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Analysis of dam-break flow with finite extent by a spatial integral model with energy equation

How Tion PUAY* and Takashi HOSODA**

*PhD Student Dept. Urban Management, Kyoto Univ.
(Kyodai Katsura, Nishikyo-ku, Kyoto 615-8540)
**Member of JSCE, Professor, Dept. Urban Management, Kyoto Univ.
(Kyodai Katsura, Nishikyo-ku, Kyoto 615-8540)

The inertial and viscous flow phases of viscous Newtonian fluid are studied in this paper by a mathematical integral model. Based on the dam-break flow of finite extent, the governing equations which are comprised of the depth-averaged continuity, momentum and energy equations are transformed from partial differential equation form to ordinary differential equation form that can be solved easily. The characteristics of the inertial and viscous flow phases are reproduced by the integral model. It is shown that the integral model is not only capable of reproducing the flow characteristic of viscous Newtonian fluid but also the transition of the flow phases.

Key Words : *dam-break flow, inertial flow phase, viscous flow phase, integral model*

1. Introduction

Studies of shallow flow of viscous fluid have shown that flow phases exhibit unique characteristics.[1),2)]. Some of the more common methods to study the characteristics of flow phases are dam-break flow model, sudden release of finite volume of higher density fluid into a much lower density ambient fluid and also constant inflow of higher density fluid into a lower density fluid environment.

The study of the characteristics of the flow phases is important as it provides an insight to the rheological properties of the fluid. For example, Hosoda[2)] investigated the rheological characteristics of fresh concrete by analyzing the characteristics of flow phases of viscous fluid.

The authors have previously used the dam-break flow of finite extent model to investigate the characteristics of flow phases[3),4)]. It was shown that in the case of viscous fluid, two flow phases exist, which define the inertial and viscous flow phases. As soon as the flow is initiated by the instantaneous release of the dam gate, the flow enters the inertial flow phase where the flow is governed by the inertia force of the motion. If the viscosity of the fluid is significant, the inertial flow phase will be followed by the viscous flow phase. The dominance of viscous flow phase depends on the viscosity of the fluid. In the viscous flow phase, the flow is governed by the viscous-pressure balance. These characteristics of flow phases were presented in the form of proportionality of h_m and L to time t. h_m is the depth at the upstream end-wall of the dam and L is the position of the wave front as shown in Fig.1. In the case of viscous Newtonian fluid, it was found that in the inertial flow phase, $h_m \propto t^{-1}$ and $L \propto t$ while in the viscous flow phase, $h_m \propto t^{-1/5}$ and $L \propto t^{1/5}$. These characteristics were derived by considering each flow phase independently. Assumption for each flow phase was made. For example, in the viscous flow phase, the inertial term can be neglected and vice-versa.

The purpose of this study is to reproduce the characteristics of the flow phases by using the integral method. The integral method is chosen as it can reproduce the characteristics of flow phases in a continuous piece-wise solution. This continuous piece-wise solution can depict the flow phases from the moment the flow is initiated by the release of the dam gate to sufficiently long time. Besides, the continuous piece-wise solution can show the transition between the flow phases. This is seen as an advantage over the method used in the authors' previous study where transition of flow phases could not be reproduced.

2. Governing Equations

A one-dimensional depth-averaged continuity and momentum equations representing the motion of dam-break flow of finite extent shown in Fig.1 can be expressed as follows,

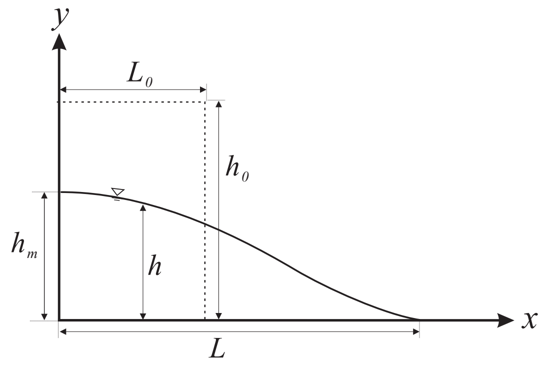

Fig. 1 Schemetic of dam-break flow model

Continuity equation,

$$\frac{\partial h}{\partial t} + \frac{\partial hU}{\partial x} = 0 \tag{1}$$

Momentum equation,

$$\frac{\partial hU}{\partial t} + \frac{\partial \beta hU^2}{\partial x} + gh\frac{\partial h}{\partial x} = -\frac{3\nu}{h}U \tag{2}$$

where g is the gravitational acceleration, ν is the kinematic viscosity, β is the momentum coefficient and the other parameters are described in the schematic of dam-break flow in Fig.1.

