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A new treatment of the exchange layer thickness to evaluate sediment sorting and armoring

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This study presents a new treatment of an exchange layer to evaluate the sediment sorting and armoring process. In this treatment, a thickness of bed load layer is evaluated as a function of bed shear stress, and is introduced into a well known exchange layer model which firstly proposed by Hirano (1971). The results predicted by the present method on sediment sorting, armoring and associated bed degradation accord with flume data. This suggests that sediment sorting and associated problems can be predicted well using an automatically determined thickness of bed -load layer, although the well known constant layer model can give good results on the problems under consideration if one determines the thickness properly by try and error.

Key Words: *Sediment armoring, sediment sorting, exchange layer, bed-load layer, sediment transport*

1. Introduction

Sediment sorting and armoring has long been studied by many researchers in relation to river bed stability, sediment transportation and associated bed evolution, longitudinal and transverse bed topography, etc. (Raudkivi et al., 1982[1]; Shen et al.,1983[2]; Ashida et al, 1971[3]; Suzuki et al., 1988[4]; Parker, 1990[5] and Egashira et al., 1990[6]). As far as bed-load and corresponding problems are concerned, these have been solved numerically using governing equations for water as well as for a bed load equation of non-uniform sediment and mass conservation equations of bed sediment. In the methods employed therein, it is very important how to evaluate bed-load transport rate of each grain size and sediment size distribution of bed surface.

The study of bed armoring has progressed rapidly in terms of "so called exchange layer" proposed first by Hirano[7] (1971). He introduced an exchange layer thickness in order to develop the mass conservation equation of each grain size in the bed surface layer, which enables us to compute sediment size distribution and bed load rates of non-uniform sediment bed. In addition to this, Egiazaroff's formula[8] (1965) and its modified formula by Ashida and Michiue[9] (1972) which predict incipient motion of individual particles were a key for developing these studies. Since then, many valuable results have been proposed, and are illustrated in many textbooks, Walter[10] (1984), Ning[11] (1999).

In above mentioned studies, the exchange layer thickness is treated as a constant value in relation to reference sediment size; i.e. maximum grain size, although it might be specified sometimes in situ problems, referring to height of sand waves. Nevertheless, we have obtained valuable results on sediment armoring and sorting, lowering processes, etc. However, most researchers may not think that "constant value" for the exchange layer thickness is reasonable from a view of sediment dynamics principle, because the bed-load layer thickness changes with bed shear stress. In fact, it is well known that propagation speed of armor coat and lowering process of bed elevation (Hirano, 1971[7]; Garde et al., 1977[12] and Little et al, 1976[13]), formation process of sand bars (Jeaggi et al, 1982[14]; Parker, 1991[15] and Takebayashi et al 1997[16]), stability of river bends (Bridge, 1992[17]) etc. are very sensitive to the thickness of the exchange layer.

The present study tries to introduce a new idea, a temporally and spatially changing bed-load layer instead of the constant exchange layer, into a general method, and tests its applicability, showing data obtained from a flume test and results predicted using the general exchange layer model and the present model.

2. Description of Model

2.1 Exchange Layer Model

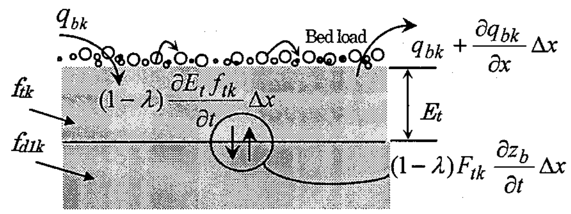

Fig. 1 Schematization of sediment exchange process on channel bed in an exchange layer model

In treating the sediment sorting numerically, a concept of the exchange layer has been widely used in mobile-bed with sediment mixtures. In this model, as shown in **Fig.1**, it is assumed that all material in the exchange layer (denoted by E_t) is homogeneously mixed. Many experiences suggest that the general method predicts experimental data well when the exchange layer thickness is specified properly, i.e. to be equal to the maximum grain diameter, although there is not a universal criterion for determining it.

