Dynamic response and reliability analysis of offshore platform with porosity
due to wave and seismic forces

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Offshore platform with porosity is a new offshore structure concept in industry where the large scale offshore structure can develop the available ocean space several activities. Especially, effects on the diffraction in each fluid region are expressed by an eigenfunction expansion method. In the present study, the dynamic response of the offshore platform with porosity is examined by using the seismic and wave forces, and the reliability evaluation on the offshore platform with porosity subjected to these forces is carried out using the MCS approach. It is suggested that the offshore platform with porosity is very efficient to reduce effects of wave forces and wave run up. In order to carry out the reliable design of the offshore platform with porosity, it is important to verify the response properties on uncertainties to dynamic forces and structural properties.

Key Words: eigenfunction expansion method, offshore platform, wave and seismic force, MCS method, reliability index

1. Introduction

Offshore platform with a large deck area may provide great possibilities to develop ocean spaces which may be used as resident areas, airports, power station, etc. For the design of an offshore structure, it is necessary to examine about severe wave force and seismic force situation. To perform the reliable design of an offshore structure on ocean, it is very important to evaluate effects on the wave and seismic forces acting on the offshore structure. If the severe reaction forces of the offshore structure can be reduced by the way such as large diameter members of buoyancy-type or having porosity, it is very helpful to develop the large scale offshore structure.

On the other hand, the uncertainty on dynamic forces and structural properties plays the important roles on the reliability of the structure. If the uncertainty is limited within small variations, the sensitivity on uncertainties can be effectively evaluated by the perturbation method. If the uncertainty involves relatively large variations and the structure response is caused to be nonlinear motions, the Monte Carlo Simulation (MCS) method would be very effective to account for these influences (Guan, et al(2000), Kawano, et al(2004)). While the reliability estimation has been performed with the static response analysis, there are few examinations on the uncertainty estimation to the dynamic force situation (Kardeniz (2005)). Since the offshore platform system subjected to dynamic forces involves the uncertainty effects, it is important for the reliability estimation to clarify the uncertainty effects on the dynamic forces.

In the present study, the eigenfunction method after Williams & Li(2000) is applied to evaluate the wave force on the offshore structure with \( N \) porous cylindrical structure. Numerical results are presented with the effects of various wave forces and wave run up while the porosity of structure is changed. It is found that the increasing porosity of structure is significantly effective to reduce the effects on the wave forces and wave run up. If the wave force by the present method can be applied to an offshore platform with porosity, the dynamic response of the offshore platform is carried out using the modal analysis that can be solved by step-by-step integration such as Newmark \( \beta \) method(Kawano, et al(1990)). The importance of dynamic soil-structure interaction for several structures founded on soft soils has been well recognized. For an idealized two dimensional offshore platform with the pile-soil foundation system, the dynamic response evaluation is carried out using the seismic forces and the wave
forces. The MCS approach is applied to evaluate the effect of uncertainty on an idealized two-dimensional offshore platform. The dynamic responses of the offshore platform with porosity subjected to severe wave and seismic forces are examined using the reliability index. It is shown that since the uncertainty with respect to the wave and seismic force plays important effects on the response evaluation of offshore platform with porosity, it is essential for the reliable design of the offshore structure to clarify the effects due to uncertainties.

2. Formulation

2.1 The governing equation of motion

If the dynamic forces such as wave and seismic forces are given in the time domain, the dynamic response evaluation of the offshore structure can be implemented with the modal analysis.

In the present study, the dynamic response analysis is carried out for the idealized offshore platform with porosity as shown in Fig. 1. Applying the Airy wave theory and the diffraction results, the wave force on structural members with various diameters can be determined using the Morison’s equation. The superstructure is represented with the framed structure subsystem and supported with pile foundation subsystem on the sea bed. The two subsystems are connected at nodal points between the pile heads of the foundation and the bottom of the super structure. The interaction of these two subsystems is transmitted through the displacements and the dynamic forces of the pile heads and the column bases. The governing equation of motion of the superstructure is formulated with the finite element method. For the offshore platform with porosity subjected to wave forces and seismic forces, the governing equation of motion can be expressed with

\[
\begin{align*}
\begin{bmatrix}
[C_m] & [K_p] & [G] \\
\end{bmatrix}\begin{bmatrix}
\ddot{x} \\
\dot{v} \\
\dot{v} \\
\end{bmatrix} &= \begin{bmatrix}
0 \\
0 \\
0 \\
\end{bmatrix} + \begin{bmatrix}
F_p \\
F_p \\
G \end{bmatrix} \\
\end{align*}
\]