In the case of viscous fluid, it is assumed that the depth h and velocity U exhibit self-similarity[1),2)]. Therefore, h and U are expressible in similarity form as follows,

$$h = h_m(t)p(\xi) \tag{3}$$

$$U = U_m(t)q(\xi) \tag{4}$$

with

$$\xi = x/L \tag{5}$$

$U_m(t)$ is the representative velocity, $h_m(t)$ is the representative depth, which is also the depth at the upstream end-wall. L is the position of the wave front measured from the origin of the coordinate system $x = 0$. $p(\xi)$ and $q(\xi)$ are shape functions (also known as similarity functions) for h and U respectively. For simplicity, $h_m(t)$ and $U_m(t)$ are written as h_m and U_m respectively, while $p(\xi)$ and $q(\xi)$ are written as p and q.

The shape functions p and q satisfy the following boundary conditions,

$$\begin{aligned} &p(\xi = 1) = 0, \quad p(\xi = 0) = 1 \\ &q(\xi = 0) = 0 \end{aligned} \tag{6}$$

The unknown variables L, U_m and h_m cannot be solved by using only the momentum and continuity equations because there are only two independent equations available to solve three unknown variables. We therefore proposed to consider the conservation of energy in the system. The energy equations for the system can be written as follows,

Energy equation,

$$\begin{aligned} &\frac{\partial}{\partial t}\left(\frac{hU^2}{2}\right) + \frac{\partial}{\partial x}\left(hU\frac{U^2}{2}\right) + U\frac{\partial}{\partial x}\left[(\beta-1)hU^2\right] \\ &+g\frac{\partial}{\partial x}\left(h^2U\right) + g\frac{\partial}{\partial t}\left(\frac{h^2}{2}\right) = -3\nu\frac{U^2}{h} \end{aligned} \tag{7}$$

With the continuity, momentum and energy equations, the three unknown variables (L, U_m and h_m) can therefore be solved.

3. Integral model

The governing equations are spatially integrated for the whole flow domain from $x = 0$ to $x = L$. The integration of the governing equations is as follows,

3.1 Integration of continuity equation

$$\int_0^L \left(\frac{\partial h}{\partial t} + \frac{\partial hU}{\partial x}\right) dx = 0 \tag{8}$$

where

$$\begin{aligned} \int_0^L \frac{\partial h}{\partial t}\, dx &= \frac{d}{dt}\int_0^L h\, dx - [h]_L\frac{dL}{dt} \\ &= \frac{d}{dt}\int_0^L h dx \end{aligned} \tag{9}$$

and

$$\int_0^L \frac{\partial hU}{\partial x}\, dx = [hU]_L - [hU]_0 = 0 \tag{10}$$

Therefore, the integration of the continuity equation for the whole domain yields,

$$\begin{aligned} &\frac{d}{dt}\int_0^L h\, dx = 0 \\ \therefore\ &\int_0^L h\, dx = \text{volume} = h_0L_0 = vol \end{aligned} \tag{11}$$

h_0 and L_0 are the initial width and height of the dam as shown in Fig.1. The sign vol represents the total volume of the fluid in the dam. By substituting the expression of $h = h_m p$ into Eq.(11),

$$\begin{aligned} &\int_0^L h\, dx = \int_0^1 h_m pL\, d\xi \quad \because d\xi = dx/L \\ &h_m L\int_0^1 p\, d\xi = vol \\ &h_m L A_1 = vol \end{aligned} \tag{12}$$

where

$$A_1 = \int_0^1 p\, d\xi \tag{13}$$

3.2 Integration of momentum equation

Integration of the momentum equation with the substitution of $h = h_m p$, $U = U_m q$ and transformation to ξ coordinate system is as follows,

$$\int_0^L \left(\frac{\partial hU}{\partial t} + \frac{\partial \beta hU^2}{\partial x} + gh\frac{\partial h}{\partial x} + \frac{3\nu}{h}U\right) dx = 0 \quad (14)$$