The governing equations are described as follows.

A continuity equation of each grain size for the exchange layer is formulated as follows[7]:

$$(1-\lambda)\frac{\partial}{\partial t}(E_t f_{tk})+(1-\lambda)F_{tk}\frac{\partial z_b}{\partial t}+\frac{\partial q_{bk}}{\partial x}=0 \quad (1)$$

in which $F_{tk}=f_{d1k}, \quad (\partial z_b/\partial t\le 0)$

$$F_{tk}=f_{tk}, \quad (\partial z_b/\partial t\ge 0)$$

where f_{tk} is the fraction of size class-*k* in the exchange layer, and f_{d1k} is the fraction of size class k in the first deposited layer.

A bed elevation is estimated by means of the following formula.

$$(1-\lambda)\frac{\partial z_b}{\partial t}+\sum_{k=1}^{n}\left(\frac{\partial q_{bk}}{\partial x}\right)=0 \quad (2)$$

The bed load transport rate for sediment of size class-*k* is estimated by the following relation[3),18)].

$$q_{bk}=17\sqrt{sgd_k^3}\,(\tau_{*ek})^{3/2}\left(1-\sqrt{\frac{\tau_{*ck}}{\tau_{*k}}}\right)\left(1-\frac{\tau_{*ck}}{\tau_{*k}}\right)f_{bk} \quad (3)$$

In these equations, λ is the porosity of material of the bed layer (λ=0.4 as constant for simplify), q_{bk} is the sediment transport rate for size class-*k*, *s* is the submerged specific weight of sediment, d_k is the diameter of sediment size class-*k*, τ_{*k} is the non-dimensional shear stress of sediment size class-*k*, τ_{*ck} is the non-dimensional critical shear stress of sediment size class-*k*, τ_{*ek} is the non-dimensional effective shear stress of sediment size class-*k*.

The non-dimensional critical shear stress of size class-*k* is estimated as follows[9)].

$$\tau_{*ck}=\tau_{*cm}\left[\frac{\log_{10}19}{\log_{10}(19d_k/d_m)}\right]^2, \quad d_k/d_m\ge 0.4 \quad (4)$$

$$\tau_{*ck}=0.85\tau_{*cm}\frac{d_m}{d_k}, \quad d_k/d_m\le 0.4 \quad (5)$$

in which τ_{*cm} is the non-dimensional critical shear stress of mean grain size.

The non-dimensional effective shear stress of sediment size class-*k* is estimated as follows.

$$\tau_{*ek}=\frac{u^2}{\left(6+2.5\ln\frac{h}{d_m(1+2\tau_{*m})}\right)^2}\cdot\frac{1}{sgd_k} \quad (6)$$

2.2 Present Model

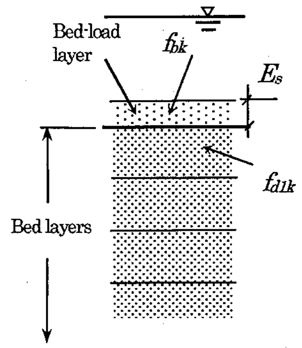

Fig. 2 Schematic diagram of the present model

In **Fig. 2**, the bed-load layer, bed surface and under-laying bed layers are shown schematically. In comparison with the exchange layer model, the present model can define the bed surface clearly as the boundary between the bed-load layer and the stationary layer, because the thickness of the bed-load layer, denoted by E_s, is evaluated to be a function of bed shear stress, using a formula proposed by Egashira and Ashida[19)] [see **Eq.(10)**].

