Influence of Mean Step Length on Sand Dune Dynamics: Nonuniform Sediment Case

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This work presents the sand dune study using the morphodynamic numerical model for nonuniform sediment. We assess the influence of mean step length, an important parameter used in sediment transport formula proposed by Tsujimoto and Motohashi, on sand dune dynamics. The equilibrium bed load transport formula is also employed in order to compare the results using different sediment transport formulae. From the results of the nonequilibrium sediment transport model, we found that the dune geometry significantly depends on the mean step length, and the appropriate value of mean step length for the simulation of nonuniform sediment is found to be 20-30 times sediment diameter. The results using the nonequilibrium sediment transport model provide better agreement with the experiments than that using the equilibrium sediment transport model. Moreover, the model sensitivity assessment, i.e. the initial perturbation and the domain length is conducted.

Key Words: mean step length, nonuniform sediment, dune formation

1. Introduction

Prediction of morphodynamic in alluvial rivers is still challenging problem in the field of river engineering. Since the bed material is composed of many sizes of grain, the bedform geometry and their evolutions are too complicated for modeling. The past studies model the bedform phenomena by using median size of sediment. As the results, some important phenomena such as bedform geometry and their migration cannot be explained sufficiently. In order to well understand the bedform characteristics of nonuniform sediment, both experimental study and numerical model study of nonuniform sediment are still needed.

The morphodynamic numerical model of nonuniform sediment recently introduces to the filed of river engineering. Most of them initiated the nonuniform sediment transport approach in the meso-scale bedform simulations. Bui and Rutschmann¹⁾ performed a 3D numerical model of nonuniform sediment transport in alluvial channel. In the calculation codes, the flow was solved by using full Reynolds-averaged Navier-Strokes

equations with k- ε turbulence model. While the sediment transport model was calculated by using a nonequilibrium sediment transport model. Fractional sediment transport and a multiple layer model were employed for sediment sorting calculation. The main features of grain sorting and bed deformation in the channel scale simulation were generally well reproduced. The model results were found to be depended on the value of the nonequilibrium adaptation length. With the use of the nonequilibrium adaptation length formula proposed by Phillips and Sutherland²⁾, the model results agreed best with the measurement. Wu³⁾ proposed a depth-averaged two-dimensional numerical modeling for unsteady flow and nonuniform sediment transport in open channels. The 2D shallow water equations were solved by the SIMPLE(C) algorithms with the Rhie and Chow's momentum interpolation technique. The sediment transport model adopted a nonequilibrium approach for nonuniform total load sediment transport and the nonequilibrium adaptation length was set to be related to length of sand dune and alternate bars. Although, the sediment module adopted a coupling procedure for the computations of sediment transport, bed deformation, sediment sorting, but flow model and sediment transport model were simulated in a decoupled manner. The model showed good agreement with the measured data in meso-scale bedforms. By the use of the nonequilibrium sediment transport model, some recent studies^{1),3)} showed that the model seemed adequately to reproduce the bed deformation and sediment sorting in the meso-scale bedforms. However, these model results were found to be sensitive with the value of the mean step length.

Not only nonequilibrium sediment transport model is widely employed in the field of numerical simulation, but the equilibrium sediment transport model is also familiar Sloff et al.4) performed a 2D in this field. morphodynamic modeling with nonuniform sediment in order to simulate typical morphological behavior for rivers characterized by nonuniform sediment. The equilibrium sediment transport model and the bed layer model were employed in the sediment transport process. The model was tested with a curved flume experimental data and the Tenryuu River observed data. The model results showed good agreement with the experiments. With the complex situations of Tenyuu River, the simulation results were consistent with the observations. Ashida et al.⁵⁾ worked on a numerical calculation of flow and bed evolution in compound channels using a two-layer flow model. The model was constructed to deal with nonuniform sediment in order to evaluate the effects of sediment sorting on bed deformation. The concept of bed layer model was proposed for grain sorting calculation. The equilibrium sediment transport approach was used in sediment transport process. Comparing the simulated results with the observed data, the model had ability to simulate bed deformation and sediment sorting phenomenon in meandering channel with flood plain flow. From previous studies, it was shown that the numerical model provided a good performance on bedform evolution and sediment sorting in the river scale simulation by using the equilibrium sediment transport approach.