The integral form of the momentum equation can be written as follows,

$$\frac{d}{dt}\left(h_m U_m L\right) A_2 - g\frac{h_m^2}{2} + 3\nu\frac{U_m L}{h_m}A_3 = 0 \quad (15)$$

where A_2 and A_3 are defined as follows,

$$A_2 = \int_0^1 pq \, d\xi \quad (16)$$

$$A_3 = \int_0^1 \frac{q}{p} \, d\xi \quad (17)$$

3.3 Integration of energy equation

The integration of the energy equation with the substitution of $h = h_m p$, $U = U_m q$ and transformation to ξ coordinate system is as follows,

$$\int_0^L \left\{ \frac{\partial}{\partial t}\left(\frac{hU^2}{2}\right) + \frac{\partial}{\partial x}\left(hU\frac{U^2}{2}\right) + U\frac{\partial}{\partial x}\left[(\beta-1)hU^2\right] + g\frac{\partial}{\partial x}\left(h^2 U\right) + g\frac{\partial}{\partial t}\left(\frac{h^2}{2}\right) + 3\nu\frac{U^2}{h} \right\} dx = 0 \quad (18)$$

The integral form of the energy equation can be summarized as follows,

$$\frac{d}{dt}\left(\frac{1}{2}h_m U_m^2 L\right) A_4 + g\frac{d}{dt}\left(\frac{h_m^2 L}{2}\right) A_5 + 3\nu\frac{U_m^2 L}{h_m}A_6 = 0 \quad (19)$$

where A_4, A_5 and A_6 are,

$$A_4 = \int_0^1 pq^2 \, d\xi \quad (20)$$

$$A_5 = \int_0^1 p^2 d\xi \quad (21)$$

$$A_6 = \int_0^1 \frac{p^2}{q} \, d\xi \quad (22)$$

The coefficients A_1 to A_6 which involve the integration of shape functions p and q are determined in a later section.

3.4 Solution of integral model

By transforming the governing equations which are in partial differential form into ordinary differential form, as in Eq.(12) , Eq.(15) and Eq.(19), the parameters h_m, U_m, and L can be solved by using the Euler method as follows,
From Eq.(12), we have

$$h_m L = \frac{vol}{A_1} \quad \text{where} \quad vol = h_0 L_0 \quad (23)$$

By substituting Eq.(23) into the momentum equation in Eq.(15), we have,

$$\therefore \quad \frac{dU_m}{dt} = \frac{A_1}{A_2 vol}\left(g\frac{h_m^2}{2} - 3\nu A_3\frac{U_m L}{h_m}\right) \quad (24)$$

By using Eq.(23), the expression for $\frac{dh_m}{dt}$ can be obtained from the energy equation in Eq.(19) as follows,

$$\frac{vol A_4}{A_1}U_m\frac{dU_m}{dt} + g\frac{vol A_5}{2A_1}\frac{dh_m}{dt} + 3\nu\frac{U_m^2 L}{h_m}A_6 = 0 \quad (25)$$

Therefore,

$$\frac{dh_m}{dt} = -\frac{2A_1}{vol A_5 g}\left[\frac{vol A_4}{A_1}U_m\frac{dU_m}{dt} + 3\nu\frac{U_m^2 L}{h_m}A_6\right] \quad (26)$$

where the term $\frac{dU_m}{dt}$ is determined from Eq.(24). If $[h_m]^n$, $[U_m]^n$ and $[L]^n$ are the values of h_m, U_m and L at time step n, then values at time step $n+1$ can be calculated from Eq.(23), Eq.(24) and Eq.(26) as follows,
From Eq.(24),

$$[U_m]^{n+1} = [U_m]^n + dt\frac{A_1}{A_2 vol}\left\{\frac{g}{2}\left[h_m^2\right]^n - 3\nu A_3\left[\frac{U_m L}{h_m}\right]^n\right\} \quad (27)$$

From Eq.(26),

$$[h_m]^{n+1} = [h_m]^n - dt\frac{2A_1}{vol A_5 G}\left\{\frac{vol A_4}{A_1}[U_m]^n\left[\frac{dU_m}{dt}\right]^n + 3\nu\left[\frac{U_m^2 L}{h_m}\right]^n A_6\right\} \quad (28)$$

From Eq.(23),

$$[L]^{n+1} = \frac{vol}{A_1[h_m]^{n+1}} \quad (29)$$

where dt is the time increment in each n steps.