The continuity equation of each grain size for the bed load layer is given as follows.

$$c_b\frac{\partial f_{bk}E_s}{\partial t}+(1-\lambda)F_{bk}\frac{\partial z_b}{\partial t}+\left(\frac{\partial q_{bk}}{\partial x}\right)=0 \quad (7)$$

in which $F_{bk}=f_{d1k}, \quad (\partial z_b/\partial t\le 0)$

$$F_{bk}=f_{bk}, \quad (\partial z_b/\partial t\ge 0)$$

The continuity equation of each grain size for the first bed layer is formulated as follows:

$$E_{d1}\frac{\partial f_{d1k}}{\partial t}+\left(f_{d1k}-F_{dk}\right)\frac{\partial E_{d1}}{\partial t}=0 \qquad (8)$$

in which $F_{dk}=f_{d1k}$, $(\partial z_b/\partial t\le 0)$;

$$F_{dk}=f_{bk}, \quad (\partial z_b/\partial t\ge 0)$$

The bed elevation, z_b is formulated as follows, taking a temporal change of the bed-load layer into consideration.

$$(1-\lambda)\frac{\partial z_b}{\partial t}+c_b\frac{\partial E_s}{\partial t}+\sum_{k=1}^{n}\left(\frac{\partial q_{bk}}{\partial x}\right)=0 \qquad (9)$$

In these equations, f_{bk} is the fraction of size class-k in the bed load layer, E_{d1} is the thickness of first deposited layer, c_b is the sediment concentration of the bed-load layer, E_s is the bed-load layer thickness, estimated by the following equation[19]:

$$\frac{E_s}{d_m}=\frac{1}{c_b\cos\theta(\tan\phi-\tan\theta)}\tau_{*m} \qquad (10)$$

in which d_m is the mean sediment size of bed load, θ is the local bed slope, ϕ is the friction angle of sediment and τ_{*m} is the non-dimensional bed shear stress specified by d_m.

3. Experiment

To obtain flume data for testing two methods, an experiment was conducted in a straight open channel, 14m long and 0.4m wide, which is shown in **Fig.3**. An initial bed which is 12cm thick and 12m long is made smoothly, using non-uniform sediment whose size ranges from 0.5mm to 12mm. The sediment size distribution is illustrated in **Fig. 4**. A part of upstream reach is fixed to avoid disturbances from upstream entrance, and at the downstream end, a controller is mounted to obtain uniform flow. The flow condition was set so as to form parallel degradation as follows. Unit width flow discharge and the initial bed slope are 0.075 m^2/s and 0.0025, respectively. Initial bed shear stress is almost equal to τ_{*c90}, in which τ_{*c90} is the critical non-dimensional bed shear stress of d_{90}. Bed degradation processes were observed with no sediment supply.

Measurements were conducted temporally for the water and the bed surface elevations, sediment transport rate at the flume end, grain size distribution of the bed surface and so on.

The water surface elevation and the bed surface profile were measured at 1m interval along working area by using a point depth gauge with an accuracy of 0.1mm. Sediment transport rate was measured by collecting sediment at downstream end. Bed material was sampled in the surface area of about 49cm^2 (7cm x 7cm) with the thickness of 1.2cm after measuring the bed surface profile at 3m, 8m and 13m from the upstream end. The sampling tools are illustrated in **Fig. 5**.

Figure 6 shows the photos which were taken at beginning of the run and at the final stage of armored layer development.