Recently, the numerical model for micro-scale bedform simulation of nonuniform sediment was proposed by Thaisiam et al.⁶⁾ for simulating sand dune characteristics and grain sorting inside sand dunes. In the numerical model, the sediment transport model explicitly takes the turbulent flow model into account during the morphodynamic computation. The sediment transport model was composed of a nonuniform sediment transport model and a bed layer model. The concept of size fraction transport and a nonequilibrium bed load transport approach were employed in the nonuniform sediment transport model. The bed layer model was applied for grain sorting simulation inside sand dunes. In addition, the hydrodynamic model proposed by Giri and Shimizu⁷) was used for hydrodynamic simulation. This model was able to simulate the morphodynamic features and grain sorting inside sand dunes in a physically based manner.

In the present study, we perform numerical studies in order to assess the influence of mean step length on sand dune dynamics. Two different approaches of mean step length are employed in the calculation. The equilibrium bed load transport formula is used to evaluate the performance of our morphodynamic model. In addition, the model sensitivity assessment is conducted in order to test our numerical model for nonuniform sediment with some factors. The numerical model has been tested with different domain length and initial perturbation patterns on bed.

2. Hydrodynamic model

equations The governing for unsteady two-dimensional flow in the Cartesian coordinate system (x,y,t) is transformed to the moving boundary fitted coordinate system $(\xi, \eta, \tau)^{8}$. The transformed equations are solved by splitting into a non-advection and a pure advection term. The non-advection term is solved by using central difference method. The pure advection term is calculated by using a high-order Godunov scheme known as the cubic interpolated pseudoparticle (CIP) technique. The pressure term is resolved using SOR method.

In the turbulence model, a 2^{nd} order non-linear k- ϵ turbulence closure is employed to reproduce turbulent characteristics in shear flow with separation zone. From the past study^{7,9)}, the 2^{nd} order non-linear k- ϵ turbulence closure satisfactorily predicts mean flow and turbulent properties over the bedforms.

The time-dependent water surface change computation is used for realistic reproduction of free surface flow over migrating bedforms. The model is able to accomplish stable and reasonable solutions with a free surface flow condition over the migrating bed forms. The kinematic condition is established along the water surface in order to compute water surface variation.

The boundary condition at the bed is no slip. A logarithmic expression for near-bed region is adopted. Although the logarithmic expression is found to be an inappropriate method for simulating the flow at the boundary condition with flow separation, Giri and Shimizu⁷) employed the logarithmic expression at the boundary condition with flow separation to simulate the flow over the fixed dune bed. From their study, the time averaged streamwise and vertical velocity profiles in all regions over dunes were reproduced reasonably well, although some discrepancies were noticed with respect to streamwise normal stress, particularly in separation region behind the dune crest. However, the sophisticated model for calculating flow in the separation zone over the bedform migration is needed in the further study. The periodic boundary condition is employed in the computation domain in which output at the downstream end is set to be input at the upstream end.

3. Sediment transport model

The sediment transport model for nonuniform sediment was proposed for bedform evolution and grain

sorting simulations. We used both the concept of size fraction and bed load transport model of nonuniform sediment for calculating in the sediment exchange process. The grain sorting simulation was performed by using the bed layer model. The sediment transport model can be then divided into two submodels which are a nonuniform sediment transport and a bed layer model.