3.5 Determination of shape functions

Hosoda et al[2] derived the flow characteristics of viscous Newtonian fluid by assuming similarity form of the depth and velocity of flow as in Eq.(3) and Eq.(4). By substituting the similarity form of h and U, the momentum equation in Eq.(2) can be expressed as follows,

$$\frac{\partial}{\partial t}\left[h_m p U_m q\right] + \frac{\partial}{\partial x}\left[\beta h_m p {U_m}^2 q^2\right] + g{h_m}^2\frac{p}{L}\frac{\partial p}{\partial x} = -3\nu\frac{U_m q}{h_m p} \quad (30)$$

For viscous flow phase, it is assumed that pressure and viscous force are in balance. Therefore, by neglecting the inertia term, the pressure and viscous terms can be equated as follows (with transformation to ξ coordinate system),

$$g\frac{{h_m}^2}{L}p\frac{\partial p}{\partial \xi} = -3\nu\frac{U_m q}{h_m p} \tag{31}$$

The function of h_m, U_m and L can be expressed in the following power-law form[2)],

$$\begin{aligned} h_m &= \alpha h_o\left(\sqrt{g/h_o}t\right)^a \\ U_m &= \lambda\sqrt{gh_o}\left(\sqrt{g/h_o}t\right)^b \\ L &= \gamma L_o\left(\sqrt{g/h_o}t\right)^c \end{aligned} \tag{32}$$

where $\alpha, \lambda, \gamma, a, b, c$ are dimensionless coefficients. To non-dimensionalize the equations, the dimensionless form of t is introduced and defined as follows,

$$t = \sqrt{\frac{h_o}{g}}t' \tag{33}$$

By substituting the assumptions in Eq.(32) and definition of the dimensionless form of t in Eq.(33), Eq.(31) can be written in dimensionless form as follows,

$$g\frac{\alpha^2}{\gamma}\frac{h_o^2}{L_o}(t')^{2a-c}p\frac{dp}{d\xi} = -3\nu\frac{\beta}{\alpha}\sqrt{\frac{g}{h_o}}(t')^{b-a}\frac{q}{p} \tag{34}$$

In order to satisfy dimensional homogeneity, Eq.(34) can be reduced to the following form,

$$g\frac{\alpha^2}{\gamma}\frac{h_o^2}{L_o}p\frac{dp}{d\xi} = -3\nu\frac{\beta}{\alpha}\sqrt{\frac{g}{h_o}}\frac{q}{p} \tag{35}$$

By rearranging the coefficients, Eq.(35) can be rewritten as follows,

$$\frac{A}{3}\frac{d}{d\xi}\left(p^3\right) = -q \quad \text{where } A = \frac{\alpha^3 h_o^2 L_o\sqrt{gh_o}}{3\beta\gamma\nu} \tag{36}$$

Similarly, by substituting the similarity form of h and U, the continuity equation, Eq.(1) can be expressed in the following form [2)],

$$\begin{aligned} &p(\xi)\frac{dh_m}{dt} - \frac{\xi}{L}h_m\frac{dp(\xi)}{d\xi}\frac{dL}{dt} \\ &\frac{h_m V_m}{L}q\frac{dp}{d\xi} + \frac{h_m V_m}{L}p\frac{dq}{d\xi} = 0 \end{aligned} \tag{37}$$

By substituting the power-law form of h_m, U_m and L in Eq.(32), the dimensionless form of Eq.(37) can be written as follows,

$$\begin{aligned} &\alpha h_o a t'^{a-1}\sqrt{\frac{g}{h_o}}p - \alpha h_o c t'^{a-1}\xi\sqrt{\frac{g}{h_o}}\frac{dp}{d\xi} \\ &+\frac{\alpha\beta}{\gamma L_o}h_o\sqrt{gh_o}q\frac{dp}{d\xi}t'^{a+b-c} + \frac{\alpha\beta}{\gamma L_o}h_o\sqrt{gh_o}p\frac{dq}{d\xi}t'^{a+b-c} \\ &= 0 \end{aligned} \tag{38}$$

