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Recent advances on computational modeling of bed form morphodynamics

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The paper reported herein presents new advances on computational modeling technique for bed form morphodynamic simulation. A hydrodynamic model with non-hydrostatic, unsteady free surface flow condition, coupled with a pick up-deposition formulation for sediment transport, is developed. Proposed morphodynamic model simulates bed form evolution and migration process with the superimposition phenomenon realistically. Model predicts all geometric characteristics of bed form in physically based manner. Likewise, model successfully replicates temporal variation of flow-depth and form drag induced by bed form evolution. Model performance is evaluated for variety of turbulence closures in the context of morphodynamic simulation. It is evident that simulation result is significantly affected by vertical grid distribution pattern. Developed model is verified against laboratory observations.

Key Words: *Numerical modeling, hydrodynamics, river bed form, morphodynamics*

1. Introduction

Morphodynamics of bed form in alluvial channels is one of the challenging problems in river engineering practice, despite many invaluable efforts that have been made so far[1]. Prediction of morphodynamic feature of bed form, such as ripples and dunes is of significance in order to evaluate their evolution process as well as resistance to flow induced by them.

A comprehensive review of past investigations on river dunes has recently been done by Best[2]. He has pointed out towards the necessity for integrated and interdisciplinary studies in order to improve existing understanding on the kinematics of dune morphology and flow. Most past observations evinced strong correlation between turbulence structures induced by flow separation, bed resistance and sediment transport. Consequently, all these aspects are equally important for quantitative determination of bed form morphodynamics. Of most past investigations, significant attempts were made to analyze flow and turbulence structures over fixed bed form. Nelson et al.[3] conducted flow and turbulence measurement over a two-dimensional dune using laser-Doppler Velocimeter. The coupling between mean flow and turbulence was investigated. Furthermore, effects on bed form instability and finite amplitude stability were examined. Best et al.[4] carried out laboratory measurements to evaluate influence of sediment transport on turbulence production at low transport rate. Schmeeckle et al.[5] conducted laboratory visualization of turbulence and suspended sediment transport over two-dimensional dunes. Schindler and Robert[6] examined flow and turbulence characteristics across the ripple-dune transition in laboratory flume. These observations have corroborated the significance of fluid dynamics of bed form. Variety of approaches were developed in order to analyze bed form-induced morphology in alluvial channels and development of dunes[7, 8, 9, 10]. All these studies revealed influence of temporally dynamic parameter contributing to bed form development.

Recently, owing to the high performance computers and reducing computational expense, application of numerical models to such problem has become more reliable, particularly for modeling flow and turbulence characteristics over bed form. For instance, Shimizu et al.[11] developed computational codes for a three-dimensional direct numerical simulation of flow and turbulence over two-dimensional fixed dunes. The numerical model was able to reproduce coherent structures induced by dune crest adequately. Numerical computations, carried out by Richards and Taylor[12], Mendoza-Cabrales[13], Yoon and Patel[14] and Barr et al.[15] also yielded realistic prediction of flow field and turbulence over bed form. However, these numerical studies do not comprise bed form dynamics.

Morphologic numerical models are being increasingly employed these days in various river engineering issues[16), 17)]. Despite considerable amount of works, rather sluggish progress has been made so far regarding numerical computation of bed form dynamics. Notably, an attempt was made by Onda and Hosoda[18)] to simulate development of bed form numerically. They proposed a depth-averaged flow model considering vertical acceleration with the description of bottom shear stress based on potential flow analysis. They used sediment pick up and deposition function to simulate sand dune formation. However, this model is incapable to reproduce separation behind dune crest and, thereby, its effect on flow field and sediment transport. Recently, Jerolmack and Mohrig[19)] proposed a nonlinear stochastic surface evolution model to simulate bed form growth. In this model, they treated temporal variation of bed shear stress as stochastic variable and explored its morphodynamic importance by adding a noise term. They succeeded to simulate qualitatively feature of bed form evolution. However, their model does not consider fluid dynamics of bed form, hence physically incomplete.