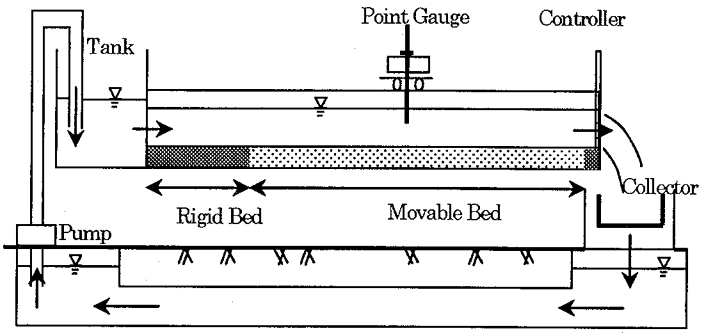

Fig. 3 Experimental channel

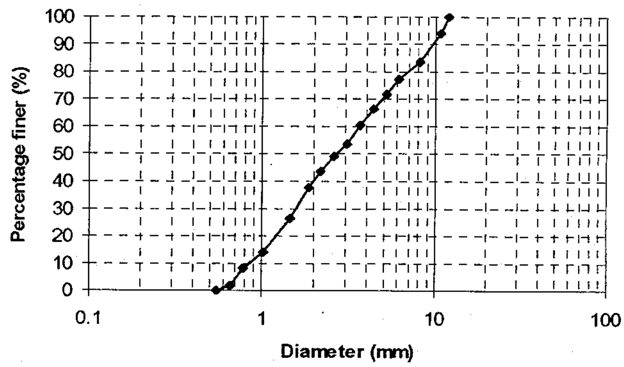

Fig. 4 Sediment size distribution

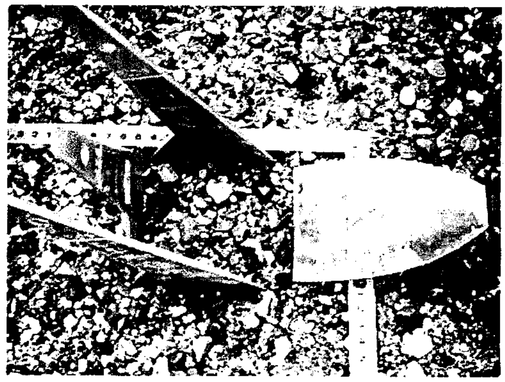

Fig. 5 Bed material sampling tools

4. Results and Discussion

A backward difference numerical scheme is employed for the governing equations of flow, and a forward difference scheme is employed for the equations associated with sediment. Numerical computations are conducted using the present model (PM for simplicity) as well as the well known exchange layer model (ELM for simplicity). In ELM method, the thickness of the exchange layer is specified as $0.5d_{max}$ (d_{max}=1.2cm), d_{max} and $2d_{max}$, respectively. The non-dimensional critical bed shear stress of d_m is specified as 0.062 in all computations. $(1-\lambda)/2$ is used as c_b in PM for simplicity.