3.1 Nonuniform sediment transport model

The concept of size fraction transport is to divide bed material into size fractions which consider each size fraction as a uniform material. The bed material transport rate can be calculated by multiplying the potential transport rate corresponding to a given size fraction with the percentage of material which can be read as follows:

$$q_{b} = \sum_{k=1}^{nk} q_{bk} = \sum_{k=1}^{nk} P_{k} \cdot q_{k}$$
(1)

where q_b is the bed load transport rate per unit width, q_{bk} is the bed load transport of sediment size fraction k per unit width, P_k is the concentration of sediment size fraction k, q_k is the potential transport rate for a given size fraction k, subscripts k and nk are the number and the total number of size fraction respectively.

In the present study, we use both the equilibrium bed load transport and nonequilibrium bed load transport approaches in our sediment transport module. We perform numerical calculations in both sediment transport approaches in order to evaluate the model performance on bedform evolution simulations. Thus, the brief description of both approaches can be described as follows.

(1) Nonequilibrium bed load transport approach

The nonequilibrium sediment transport model has been widely introduced in micro-scale bedform simulation of uniform sediment. For instant, Giri and Shimizu⁶⁾ used the nonequilibrium sediment transport approach in a vertical two-dimensional morphodynamic model for calculating sediment pickup and deposition rate in the sediment exchange process. The proposed model was found to be in good agreement with experimental data in term of bedform geometry Onda and Hosoda¹⁰⁾ proposed a depth prediction. averaged flow model combined with a nonequilibrium sediment transport model. They succeed to reproduce the formation process of micro-scale sand waves and the shape characteristics of dune. By using the nonequilibrium sediment transport approach in the computation model, it seems adequately to reproduce bedform evolution of uniform sediment.

For uniform sediment, Nakagawa and Tsujimoto¹¹⁾ proposed a nonequilibrium transport model characterized by pickup rate and step length, and applied it successfully to explain several alluvial processes such as the micro-scale bed deformation. Based on the uniform material concept, Tsujimoto and Motohashi¹²⁾ advanced a

pickup rate formula for nonuniform sediment by using the critical tractive force for each sediment size fraction. The pickup rate for sediment size fraction k can be written as follow:

$$p_{sk} = \frac{0.03\tau_{*k} \left(1 - 0.7\tau_{*ck} / \tau_{*k}\right)^3}{\sqrt{d_k / (\rho_s / \rho - 1)g}}$$
(2)

where p_{sk} is the pickup rate of sediment size fraction k, ρ and ρ_s are fluid and sediment density respectively, τ_{*k} is dimensionless local bed shear stress of sediment size fraction k, and τ_{*ck} is dimensionless critical bed shear stress of sediment size fraction k.

The sediment deposition rate is expressed as:

$$p_{dk} = \int p_{sk} f_s(s) ds \tag{3}$$

where p_{dk} is the deposition rate of sediment size fraction k and $f_s(s)$ is the distribution function of step length.

The exponential distribution function of step length of the k^{th} size fraction of bed material by Nakagawa and Tsujimoto¹¹⁾ applied for nonuniform sediment is written as:

$$f_{sk}(s) = \frac{1}{\Lambda} \exp\left(-\frac{s}{\Lambda}\right) \tag{4}$$

where Λ is the mean step length and *s* is the distance of sediment motion from the pickup point. On the basis of probability theory, Einstein¹³⁾ proposed $\Lambda = \alpha d$, in which α is an empirical constant.

The non-equilibrium bed load transport potential rate by Tsujimoto and Motohashi¹²⁾ for each sediment size fraction is written as follows:

$$q_{k}(x) = \frac{A_{3}}{A_{2}} d_{k} \int_{0}^{\infty} p_{sk}(x - x') \int_{x'}^{\infty} f_{sk}(s) ds dx'$$
(5)

where A_2 and A_3 are geometric coefficients of sediment particles.