Despite significant amount of past investigations, almost all of them are unable to tackle the complexity of temporal bed evolution and associated flow field modification. Recently, we proposed a vertical two dimensional flow model with sediment pick up-deposition function[20)]. It was shown that model possesses capability to simulate flow and turbulence field over fixed dunes. Moreover, model was able to replicate bed form evolution from an initial perturbation. Most field and laboratory observations on bed form development have revealed the dynamic pattern of bed form evolution with superimposition or amalgamation that could not be replicated by the model. In this study, we continue our study[21)] to confirm that the simulation of bed form evolution mechanism and their geometric characteristics can significantly be improved by using non-uniform pattern of vertical grid distribution. Furthermore, effect of turbulence models on simulation results is also evaluated and discussed in present study. Additionally, we demonstrate the model capability to replicate evolution of drag and flow depth induced by bed forms.

2. Morphodynamic Model

A three-dimensional computation with direct numerical simulation as proposed by Shimizu et al.[11)] seems to be more complete approach for computation of flow and turbulence over bed form. However, implication of this flow solver with a sediment transport model for morphodynamic simulation may inhibit it from being applied efficiently because of extremely high computational effort. Consequently, aforementioned three-dimensional hydrodynamic model was simplified to be a vertical two-dimensional and enhanced by imposing non-hydrostatic, free surface flow condition and, subsequently, coupled with appropriate sediment transport module. The governing equations for this model are presented in **Appendix**.

Computation of time-dependent water surface change is of importance for realistic reproduction of free surface flow over migrating bed form. Basically, majority of morphologic numerical models developed earlier assume rigid lid water surface condition to achieve numerically stable solution. Such model cannot reproduce properly flow characteristics and water depth variation induced by bed form. In present study, we succeeded to achieve stable and reasonable solution under free surface flow condition over migrating bed form. The kinematic condition, which constraints fluid particles to remain on the water surface at any time following the local flow velocity, is imposed along the water surface in order to compute water surface variation (see **Appendix**).

The boundary condition at the bed is no slip. A logarithmic expression for near-bed region was adopted[21)]. The periodic boundary condition was employed in this computation, that is, out put at the downstream end is set to be input at the upstream. The computation was performed for different length of calculation domain in order to assess the effect of periodic boundary condition.

The equations were transformed into a boundary fitted coordinate system. Transformed equations were numerically solved by splitting them into non-advection and pure advection phase. Non-advection phase was computed using central difference method. The pressure term was resolved using SOR method. Advection phase was calculated using a high-order Godunov scheme known as CIP method[21)]. In this scheme, at very small time increment, the temporal change of velocity components at a point in space can be split into the time evolution of the inhomogeneous terms and the time evolution at a point due to the advection of the field. More detail on solution procedure can be found elsewhere[21)].

2.1 Turbulence Models

In our previous study[21)], a nonlinear k-ε turbulence closure was employed that enables the anisotropy of Reynolds stresses to be considered to some extent. In conventional k-ε model turbulence stress tensors are evaluated using linear relationship. In order to reproduce turbulence characteristics more precisely in shear flow with separation zone, a nonlinear term is added to the standard k-ε model. Kimura and Hosoda[22)] made detailed analysis and comparison of nonlinear k-ε model with other turbulence closures and found it more efficient than RSM or LES model in terms of CPU time. In present study, we have tested the performance of a standard k-ε model also so as to assess the significance of turbulence asymmetry offered by non-linear k-ε model in the context of morphodynamics simulation. Moreover, we performed similar computation with a simplest turbulence closure known as zero-equation model for the sake of comparison.

The expression for turbulence tensors considering nonlinear term is written as follows:

$$-\overline{u_i'u_j'} = \nu_t S_{ij} - \frac{2}{3}k\delta_{ij} - \frac{k}{\varepsilon}\nu_t \sum_{\beta=1}^{3} C_\beta \left(S_{\beta ij} - \frac{1}{3} S_{\beta\alpha\alpha}\delta_{ij} \right) \quad (1)$$

where ν_t = eddy viscosity coefficient, δ_{ij} = Kronecker delta, k = turbulent kinetic energy and ε = dissipation rate of k.

k and ε is computed using two more equation for energy production and dissipation[21].