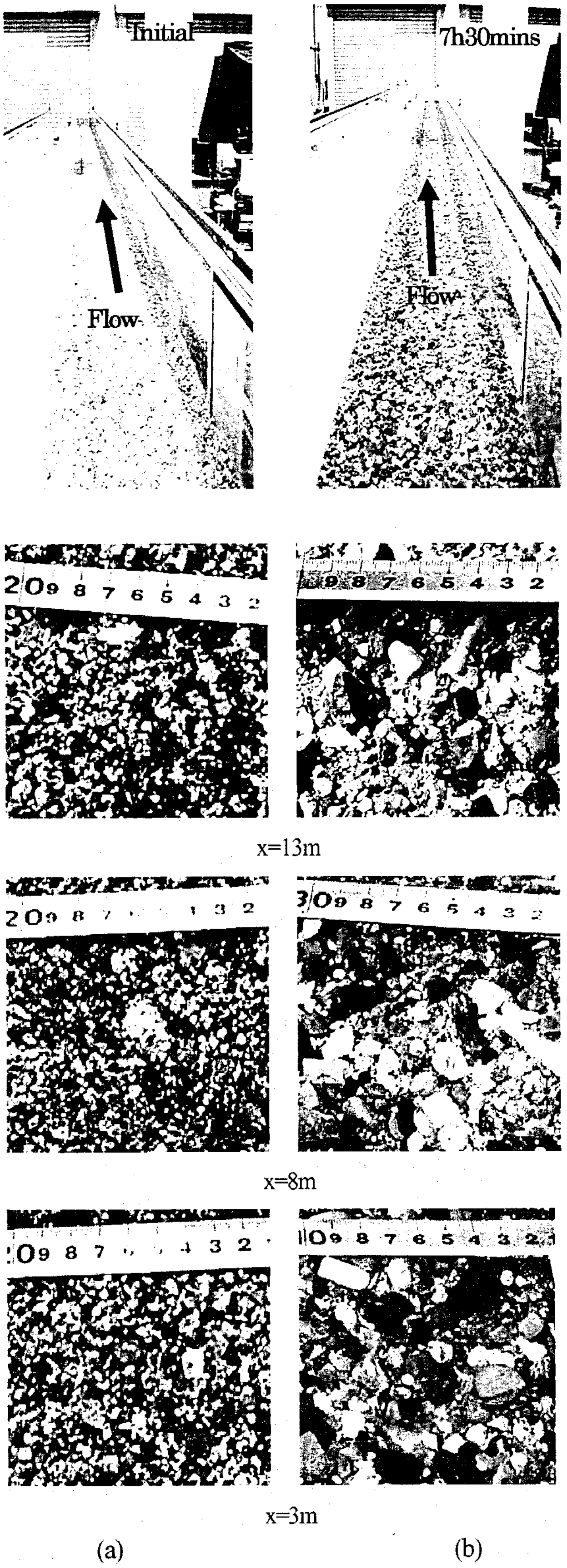

Fig. 6 Photos of flume and bed surface material taken at 13m, 8m and 3m from upstream end, respectively.
(a) Initial stage; (b) Final stage

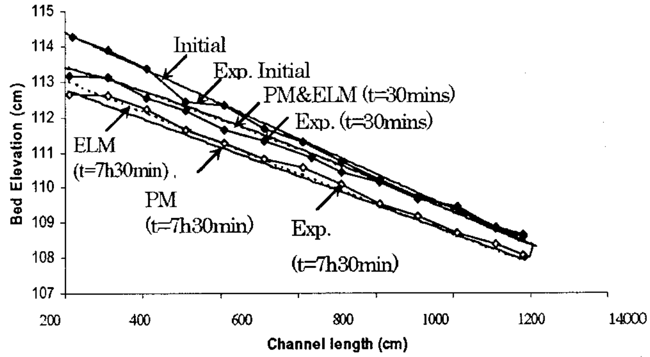

Fig. 7 Channel bed degradation (E_t=d_{max} in ELM method)

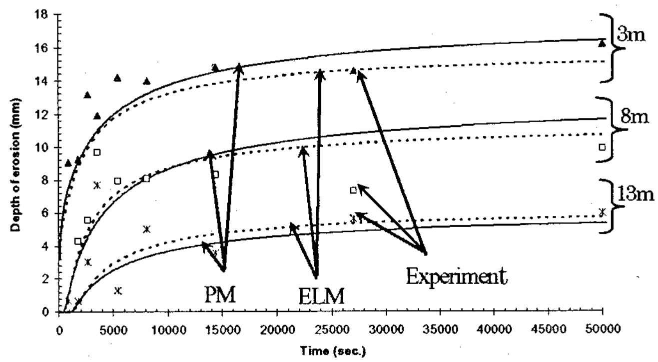

Fig. 8 Temporal change of the bed elevation at three sections (E_t=d_{max} in ELM method)

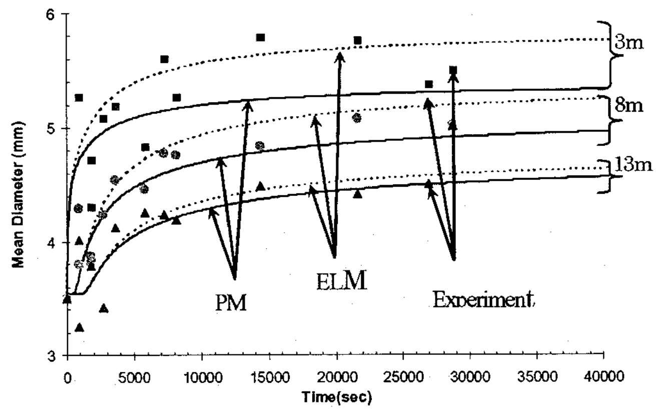

Fig. 9 Temporal change of the mean size of bed surface material at three sections (E_t=d_{max} in ELM method)