(2) Equilibrium bed load transport approach

The equilibrium bed load transport formula proposed by Ashida and Michiue¹⁴⁾ can be expressed as:

$$q_{k} = 17\sqrt{Rgd_{k}^{3}}\tau_{*ek}^{3/2} \left(1 - K_{c}\frac{\tau_{*ck0}}{\tau_{*fk}}\right) \left(1 - \sqrt{K_{c}\frac{\tau_{*ck0}}{\tau_{*fk}}}\right)$$
(6)

where d_k is the characteristic diameter of sediment size fraction k, g is the gravitational constant, R is the relative density of the sediment $(R = \rho_s / \rho - 1)$, τ_{*jk} is the dimensionless shear stress, τ_{*ek} is the effective non-dimensional shear stress, τ_{*ck0} is the dimensionless critical shear stress and K_c is a correction factor on the magnitude of the transport rate for the influence of bed slope. The dimensionless effective shear stress for each size fraction can be calculated by using the representative grain size d_k and the shear velocity as follows:

$$\tau_{*ek} = \frac{u_*^2}{Rgd_k} \frac{1}{\left(6 + \frac{1}{\kappa} \ln\left(\frac{h}{d_m(1 + 2\tau_{*m})}\right)\right)}$$
$$\tau_{*m} = \frac{u_*^2}{Rgd_m}; \tau_{*fk} = \frac{u_*^2}{Rgd_k}; \tau_{ck0} = \zeta_i \tau_{*cm}$$
(7)

where *h* is the local water depth, d_m is the mean grain size of bed material, τ_{*cm} is the dimensionless critical shear stress for grain size dm, ζ_i is the coefficient for hiding and exposure which is common to use the Egiazaroff's formular adjusted by Ashida and Michiue¹⁴

$$\zeta_{i} = \left[\frac{\log_{10}(19)}{\log_{10}(19d_{i}/d_{m})}\right] \quad \text{for } \frac{d_{i}}{d_{m}} \ge 0.4,$$

$$\zeta_{i} = 0.85 \frac{d_{m}}{d_{i}} \quad \text{for } \frac{d_{i}}{d_{m}} \le 0.4 \quad (8)$$

The slope effect on sediment transport magnitude *Kc* can be express as:

$$K_{c} = 1 + \frac{1}{\mu_{s}} \left[\left(1 + \frac{1}{R} \right) \cos(\alpha) \frac{\partial z}{\partial s} + \sin(\alpha) \frac{\partial z}{\partial n} \right] \quad (9)$$

where α is the flow direction near the bed and μ_s is the static friction coefficient for sediment.

Then, the bed deformation is computed by using the sediment continuity equation which is

$$\frac{\partial z_b}{\partial t} + \frac{1}{1 - \lambda} \cdot \frac{\partial}{\partial x} \left(\sum_{k=1}^{nk} q_{bk} \right) = 0$$
(10)

where z_b is the bed elevation and λ is porosity of bed material.

3.2 The bed layer model

The bed layer model proposed by Ashida et al.⁵⁾ is employed for the grain sorting simulation of non-uniform sediment in our numerical model. In the bed layer model, bed material is divided into sublayers which are a mixed layer, a transition layer and a deposited layer.

The mixed layer represents the exchange layer or top layer containing the bed materials which is active to the transport process. The mixed layer thickness is assumed to be constant and equivalent to the size d_{90} of initial bed material distribution⁵⁾. The transition layer acts as a buffer layer between the mixed layer and the deposited layer. The thickness of transition layer is a function of time and streamwise direction and is restricted between 0 $< E_t \le E_d$, where E_t is the thickness of transition layer and E_d is the thickness of multiple layers. The deposited layer is divided into N_b layers in which the thickness of sublayers is equal to E_d . Therefore, thickness of the deposition layer is equal to the multiplication of N_b and E_d .