(1)

in which,

\[
\begin{align*}
[M] &= [M] + [C_m], & [C] = [C] + [C_D] \\
\end{align*}
\]

The vector \(F_p\) denotes the reaction force caused by the dynamic soil structure interaction involving seismic motions and \(\{x_a\}\) the displacement vector. The \(\ddot{v}_a\) and \(\dot{v}_a\) denote the acceleration and velocity of water particle motions. The suffix ‘a’ corresponds to the unconstrained nodal point and the suffix ‘b’ denotes the nodal point connecting to the soil foundation system. \([M_a]\), \([K_a]\) and \([C_a]\) denote the mass matrix, the stiffness matrix, and damping matrix of the structure, respectively. The matrix \([C_a]\) and \([C_D]\) denotes the coefficient matrix on the added mass due to inertia force of water and the hydrodynamic damping due to drag force.

On the other hand, increasing the mass of the size of the offshore structure, it is expected that the dynamic soil-structure interaction has generally important contributions on the dynamic response evaluation of the total system. The dynamic characteristics of the pile-soil foundation system can be expressed with the impedance function model and the governing equation of motion is as follows (Yamada, et al(1988))

\[
[M_p]\ddot{x}_p + [C_p]\dot{x}_p + [K_p]x_p = [R_p]
\]

(2)

If the substructure method is applied to the offshore structure involving the pile-soil foundation system, the governing equation of motion of the total system can be obtained by combining the Eq (1) and Eq (2). The governing equation of motion for the total system is then given by

\[
\begin{align*}
\begin{bmatrix}
[M_m] & [C_m] & [G] \\
[C_m] & [M_p] & [G] \\
\end{bmatrix}\begin{bmatrix}
\ddot{x}_m \\
\dot{v}_p \\
\dot{v}_p \\
\end{bmatrix} &= \begin{bmatrix}
0 \\
0 \\
0 \\
\end{bmatrix} + \begin{bmatrix}
F_p \\
F_p \\
G \end{bmatrix} \\
\end{align*}
\]

(3)

in which

\[
\begin{align*}
\{p_a\} &= \{C_M\}[\ddot{v}_a] + [C_D][\dot{v}_a] - [M_p][\ddot{z}_g] - [C_p][\dot{z}_g] \\
\{p_p\} &= \begin{bmatrix}
\begin{bmatrix}
-M_p \ddot{z}_g \\
-G \ddot{z}_g \\
\end{bmatrix} + [G][M_p][\ddot{x}_p] + [G][C_p][\dot{x}_p] + [G][C_D][\dot{x}_p] \\
\end{bmatrix}
\end{align*}
\]

and the suffix \(p\) denotes the pile-soil foundation, the matrix \(G\) relates the displacements of the base nodal points to those of the pile-soil foundation, \(x_p\) denotes the displacements of the pile-soil foundation system, and \(z_g\) is the ground acceleration. Given the water particle velocity and acceleration at each nodal point, the equation can be solved by time integration method such as the linear acceleration method. Since the several lower dominating vibration modes have significant contributions on the dynamic responses of the superstructure, the governing equation of motion of the total system can be expressed with these modes(Yamada, et al(1988)). If the eigenvalue analysis is carried out for the governing equation of motion of the total system, it can be expressed with the generalized coordinate system \(\{\gamma\}\). Finally the
The external force term \([\vec{p}]_w\) and \([\vec{p}]_s\) denote the force due to a wave force and a seismic force, respectively. The vector \(\{\psi\}\) denotes the modal matrix and \(\{\ddot{z}_g\}\) the acceleration of the ground motion.

By the way, the response analysis is carried out by assumption that the wave force and seismic force can be applied independently to the structure in the present study.