With $a = -1/5$, $b = -4/5$ and $c = 1/5$ for viscous Newtonian fluid[2)], Eq.(38) can be reduced to the following form,

$$-\frac{1}{5}\sqrt{\frac{g}{h_o}}\left[\frac{d}{d\xi}(p\xi)\right] + \frac{\beta}{\gamma L_o}\left[\frac{d(pq)}{d\xi}\right] = 0 \tag{39}$$

Integrating Eq.(39) yields,

$$-\frac{1}{5}\sqrt{\frac{g}{h_o}}\cdot p\xi + \frac{\beta}{\gamma L_o}\sqrt{gh_o}pq + C = 0 \tag{40}$$

where C is a constant. At $\xi = 1$, $p = 0$, therefore, $C = 0$. The shape function q can therefore be determined from Eq.(40) as follows,

$$q = E\xi \quad \text{where} \quad E = \frac{1}{5}\frac{\gamma L_o}{\beta h_o} \tag{41}$$

By substituting Eq.(41) into Eq.(36), and integrating it from $\xi = 0$ to ξ, the shape function p can be determined as follows,

$$\begin{aligned} \frac{A}{3}\frac{d}{d\xi}\left(p^3\right) &= -E\xi \\ \int_0^\xi \frac{A}{3}\frac{d}{d\xi}\left(p^3\right)d\xi &= -\int_0^\xi E\xi\, d\xi \\ p &= \sqrt[3]{1 - \frac{3E}{2A}\xi^2} \end{aligned} \tag{42}$$

The governing equations in Eq.(1), Eq.(2) and Eq.(7) are transformed into the ordinary differential form as in Eq.(23), Eq.(24) and Eq.(26) where the solution of U_m, h_m and L can be determined from Eq.(27), Eq.(28) and Eq.(29).

3.6 Initial conditions of calculation

The initial conditions used in the integral model are as follows,

$$\begin{aligned} h_m &= h_o = 0.5m \\ L &= L_o = 0.5m \\ U_m &= 0.0ms^{-1} \end{aligned} \tag{43}$$

In the case of inviscid fluid, the value of $\nu = 0.0\, m^2s^{-1}$ is used. For viscous Newtonian fluid, four values of kinematic viscosity are used, $\nu = 0.00005\, m^2s^{-1}$, $\nu = 0.0005\, m^2s^{-1}$, $\nu = 0.005\, m^2s^{-1}$ and $\nu = 0.05\, m^2s^{-1}$. Time step dt is set as $0.0001\, s$ in the calculation. The value of coefficient α, λ and γ in the shape function p and q are set to unity for simplicity.

3.7 Results

(1) Inviscid fluid

The author previously used depth at the upstream end wall, h_m as parameter to describe the characteristic of inertial flow phase[3)] where,

$$h_m \propto t^{-1} \tag{44}$$

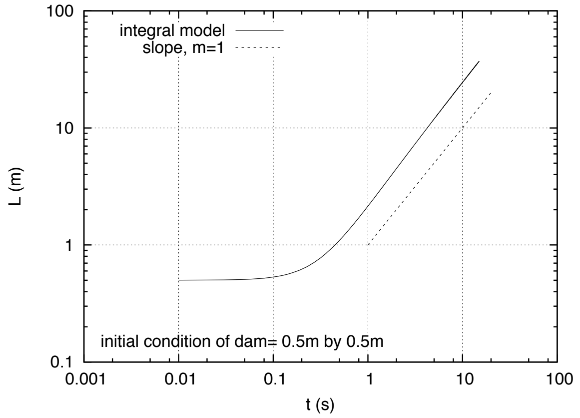

Fig. 2 Temporal variation of wave front L in the case of inviscid fluid

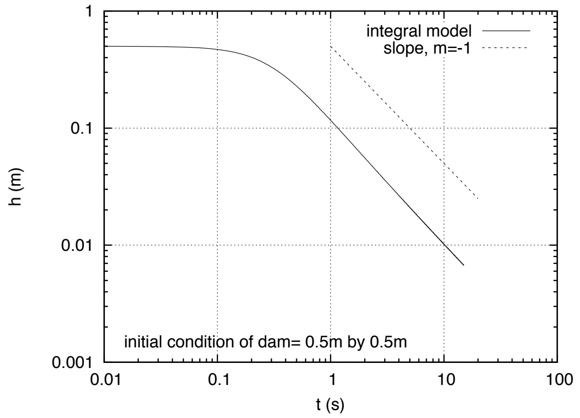

Fig. 3 Temporal variation of depth at the upstream end-wall h_m in the case of inviscid fluid