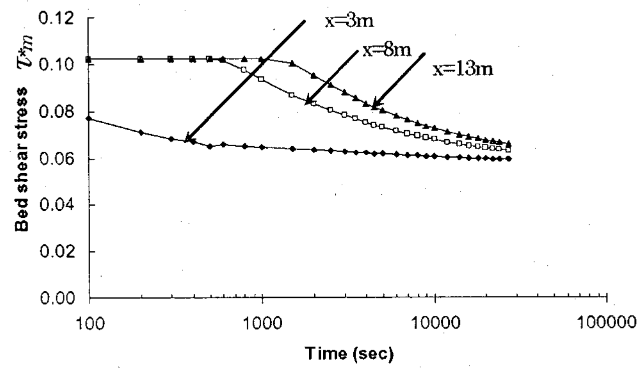

Fig.10 Temporal change of bed shear stress at three sections predicted by PM

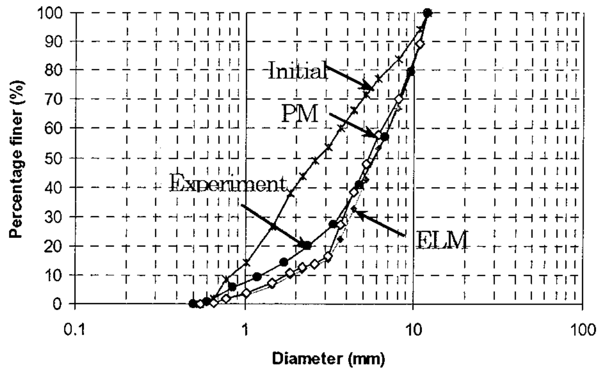

Fig. 11 Grain size distributions at x=3m from the upstream end after 7h30min ($E_t=d_{max}$ in ELM method)

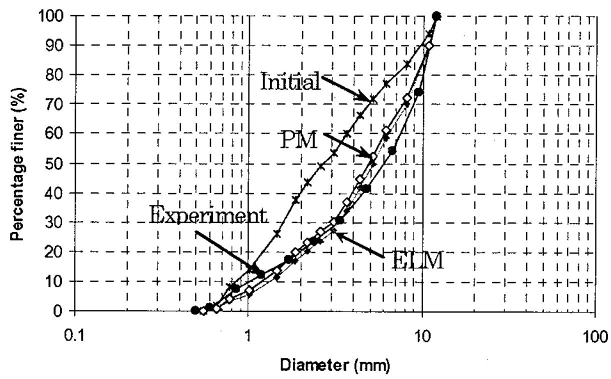

Fig. 12 Grain size distributions at x=8m from the upstream end after 7h30min ($E_t=d_{max}$ in ELM method)

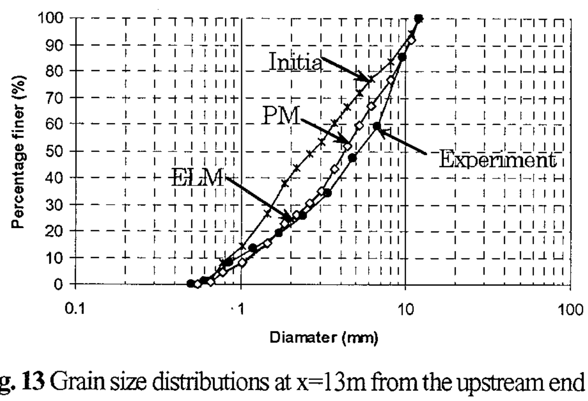

Fig. 13 Grain size distributions at x=13m from the upstream end after 7h30min ($E_t=d_{max}$ in ELM method)