### 2.2 Wave force evaluation

It is assumed that the fluid is inviscid, and incompressible, and its motion is irrotational. An arbitrary array of \(N\) porous circular cylinders is situated in water of uniform depth \(h\). The radius of the \(j\)th cylinder is \(a_j\). The global Cartesian coordinate system \((x, y, z)\) is defined with an origin located on the still-water level with the \(z\)-axis directed vertically upwards. The center of each cylinder at \((x_j, y_j)\) is taken as the origin of a local polar coordinate system \((r_j, \theta_j)\), where \(\theta_j\) is measured counterclockwise from the positive \(x\)-axis. The center of the \(k\)th cylinder has polar coordinates \((R_{kk}, \phi_{kk})\) relative to the \(j\)th cylinder. The coordinate relationship between the \(j\)th and \(k\)th cylinders is shown in Fig. 2.

The interaction effects between wave and porous cylinders can be calculated with the eigenfunction expansion method (Williams & Li (2000) and Park & Kawano (2008)). The wave force on the \(j\)th porous cylinder is determined by integrating the pressure over the surface of the cylinder. Performing the integration, the wave force subjected to the \(j\)th porous cylinder can be expressed with

\[
\{F_j\} = -\frac{i \rho g H}{k^2 H'} \left( A'_j - A_i \right)
\]

(5)

Moreover, the wave run up of the \(j\)th porous cylinder is calculated as follow;

\[
H(x, y) = \frac{H}{2} e^{\frac{\alpha x}{k}} + \sum_{j=1}^{N} A'_j Z_n k H_n (kr) e^{i \omega t}
\]

(6)

For applying the wave force interaction to the inertia force in modified Morison equation, the acceleration \(\{\ddot{v}_a\}\) can be alternated with \(\{\ddot{v}_M\}\) as follow;

\[
\{F_j\} = C_m \rho \frac{2D^2}{4} \{\ddot{v}_M\}
\]

(7)

Using equation (5), the water particle acceleration including the wave force interaction can be expressed with

\[
\ddot{v}_{MM} = -\frac{\rho g H}{k} \frac{1}{\cosh k h H' (ka_j)} \left( A'_j - A_i \right) \cos(\omega t - \alpha_j) - \frac{1}{C_m \rho \sigma^2}
\]

(8)

### 2.3 Uncertain parameters and reliability index

If the extent of the uncertainty becomes relatively wide range of variation, the MCS method can be applied to the dynamic response estimation in spite of the extent of the uncertainty. For the linear dynamic response to uncertain parameters, the reliability evaluation can be carried out the maximum response evaluation with the MCS. Emphasis is placed upon evaluations of the reliability on uncertain parameter effects. It is considered that the wave period, wave height, and strength of the structure have some uncertainties for the dynamic response estimation. For example, an uncertain parameter is denoted with \(q\) and is assumed to be a normal distribution, it can be expressed with the mean \(\bar{q}\) and the coefficient of variation \(\delta\) as follows

\[
q = \bar{q} (1 + \delta \bar{q})
\]

(9)

in which \(\delta\) is a random number with the normal distribution, which the mean value is 0 and its variance is 1. Assigning some uncertain parameter to the variable, the dynamic response analysis can be carried out with the equation (4). The response quantities such as the mean value of the maximum displacement and bending stress can be determined using the results from the simulation. If the limit state function is given by the most critical situation of the response such as the maximum bending stress, the reliability index can be expressed with the results as follows

\[
\beta_R = \frac{\bar{R} - \bar{S}}{\sqrt{\sigma_s^2 + \sigma_R^2}}
\]

(10)

in which

\[
\sigma_R = \delta_R \times \bar{R}
\]

and \(\delta_R\) denotes the coefficient of variation for strength of member. Assuming that the mean value \(\bar{R}\) and its standard deviation \(\sigma_R\) are independent of time, it can be given by constant values. The stress \(\bar{S}\) and its standard deviation are function of time. For applying the performance based on the evaluation of the offshore structure to the limit state function, it is closely related to the reliability index.