The propagation of wave front L for inertial flow phase can be presented by the Ritter's solution for dam-break flow of infinite extent [3),4)], where

$$L \propto t \tag{45}$$

The attenuation of wave front L and depth at the upstream end-wall h_m for the solution of inviscid fluid with the integral model are plotted in Fig.2 and Fig.3 respectively. It can be seen from both Fig.2 and Fig.3 that the inertia region can be reproduced well in both the attenuation of depth at the upstream end-wall and the wave front by using the solution of the integral model.

(2) Viscous Newtonian fluid

The viscous flow phase characteristics for Newtonian fluid can be represented by the following characteristics of the attenuation of depth at the upstream end-wall, h_m and wave front propagation, L [3)].

$$h_m \propto t^{-\frac{1}{5}} \tag{46}$$

$$L \propto t^{\frac{1}{5}} \tag{47}$$

The results for viscous fluid are shown in Fig.4 to Fig.11, with different kinematic viscosity. In the

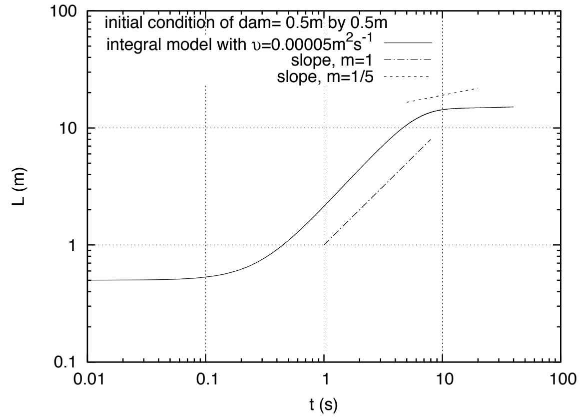

Fig. 4 Temporal variation of wave front L in the case of viscous fluid with kinematic viscosity $\nu = 0.00005 m^2 s^{-1}$

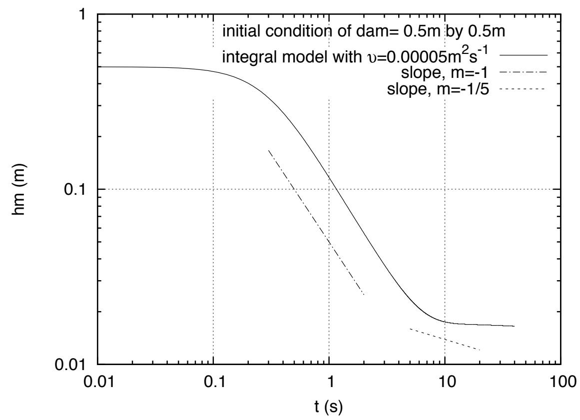

Fig. 5 Temporal variation of depth at the upstream end-wall h_m in the case of viscous fluid with kinematic viscosity $\nu = 0.00005 m^2 s^{-1}$

case of low viscosity such as for the case of $\nu = 0.00005\, m^2 s^{-1}$ and $\nu = 0.0005\, m^2 s^{-1}$, it can be seen from Fig.4 to Fig. 7 that the inertia flow phase is dominant in the flow. The viscous flow phase characteristic is hardly detectable as the viscosity is not dominant due to low viscosity.

However, when kinematic viscosity is increased to $\nu = 0.005\, m^2 s^{-1}$, the viscous flow phase characteristic can be observed clearly in both attenuation of the depth at the upstream end-wall h_m and wave front L as shown in Fig.8 and Fig.9. It can be seen from both figures that the transition of the flow phases is not abrupt. The viscous flow phase characteristic slowly developed after the inertia flow phase and fully dominates the flow almost at the same time for the attenuation of the depth at the upstream end-wall h_m and the wave front L.