As to the initial nonuniform bed, the bed elevation is calculated by

$$z_{b} = E_{m} + E_{t} + N_{b} \cdot E_{d} + z_{0}$$
(11)

where z_b is the bed elevation, E_m is the thickness of mixed Table 1 Cases and conditions for sediment size fraction concentration calculation

Cases	Bed deformation (Δz_b)	Conditions	
А	aggradation	$E_t^n + \Delta z_b \leq E_d$	
В	aggradation	$E_t^n + z_b > E_d$	
С	degradation	$E_t^n + \Delta z_b > 0$	
D	degradation	$E_t^n + \Delta z_b \leq 0$	

layer, N_b is the total number of sub-layers in the deposited layer, and z_0 is the datum elevation.

In our calculation codes, the nonuniform sediment transport model is employed to calculate sediment pickup rate, sediment deposition rate and bed deformation in each calculated cell. Then, the concentration of sediment size fractions can be calculated from the bed layer model. Details of size fraction concentration calculation can be expressed in four cases as shown in Table 1.

4. Experiment

Miwa and Daido¹⁵⁾ conducted experiments of sediment. nonuniform Both the sand wave characteristic and sediment sorting inside sand waves were observed in order to study the interaction between sediment sorting and formation of sand waves. The experiment of uniform sediment was also carried out in order to compare with the nonuniform sediment case. Experiments were conducted in a straight rectangular open channel, 6.5 m long, 0.2 m wide and 0.3 m deep. Four sets of nonuniform sediment were used. The observed data from five cases of a nonuniform sediment experimental study are shown in Table 2. Hydraulic conditions of all experiments are given in the table in which all parameters were measured after bed forms reached to their equilibrium stage. In the table, qw is water discharge per unit width, Ie is the energy slope, u* is the mean shear velocity, L and Δ are wave length and wave height of sand dune, respectively.

5. Model validation and sensitivity study

In present study, we have conducted numerical simulations in order to validate and test the sensitivity of our morphodynamic model for nonuniform sediment. The details of studied result can be drawn as follow:

(1) The influence of mean step length on bedform characteristics and their evolution

With the use of nonequilibrium sediment transport formula proposed by Tsujimoto and Motohashi¹²⁾ on nonuniform sediment, the mean step length is one of the significant parameter that affects the bed deformation and bedform geometry. Many researchers have been applied and proposed various values of mean step length in their studies. For instant, Wu et al.^{2,16)} used the wave

Run no.	$q_{\rm w}$	Water	Sediment	Ie	u*	L	Δ
	(cm^2/s)	depth	mean diameter	$(x10^{-3})$	(cm/s)	(cm)	(cm)
		(cm)	(cm)				
Run-1	350	8.21	0.070	3.50	3.31	30.70	0.67
Run-2	400	8.87	0.070	4.00	3.69	54.00	1.16
Run-3	400	8.83	0.076	2.50	4.50	42.49	1.03
Run-4	350	8.38	0.073	2.50	4.18	42.45	0.88
Run-5	450	8.86	0.075	2.50	5.08	46.50	0.81

Table 2 Experimental cases and conditions

Table 3 The mean step length value for numerical simulation cases

Case	Mean step length	Remark
А	Dune length	Van Rijn
B1	10d	Nakagawa et al. ¹⁸⁾
B2	20d	Nakagawa et al. 18)
B3	30d	Nakagawa et al. 18)
B4	35d	Nakagawa et al. ¹⁸⁾
B5	50d	Nakagawa et al. ¹⁸⁾

length of sand dunes on the bed as the length of mean step length in the calculation. From the study of Van Rijn¹⁷⁾, when sand dunes are a dominant bedform, the mean step length may be taken as a wave length of sand dune. Based on a large number of reliable flume and field data, Van Rijn proposed the following expression for the bedform length:

$$L_s = 7.3h \tag{12}$$

where L_s is the bedform length and *h* is water depth. Moreover, the mean step length of nonuniform material was experimentally investigated by Nakagawa et al.¹⁸. It was found that the distribution of mean step length for each grain size was approximated by an exponential distribution. The empirical constant (α) for each grain size was almost constant and was proposed to be 10-30.