### 3. Numerical results and discussion

#### 3.1 Wave and seismic force
The wave force interaction acting on arrays of vertical circular cylinder is examined with changing the porosity of structure. The wave force interaction on the structures is examined about nine cylinders arranged at the vertices of a square of side distance (50m) as shown in Fig. 3. The wave forces are non-dimensionalized by \( \rho g H a^2 \), and the abscissa denotes the wave period. Each parameter is assigned to \( a=5m \), \( h=80m \), and \( \beta=0 \). The locations (m) of each cylinder are situated at (-50, 50), (0, 50), (50, 50), (-50, 0), (0, 0), (50, 0), (-50, -50), (0, -50) and (50, -50) respectively. Wave impacts on the right angle direction of cylinder 1, 4 and 7, as shown in Fig. 3. The wave forces from 1 to 3 and from 7 to 9 have the same values. Thus, the wave forces of cylinder from 7 to 9 are not shown in Fig. 3. It is understood that when the porosity is zero, there are interaction effects between wave and cylinders, and wave forces are remarkably influenced by the interaction effects as the wave period becomes shorter. The wave forces of all cylinders are significantly decreased, and tend to have similar values as the porosity increases. It is noted that the interaction effects are closely depended on the relationship between diameter of cylinder and wave length, and the wave forces of cylinders with porosity are significantly reduced with an increase of the porosity, and slightly influenced by the interaction effects.

Porosity rate=0%

\[ \beta=0 \]

Fig.3 Dimensionless wave forces on nine circular cylinders

Porosity rate=25%

\[ \beta=0 \]

Fig.4 The ratio of wave run-up on nine circular cylinders for \( \beta=0 \) and \( T=7sec \)

The influence of the porosity on the wave run up in the vicinity of a nine cylinders array is presented in Fig. 4. The vertical axis denotes the ratio of wave run up. For the structure with porosity, the wave run up is significantly decreased as the porosity is increased. Because the reflection wave from cylinders is rapidly decreased by the damping effect of energy that is produced as the incident wave passes through the permeable cylinders. In order to clarify the wave run-up in more detail, Fig. 5 presents free-surface elevation around nine circular cylinders for wave period, 7sec, and \( \beta=45^\circ \). When the porosity is zero as shown in the upper figure, the free-surface elevation of cylinder 7 is the largest because of the interaction effects between wave and cylinders. But when the cylinders have the porosity (25%) as shown in the bottom figure, every cylinder has the very similar free-surface elevation in spite of the changing of incident wave direction. It is understood that the porosity plays the significant roles on the reduction of the wave run up and the interaction effects between wave and cylinders. It is suggested that the offshore structure with porosity is especially efficient to reduce the possibility of excess on the wave run-up.

Porosity rate=0%

\[ \beta=45^\circ \]

Fig.5 Dimensionless free-surface elevation on nine circular cylinders for \( \beta=45^\circ \) and \( T=7sec \)

Porosity rate=25%

\[ \beta=45^\circ \]

Fig.6 Acceleration response spectra of seismic motions
Fig. 6 shows the acceleration response spectra of seismic motions (x-direction). The seismic motion, Kaiboku, is far field motion corresponding to ocean type, and Taka-ns is near field motion to Kobe earthquake (1995) in Japan. These seismic motions involve relatively long period components which bring about important effects on the response because the natural period of the offshore structure is relatively close to the dominant period of the seismic motion.