The detail description and formulation of strain tensors S_{ij}, $S_{\beta ij}$, $S_{\beta\alpha\alpha}$ as well as coefficient C_β can be found elsewhere[23]. The third term in right hand side of Eq. (1) is an additional nonlinear term in standard k-ε model.

Zero-equation model is expressed as follows:

$$\nu_t = A\,\frac{1}{6}\kappa u_* h \quad (2)$$

where u_* = shear velocity; κ = Karman constant, and A = weight coefficient (=1 in this study).

2.2 Formulation of Sediment Transport and Bed Evolution

With regard to sediment transport model, some investigations have revealed that spatially and temporally averaged sediment transport models are inadequate to describe the extremely inconsistent nature of sediment particle motion particularly in the region with high turbulence, viz. sediment transport over bed form[23, 24]. Direct computation of sediment particle motion, viz. Discrete Element Model (DEM), is undoubtedly more advanced approach. For instance, Schmeeckle and Nelson[23] computed the motion of large number of particles by integrating their equations of motion simultaneously, in which motion of each particles respond to the local and temporally variable flow field. However, coupling of this approach with an advanced hydrodynamic model, at present, seems to be rather sophisticated in view of computational complexity and expenses. Computationally efficient as well as reliable approach is believed to be modeling of pick up and deposition of sediment particles in terms of a stochastic formulation. There is no consistent theoretical description of this phenomenon, consequently, most proposed formulations to derive pick up and deposition functions are based on experimental or/and stochastic approach[25, 26, 27, 28]. In our numerical code, an Eulerian stochastic formulation of sediment transport proposed by Nakagawa and Tsujimoto[27] was incorporated (**Appendix**). This method was specifically developed for bed load transport which is applicable to non-equilibrium situation with sand waves. Proposed formulation of sediment pick up rate was validated with range of physical observations. Moreover, this method yielded one of the best predictions of pick up function for fine sediment grain as reported by van Rijn[28]. This approach was effectively used by Onda and Hosoda[18] as well for computation of bed form development. Proposed sediment transport computation approach explicitly considers the flow variability during morphodynamic computation. Consequently, the exchange of sediment particles, namely pick up and deposition rate at each time step can be computed by coupling sediment transport with hydrodynamic model. Detail can be seen elsewhere[21].

It should be mentioned that Nakagawa and Tsujimoto's pick up function is not valid for upper flow regime, when suspended sediment transport is dominant. In order to consider the instability of bed form in upper flow regime, a module to compute suspended sediment transport was incorporated in proposed numerical model. Considering suspended sediment transport, the bed deformation rate can be computed using following sediment continuity equation:

$$\frac{\partial y_b}{\partial t} + \frac{1}{1-\lambda}\left[\frac{A_3}{A_2}(p_d - p_s)d + (q_{sui} - w_f c_b) \right] = 0 \quad (3)$$

where y_b = bed elevation, λ= porosity of sediment particle, A_2, A_3 = shape coefficients of sand grain, p_s = sediment pick up, p_d = deposition rate, d = grain diameter, q_{sui} = upward suspended sediment flux per unit area, w_f = falling velocity and c_b = reference concentration of suspended sediment.

Itakura and Kishi[29] employed an impulse equation to describe detachment of a particle on the bed with a certain initial velocity that is influenced by lift force. This assumption is based on the idea that characteristic lift velocity of a near-bed sediment particle can be determined in terms of initial velocity of a saltating particle as in the bedload transport model. As a result, they derived a relationship to calculate suspended sediment pick up[21].