In this study, the mean step length formula proposed by Van Rijn¹⁷⁾ and Nakagawa et al.¹⁸⁾ are employed as shown in Table 3. We performed 30 cases of numerical calculation (one case of experiment was tested with six different mean step length vales).

The simulation results for all cases are shown in Table 4. We found that in the cases of the high value of mean step length (case A and case B5), the initial perturbation on bed disappears and no bedform appears with the increase of time. Whereas in the case of the smallest value of mean step length (case B1) causes the instability on bedform evolution, and dunes cannot maintain, in which small perturbations are found in the last stage. However, we found that the appropriate range of the mean step length for our simulation is found to be in the range of 20-30d, which is comparable with the study of Nakagawa et al.¹⁸.

Figure 1 shows the instantaneous bed configurations

of numerical simulation of experiment Run-2 where a, b, c, d, e and f show the simulation results in the cases of mean step length equal to dune length, 10d, 20d, 30d, 35d and 50d, respectively. Figure 1a shows simulation result in the case of mean step length equal to the length of sand dune. From the calculation result, it is found that the initial perturbations on the initial bed disappear and no bedform appears with the increase of time. Figure 1b shows simulation result in the case of mean step length equal to 10d. In the beginning of calculated time, the bedforms are generated but their evolutions are unstable with the increase of time, in which small perturbations are found in the last stage. Figure 1c shows simulation result in the case of mean step length equal to 20d. The sand dunes and grain sorting are generated with the increase of time. The sand dunes reach to the equilibrium state after 1800 seconds which are similar to the experiment. By using the mean step length equal to 20d, the numerical model provides a under predicted value of dune heights and dune lengths compared with the observed data. Figure 1d shows simulation result in case of the mean step length equal to 30d. The sand dunes appear and grain sorting is also generated with the increase time. The sand dunes reach to the equilibrium stage after 1800 seconds which are similar to the experiment. With the use of the mean step length equal to 30d, dune lengths and dune heights are in good agreement with the observed data in all cases. Figure 1e shows simulation result in the case of mean step length equal to 35d. Among six cases of experimental study, sand dunes can maintain on the bed only in the case of Run-2-B4 and small perturbation appears on the bed in other cases. In the case of Run-2-B4 the bedform migration becomes equilibrium after 3000 seconds which longer than the experiment. Figure 1f shows simulation result in the case that the mean step length is equal to 50d. The initial perturbations disappear and no bedforms appear with the increase time for all experimental cases.

With the use of the nonequilibrium sediment transport approach, Eulerian stochastic formula proposed by Tsujimoto and Motohashi¹²⁾, the model ability on bedform geometry prediction depends on the value of the mean step length. The smaller mean step length provides the smaller wave length of sand dune. The

Cases	Wave	Wave	Remark	
	length	height		
Dum 1 A	(cm)	(cm)	Elat had	
Kun-I-A	-	-	Flat bed	
Run-I-BI	-	-	Perturbation bed	
Run-1-B2	20.02	0.59	Sand dune	
Run-1-B3	30.40	0.89	Sand dune	
Run-1-B4	-	-	Small perturbation	
Run-1-B5	-	-	Flat bed	
Run-2-A	-	-	Flat bed	
Run-2-B1	-	-	Perturbation bed	
Run-2-B2	20.03	0.66	Sand dune	
Run-2-B3	27.57	0.93	Sand dune	
Run-2-B4	56.80	0.91	Sand dune	
Run-2-B5	-	-	Flat bed	
Run-3-A	-	-	Flat bed	
Run-3-B1	-	-	Perturbation bed	
Run-3-B2	19.96	0.58	Sand dune	
Run-3-B3	34.83	0.87	Sand dune	
Run-3-B4	-	-	Small perturbation	
Run-3-B5	-	-	Flat bed	
Run-4-A	-	-	Flat bed	
Run-4-B1	-	-	Perturbation bed	
Run-4-B2	19.97	0.51	Sand dune	
Run-4-B3	28.71	0.60	Sand dune	
Run-4-B4	-	-	Small perturbation	
Run-4-B5	-	-	Flat bed	
Run-5-A	-	-	Flat bed	
Run-5-B1	-	-	Perturbation bed	
Run-5-B2	29.04	0.90	Sand dune	
Run-5-B3	40.33	1.02	Sand dune	
Run-5-B4	-	-	Small perturbation	
Run-5-B5	-	-	Flat bed	