3.2 Dynamic responses of offshore platform

As previously mentioned, since the dynamic forces such as wave force and seismic force have important roles on the response evaluations of the structure for reliable design of the offshore structure, it is important to perform the dynamic response evaluations. The dynamic response of the offshore platform represented with three-dimensional model as shown in Fig. 1 is understood to govern the first few vibratory modes. The essential dynamic response characteristic could be evaluated with the two-dimensional model. In the present study, the two-dimensional model as shown in Fig. 7 is applied to figure out the dynamic response properties of the offshore platform with porosity. The height of the structure is 95m, the width 110m, and the depth of water 80m. The offshore platform consists of two parts. One part is a large size cylinder (D=10m) which can support the deck weight by having buoyancy, which is located in from 60m to 45m height, and the cylinders near the water surface, such as nodal point 1, 2, and 3 etc., are treated with permeable (porosity rate=25\%) or impermeable (porosity rate=0\%) cylinders because the wave forces are the largest at the water surface, and are influenced by the interaction wave forces. Another part, such as nodal point 5, 6, 7, and 8 etc., is a middle size pipe member (D=1m) which only supports 10\% of the deck weight, and is connected with the small size pipe member (D=0.5m) and is influenced by the Morison wave forces. The structure has the characteristic that unit weight is 77.0(kN/m^3), Young’s modulus 2.1×10^8(kN/m^2) and shear stiffness coefficient 8.1×10^7 (kN/m^2). The concentrated load on the deck is assumed to be 200KN on each top point from node 37 to 47. For the fixed foundation system, the natural periods of the first and second mode on the structure are 2.498sec and 0.685sec, respectively. For the pile-soil foundation system, the shear wave velocity of soil (Vs) 100m/s, they are 3.087sec and 0.777sec, respectively. It is understood that the natural period of structure for the pile-soil foundation system becomes longer than the fixed foundation system. It is the Rayleigh type damping which has the critical damping ratio of the first mode 2\%. The dynamic responses are primarily examined for the input motion along the x direction, which brings about the most severe response situation for this structure.

Taking into the accounts the wave forces obtained by the present method for the porosity, it is expected to have important roles on the dynamic response evaluation of the offshore platform. Fig. 8 shows the time histories of displacement response due to wave forces at the nodal point 1 for the wave height, 7m, the wave period, 9sec, the porosity rate=0\% and the porosity rate=25\% and the shear wave velocity of soil (Vs) 100m/s, respectively. For the wave forces, since the displacement responses of the fixed foundation system are very similar to ones of pile-soil foundation system, they are not expressed in this figure. It is noted that the displacement responses are significantly diminished with an increase of the porosity. It is understood that the porosity of structure plays important roles on reduction of the displacement response of the offshore platform.

Fig. 7 An idealized two dimensional offshore platform with porosity

Fig. 8 Time histories of the displacement responses due to wave forces

Fig. 9 Time histories of the displacement responses due to seismic forces
Fig. 9 shows the time histories of the responses due to seismic motion, Kaihoku, as shown in Fig. 6. The maximum acceleration of the input seismic motion is adjusted to be 500gal. The displacement responses of the fixed foundation system have about two times values of the pile-soil foundation system because it has shorter natural period of structure than the pile-soil foundation system. It is understood that the seismic response is primarily depended upon the relationship between the natural period of the structure and the dominant period of the seismic motion. Comparing the response due to wave forces with seismic forces, the displacement responses due to seismic forces are considerably larger than ones due to wave forces.

Fig. 10 shows maximum bending stress responses corresponding to the shear wave velocity of soil(Vs) due to seismic forces. The maximum bending stress responses of the fixed foundation system are very similar to one of the pile-soil foundation system except for the bottom member such as nodal point 7 and 8. By comparing them, the dynamic responses are primary influenced by the shear wave velocity of soil(Vs). The bending stresses for the fixed foundation system are very larger than ones for pile-soil foundation system. As previously mentioned, the natural period of offshore platform is considerably influenced by the effects on pile-soil foundation system. In other words, the dynamic responses of structure having the long natural period could be worked out the reduction of the effects for seismic forces. However, if the natural period of structure comes close to the dominant wave period, the dynamic responses due to wave forces would be significantly influenced.

Fig. 11 shows maximum bending stress responses due to wave and seismic forces for pile-soil interaction system. The maximum bending stress is evaluated at the node 3, which is one of the most severe responses due to wave and seismic forces, Taka-ns. For the response due to the wave forces, there are significant differences between with porosity and without porosity, and it gradually increases as the wave period becomes shorter. It is noted that the porosity of structure plays very important roles on reduction of wave force effects. The three horizontal lines denote the maximum bending stress due to the maximum acceleration from 300gal to 800gal for Taka-ns. The bending stress due to seismic forces is diminished as decrease of the maximum input acceleration. The bending stress is remarkably depended upon the wave force condition such as the significant wave height and the significant wave period. If the maximum input acceleration of seismic motion is about 500gal, the response due to seismic forces is compatible to the response due to wave forces for the significant wave height, 10m. Generally speaking, it is noted that the offshore platform is significantly influenced by seismic forces than wave forces.