In an extreme case where the kinematic viscosity is increased to $\nu = 0.05\, m^2 s^{-1}$ as in Fig.10 and Fig.11, the inertial flow phase cannot be observed. It is thought that for highly viscous fluid, pure in-

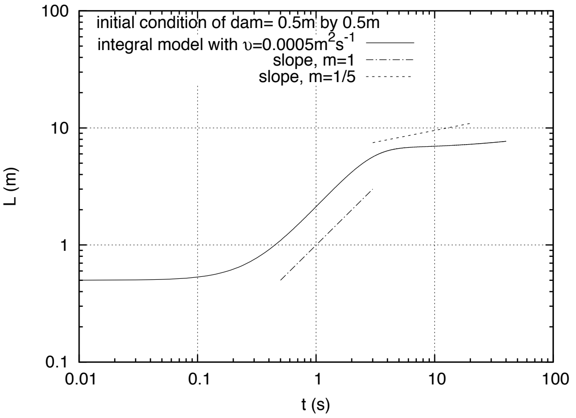

Fig. 6 Temporal variation of wave front L in the case of viscous fluid with kinematic viscosity $\nu = 0.0005m^2s^{-1}$

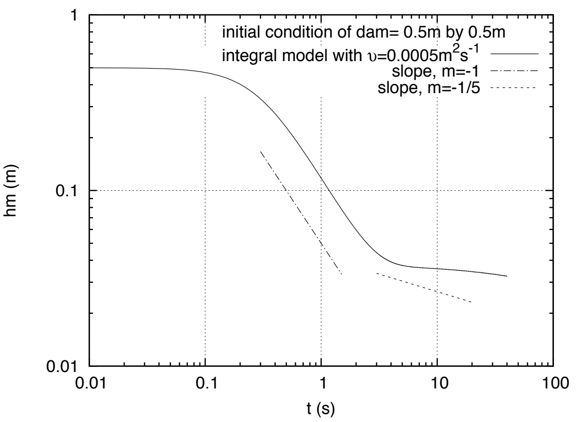

Fig. 7 Temporal variation of depth at the upstream end-wall h_m in the case of viscous fluid with kinematic viscosity $\nu = 0.0005m^2s^{-1}$

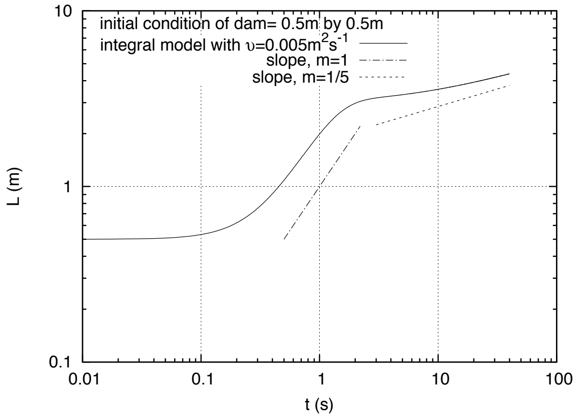

Fig. 8 Temporal variation of wave front L in the case of viscous fluid with kinematic viscosity $\nu = 0.005m^2s^{-1}$

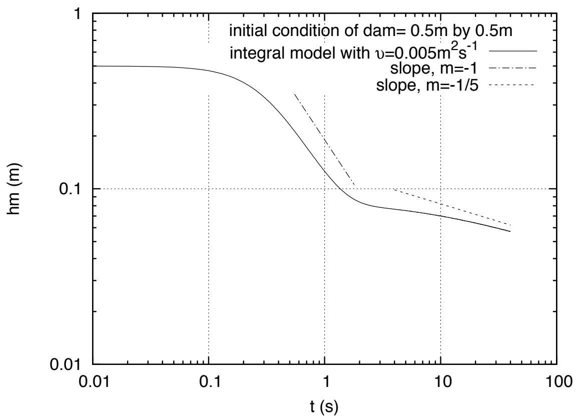

Fig. 9 Temporal variation of depth at the upstream end-wall h_m in the case of viscous fluid with kinematic viscosity $\nu = 0.005m^2s^{-1}$

ertial flow phase does not exist. The flow enters a self-similar inertial phase (also known as inviscid self-similar phase)[5] instead of a pure inertial phase. This self-similar inertial phase is characterized by the follow relations,

$$h_m \propto t^{-\frac{2}{3}} \tag{48}$$

$$L \propto t^{\frac{2}{3}} \tag{49}$$

After a short self-similar inertial phase, the viscous flow phase almost immediately dominates the flow as shown in both Fig.10 and Fig.11.