Table 4 Simulation results of various mean step length cases





a. Shows simulation results of Run-2-A (mean step length= dune length)



b. Shows simulation results of Run-2-B1 (mean step length= 10d)



c. Shows simulation results of Run-2-B2 (mean step length= 20d)



d. Shows simulation results of Run-2-B3 (mean step length= 30d)



e. Shows simulation results of Run-2-B4 (mean step length= 35d)



f. Shows simulation results of Run-2-B5 (mean step length= 50d)

Fig. 1 The instantaneous bed configurations of numerical simulation result of experiment Run-2



a. The comparison of grain size distributions (Run-2-B3) where the locations of measured mixture are shown as a small figure.

2.0

1.5

1.0



Experimental wave height (cm) 0.5 0.0 0.0 0.5 1.0 1.5 Simulated wave height (cm)

Run-1-B3

Run-2-B3

Run-3-B3

Run-4-B3

Run-5-B3

۸

×

-30%

+30%

2.0

b. The comparison between the experimental results and the simulated results of the wave length of sand dunes.

c. The comparison between the experimental results and the simulated results of the wave height of sand dunes.

Fig. 2 The comparison of sand dune geometries between the experimental results and the simulated results (the mean step length = 30d)

model results are comparable with the study of Yamaguchi and Izumi¹⁹⁾. They performed a linear stability analysis by the use of the stochastic model with two-dimensional Reynolds equation on dune formation of uniform sediment. They found that the case of small value of mean step length, sand dunes can be generated in the large range of unstable wave number therefore the bed is composed of many sizes of wave length and then causes a perturbation bed. The unstable area of dune formation is large. In the case of high value of mean step length, the range of unstable wave number for generating instability becomes small therefore only large

wave length of sand dune can be generated and maintained on the bed. The unstable area of dune formation becomes small. Moreover, some recent studies on numerical model of the uniform sediment using the Eulerian stochastic formula, Giri and Shimizu²⁰⁾ and Toyama et al.²¹⁾, showed that the simulation results of the bedform evolution are sensitive to the value of mean step length. However, the influence of mean step length on dune evolution in the case of nonuniform sediment needs the further study in order to clarify their effects on dune formation.



Fig. 3 Instantaneous feature of numerical simulation result with equilibrium sediment transport model (top) and nonequilibrium sediment transport model (bottom)



Fig. 4 Simulation results with random field shape of initial perturbation (top) and cosine shape of initial perturbation (bottom)



Fig. 5 Instantaneous simulation results with 2 m. domain length (top) and 4 m. domain length (bottom)

Among six cases of various mean step length which are applied to all experimental cases, it is found that the numerical model is in best agreement with the experiments in geometry and grain sorting when mean step length is equal to 30d as shown in Figure 2. Figure 2 shows the comparison between the simulated results and the experimental results of sand dune geometry and grain sorting inside the sand dunes using the mean step length = 30d.

Figure 2a show the comparison between the simulated results and the experimental results on grain sorting at various locations inside the sand dunes of the case Run-2-B3, and a small figure shows three locations of the measured mixture as Layer I, II and III. Layer I denotes the mixture on the upstream part of dune crests, Layer II denotes the mixture in the trough of dunes, and Layer III denotes the mixture in the substrate layer. From the experimental results, it was found that Layer I provides the finest mixture among three layers. Layer II which is coarser than Layer I and III because coarse grains was deposited at the lee side, whereas the grain size distribution in Layer III shows no significant change because it does not strongly participate with the sediment transport and bed evolution. Comparing with the simulated results, the grain size distribution of bed material inside the sand dunes in layer II and layer III are reproduced very well by our numerical model.