For the design of the offshore platform based on the reliability evaluation, it is required to figure out the exact estimation of the maximum response quantities for the structure with some uncertainties. The effects of uncertainties for response evaluations may be efficiently evaluated with the MCS method.
simulation over about 300 times. From the demonstration of the simulation, uncertain parameter effects on the dynamic response quantities can be efficiently evaluated with the appropriate simulation.

Fig. 13 also shows the convergence on the maximum bending stress of the offshore platform with porosity due to the seismic forces, Kaihoku, for variations to the Young’s modules and the maximum input acceleration of seismic motion. The mean value of the maximum input acceleration is 500gal and the coefficient of variation 10%. While the mean value of the maximum bending stress has a slight fluctuation for the convergence, it is gradually convergent to allowable one as increasing the simulation. For the simulation of about 400 times, the response is definitely achieved to the convergence of the allowable one with respects to these uncertainties.

![Fig. 13](image)

Fig. 13  Convergence of bending stress due to seismic forces

forces, the maximum displacement is significantly affected by variation of the wave height and the Young’s modules, and it is very important to figure out the variation of them.

Fig. 15 also shows the cumulative probability of the maximum bending stress to variations on the Young’s modules and the maximum acceleration of the seismic motion, Kaihoku and Taka-ns, respectively. The coefficient of variation of the maximum input seismic acceleration is 10% and its mean value is 500gal. Although there are little differences on the cumulative probability of the bending stress to the variation of the seismic forces, Kaihoku and Taka-ns, the distributions becomes to be wide bounds except for the Young’s modules. It is understood that the variation of seismic forces gives considerable effects on the cumulative probability distribution of maximum bending stress responses.

![Fig. 15](image)

Fig. 15  Cumulative probability of bending stresses due to seismic forces for pile-soil interaction system

The reliability index, which can be expressed with the second moment of the response quantity, may be one of the most suitable evaluations on these uncertainties. Since the second moment quantity can be determined with the MCS method, it may be available to carry out the reliability evaluation of the structure with respect to these uncertainties. It is assumed that the mean value of the bending stress of the present structure is 240Mpa, and the coefficient of variation is 10%.

![Fig. 16](image)

Fig. 16  Reliability index on pile-soil interaction system due to wave forces
Young’s modulus is a coefficient of the maximum input seismic forces and the one of the most severe responses to seismic forces. The variation of Young’s modulus is closely related to the dominant frequency of the seismic motion and the natural period of structure. It is important to figure out the effects on the pile-soil foundation system in seismic active areas.

The dynamic response characteristics of the offshore platform with porosity subjected to wave and seismic forces are examined. Therefore, it is important to figure out the effects on the pile-soil foundation system to seismic forces, the former can significantly reduce the dynamic responses of the offshore platform by changing the natural frequency of structure. It is important to figure out the effects on the pile-soil foundation system in seismic active areas.

For the offshore structure with pile-soil foundation system, it is important for the reliability estimations to figure out the effects on the seismic force in which the dynamic response is closely related to the relationship between the dominant frequency of the seismic motion and the natural period of the structure system.

The results are summarized as follows:

1) It is shown that the dynamic responses of the offshore platform with porosity are considerably decreased with an increase of the porosity. Thus, it is suggested that the present offshore platform with porosity is very efficient to reduce effects of wave forces and wave run up.

2) Comparing the pile-soil foundation system with the fixed foundation system to seismic forces, the former can significantly reduce the dynamic responses of the offshore platform by changing the natural frequency of structure. It is important to figure out the effects on the pile-soil foundation system in seismic active areas.

3) By the application of the reliability index evaluated with the MCS, it can be carried out the available estimation of the safety on offshore platform with porosity subjected to dynamic forces such as wave and seismic forces with considerably different characteristics.

4) For the offshore structure with pile-soil foundation system, it is important for the reliability estimations to figure out the effects on the seismic force in which the dynamic response is closely related to the relationship between the dominant frequency of the seismic motion and the natural period of the structure system.

**References**


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