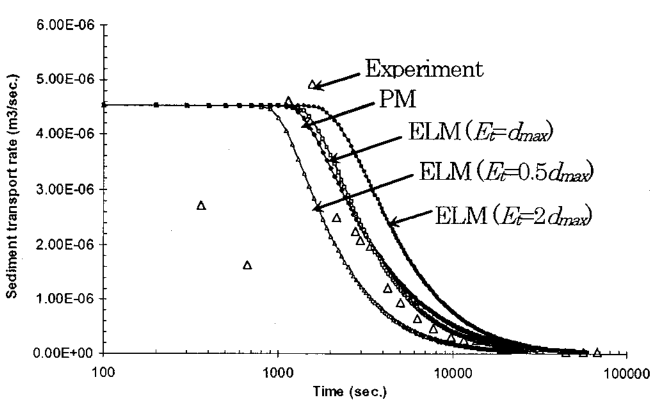

Fig.14 Sediment transport rates at the downstream end

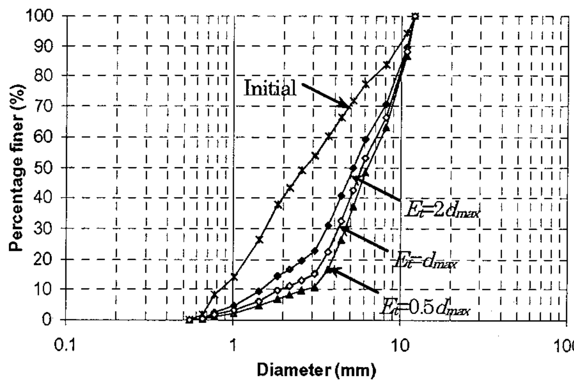

Fig. 15 Grain size distributions predicted by ELM at x=3m with different E_t after 7h30min.

In **Figs. 7** and **8**, the results computed by two methods are shown with flume data for the longitudinal bed profile and the temporal change of the bed elevation at three sections, in which the exchange layer thickness, E_t in ELM is specified as d_{max}. It is shown that both methods can predict flume data well.

In **Fig. 9**, the computed results are shown with flume data for the temporal change of the mean size of bed surface sediment at three sections. The temporal change of the bed shear stress which is computed in PM is illustrated in **Fig. 10**. The mean sediment size is well predicted by two methods although there is some discrepancy between computed results at downstream sections. At three sections, the mean size tends to increase slightly at the end of the computed period. According to the bed shear stress computed at three sections by PM, it is decreasing to the critical value at the section 3m from the upstream and at other two sections are still above the critical value. Therefore, it is expected that the grain size tends to increase.

Figures 11, **12** and **13** show the sediment size distributions of the bed surface after 7h30min from the start at each cross section. As can be expected from the results shown in **Fig.9**, two methods predict well flume data, too. There are not accurate differences among the results predicted by both methods, PM with automatically determined thickness of bed load layer and ELM with constant exchange layer thickness.

In order to illustrate the differences between the results obtained from two methods, the computed sediment transport rates are shown with flume data in **Fig.14**, in which the exchange layer thickness is specified as $0.5d_{max}$, d_{max} and $2d_{max}$ in ELM, and in addition, the sediment size distributions of bed surface which is predicted by ELM with three cases of the exchange layer thickness are shown in **Fig.15**. It is clearly seen that in ELM, the sediment transport rate is influenced by the thickness of exchange layer and decreases rapidly with decrease of the layer thickness, which corresponds to the developing speed of amour coat. While PM has a unique solution, although its applicability should be investigated further.

5. Conclusion

In order to remove ambiguity for determining an exchange layer thickness, a bed load layer thickness which is a function of bed shear stress is introduced instead of a constant exchange layer into a general, classic method. The proposed method can predict flume data well on sediment sorting and armoring, sediment transportation and lowering of bed elevation although the validity of the method is not tested widely.

The authors will apply the method to corresponding several problems in order to assure its applicability.

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