Figure 2b shows the comparison between the simulated results and the experimental results of the wave length of sand dunes. The results show that the numerical model provides the under predicted wave lengths of sand dunes with the discrepancy 30% in most cases (3 in 5 cases). We found that the under predicted wave lengths of sand dunes are affected by the water depth. The present model provides the under estimated flow depth, therefore causes the under prediction of wave length. The model shows a fair agreement on the wave length prediction. However, some improvement on flow model is needed in order to well predict the flow depth and the wave length of sand dunes.

Figure 2c shows the comparison of the wave heights of sand dunes between the simulated results and the experimental results. It is found that the numerical model shows a satisfactory agreement on the wave height predictions with the discrepancy $\pm 30\%$ in most cases.

(2) Influence of sediment transport model on bedform configuration simulation

In this study, we attempt to evaluate the performance of our morphodynamic model for nonuniform sediment by using the different sediment transport approach. The equilibrium sediment transport approach, a bed load transport formula proposed by Ashida and Michiue¹⁴, is employed in the nonuniform sediment transport module. The instantaneous features of simulation result of Run-2 are depicted in Figure 3. With the use of the bed load transport formula proposed by Ashida and Michiue¹⁴, it is showed that the perturbation grows rapidly and the sand dune cannot be generated on the bed. The grain sorting simulation appears to be incompatible with the physical observation. The flow depth is also overestimated. Whereas, the simulation results using the nonequilibrium sediment transport model show a satisfactory agreement in bedform geometry and grain sorting prediction⁵⁾. From the simulation result, it can concluded that the use of nonequilibrium sediment transport model is an appropriate approach for simulating micro-scale bedform dynamics, because it can reproduce the phase lag between bed shear stress and the particle transport by incorporating the step length.

(3) Model sensitivity to initial perturbation

In the computational method, we put small perturbations on the initial bed condition and then allow it to grow using our simulation technique. Two different perturbation shapes, random shape and cosine shape, are used in our model. It is necessary to assess the effect of the initial perturbation shape on bedform evolution. We test both types of initial perturbation shape in experiment case of Run-3. The instantaneous bedform features from simulation result are illustrated in Figure 4. Some small alteration can be noticed in size of bedform. However, no any significant quantitative distinction can be made between these two of cases.

(4) Model sensitivity to domain length

The periodic boundary condition was employed in our calculation in order to reduce the computational time. It is necessary to assess the effect of calculation domain length in order to confirm the periodic phenomenon. We used two different domain lengths with all experimental cases, namely 2m and 4m. The quantitative comparison of an instantaneous bedform configuration of Run 3 is shown in Figure 5. The comparison result shows that the numerical model is sufficient to simulate the experimental cases. However, the full length of flume should be investigated in the further study.

6. Conclusions

The numerical simulation studies were conducted in order to investigate on the morphodynamic model for nonuniform sediment. From this study, some conclusions can be drawn as follows:

1) The appropriate mean step length for nonuniform sediment is found to be in the range of 20-30d. By using the mean step length equal 30d, it is found that our morphodynamic model is in good agreement of bedform geometry and grain sorting predictions.

2) With the use of nonequilibrium sediment transport model, Eulerian stochastic formula by Tsujimoto and Motohashi, the model ability on dune formation prediction appears to be depending on the mean step length.

3) The model performance on sand dune dynamics and grain sorting simulation differs based on the sediment transport model. A nonequilibrium sediment transport model seems to be more appropriate than an equilibrium model on micro-scale of bedform simulation. Although the equilibrium sediment transport model proposed by Ashida and Michiue seems adequately to reproduce the bed deformation on meso-scale of bedform simulation, the model shows poor results on micro-scale of bedform simulation.

4) The morphordynamic model appears to be insensitive with the initial perturbation.

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