performance of the $k-\omega$ model to work independently for assessing the bed stress in the boundary layer. The SWE model with Manning approach was verified with experimental data of canonical problem from previous study with relatively good agreement.

The new method was tested to simulate canonical problems. The results show improvement as compare to the SWE with Manning method. Momentum balance analysis was carried out in order to investigate the effect of bed stress during wave runup. It was shown that the Manning method tends to provide higher bed stress magnitude in shallow area and lower

magnitude in the deep area. This is caused by the Manning assumption of bed stress relation to the square of velocity divided by depth. On the contrary, the new method approaches the bed stress directly from the boundary layer using pressure gradient related to acceleration.

Further analysis from the momentum balance showed that at the deeper area, the effect of bed stress in the momentum equation is very small with the exception of points located around the initial slope. The effect becomes higher as the wave travels to shallow area. The location of the separation point does not correspond to the proposed non dimensional parameter (ratio of wave height to depth). However, the separation point is situated around the wave peak, relative to the wave propagation.

Though the proposed method has been showing promising results, further improvement and application to various cases with different bed slope and different wave height should be conducted.

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