4. Conclusion

In this study, the inertial and viscous flow characteristics are reproduced by a mathematical integral model. By transforming the governing equations which are comprised of the depth-averaged continuity, momentum and energy equations into ordinary differential equation form, the solution for the attenuation of depth at the upstream end-wall, h_m and propagation of wave front L are obtained.

In the case of viscous fluid, the depth at the upstream end-wall h_m and wave front L are assumed to be expressible in similarity form. The shape functions p and q (also known as similarity functions) are derived based on earlier studies by Hosoda et al[2].

In the case of inviscid fluid, the inertial flow phase characteristic is satisfactorily reproduced. Although the shape functions p and q are derived based on viscous fluid, it is shown that the integral model can still reproduced the characteristic of inertial flow phase in the case of inviscid fluid.

In the case of viscous fluid, two flow phases are reproduced by the integral model. It is shown that for low viscosity fluid, the viscous flow phase is not dominant, preceded by a dominant inertial flow phase. As the fluid viscosity is increased, the viscous flow phase dominance increases. It is worth to note that in the case of highly viscous flow, an initial region known as the self-similar inertial phase or inviscid self-similar

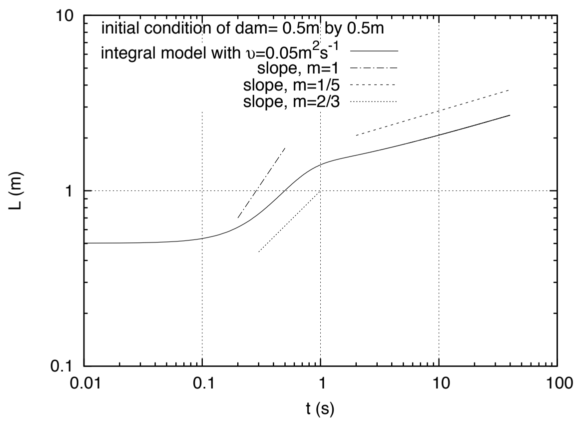

Fig. 10 Temporal variation of wave front L in the case of viscous fluid with kinematic viscosity $\nu = 0.05m^2s^{-1}$

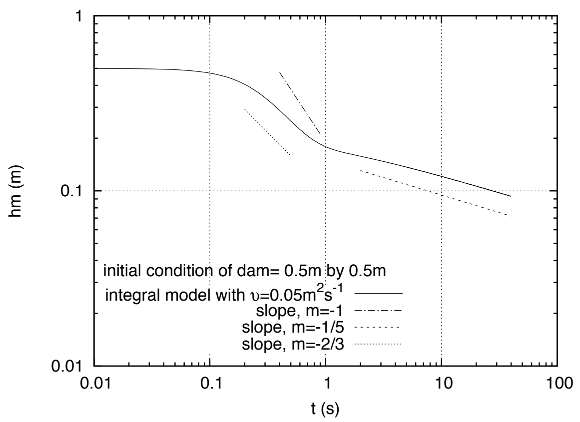

Fig. 11 Temporal variation of depth at the upstream end-wall h_m in the case of viscous fluid with kinematic viscosity $\nu = 0.05m^2s^{-1}$

phase is reproduced by the integral model.

The integral model developed in this study is shown to be capable of producing a continuous piece-wise solution describing the flow phases characteristic from the moment the flow is initiated until sufficiently long time. The development of an integral model is vital as an alternative to further clarify the existence and characteristic of flow phases. The capability of the integral model to reproduce a smooth piece-wise solution bridging the inertial and viscous flow phases is worth noting.

The author plans to extend the integral model to reproduce the flow phases characteristic of non-Newtonian fluid and investigate its rheological properties using the findings in this study.

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