3. Model Performance and Discussion

Proposed numerical model was validated using laboratory observations on bed form evolution process and their physical characteristics. Numerical model was verified against movable bed experiments in order to assess model performance in simulating bed form evolution and predicting geometric characteristics. We used movable bed experiments performed by Venditti et al.[30] (**Table 1**). In their experiments, they observed five different flow stages that were both sub-critical and fully turbulent. The two lower stages are at or below the threshold for sediment motion and plot near the boundary between ripples and dunes on bed form existence diagram proposed by Southard and Boguchwal[30]. The three higher flows are well above the threshold and fall within the range in which dunes are predicted to occur. They recorded the migration rate of bed forms as well. They found that spatially averaged crest migration rate shows a decrease during bed

Table 1. Experimental conditions and results

Case	Duration [min]	Discharge [l/s]	Flow-depth [cm]	Slope	Grain size [mm]	Froude number	Dune length [cm]	Dune height [cm]	Migration rate [cm/min]
A	90	75.9	15.2	0.0012	0.5	0.41	117.2	4.8	3.9
B	90	72.3	15.2	0.0011	0.5	0.39	86.0	4.2	2.2
C	90	69.6	15.3	0.0007	0.5	0.37	95.4	3.6	2.0
D	90	61.1	15.3	0.00055	0.5	0.33	38.3	2.2	1.0
E	90	54.6	15.3	0.00055	0.5	0.29	30.0	2.0	0.6

evolution at the two largest flow strengths (Case A and B). The experimental results are presented in **Table 1**.Bed form characteristics presented in **Table 1** were recorded once they attained nearly equilibrium state. All these equilibrium values were found to be reached after all but 1.5 hours.

In our previous study [21] we performed computation of flow and turbulence characteristics over a fixed dune in order to validate the hydrodynamic module. Effect of vertical grid spacing on flow and turbulence characteristics was examined. It was found that the time-averaged streamwise and vertical velocity profiles in all regions over dunes are well reproduced. While some discrepancies were noticed in respect of streamwise normal stress, particularly in separation region behind the dune crest. Model severely underpredicted this component. It appeared that model is unable to resolve turbulence in small scales when compare to three-dimensional solver. According to some researchers, LES model could be more accurate; however it is extremely computationally intensive, therefore inappropriate to be applied for morphodynamic computation. We attempted to make some improvement using exponentially stretched grid in vertical direction preserving same number of grids as in case of uniform spacing. Consequently, alteration in grid distribution pattern does not affect computational effort. As a result, prediction of velocity components was somewhat improved. Particularly, components of turbulence stresses appeared to be reproduced adequately by the model with vertically stretched grid spacing[21].

In present study, it is additionally shown that model with non-uniform grid spacing significantly improves the results on bed form evolution. Furthermore, we have tested the performance of model with different turbulence closures. Moreover, the computation has been conducted to assess the effect of calculation domain and initial perturbation.

3.1 Effect of grid distribution pattern on bed form evolution

An attempt was made to simulate sand wave formation and migration process from a small initial perturbation to examine model performance for different cases of vertical grid spacing. It is revealed that vertical grid spacing pattern possesses significant impact on model performance, particularly for morphodynamic simulation. Bed form evolution was poorly simulated by numerical model for the case when vertical grid was distributed uniformly. On the other hand, when same number of grid was stretched exponentially in vertical direction with fine spacing near the bed, the result was found to be improved dramatically. Bed form evolution and geometric characteristics appeared to be more realistic and sustainable. Simulation of bed form evolution showed that computation with stretched grid spacing can replicate realistically some key physical features observed in natural rivers during bed form evolution, namely amalgamation, asymmetric dune shapes and celerity variation. A typical example of some selected instantaneous bed form evolution pattern simulated by proposed model is depicted in **Fig.1**. Likewise, a qualitative comparison between bed forms evolution for a typical case, simulated with both grid-spacing patterns can be made from the same figure. Simulation results clearly showed qualitatively and quantitatively improved bed form evolution features for the case with non-uniform vertical grid.

3.2 Simulation of bed form geometry and migration

Numerical computation of bed form growth and geometry was performed for all experimental cases and compared with observed quantities. Simulation results for some selected cases are illustrated in **Fig.2**. In addition, quantitative comparison between observed and computed bed form characteristics (wavelength and height) for all experimental cases after analogous time increment is depicted in **Fig.3** and **4**. The verification of numerical results against observed quantities yielded reasonable agreement.

Model simulation on bed form migration also showed satisfactory result. The computation result of dune migration in equilibrium state showed agreement with their experiments except for the case with low flow stages (**Fig.5**). The reason is not clear why it possesses inaccuracy for lower stage. From the

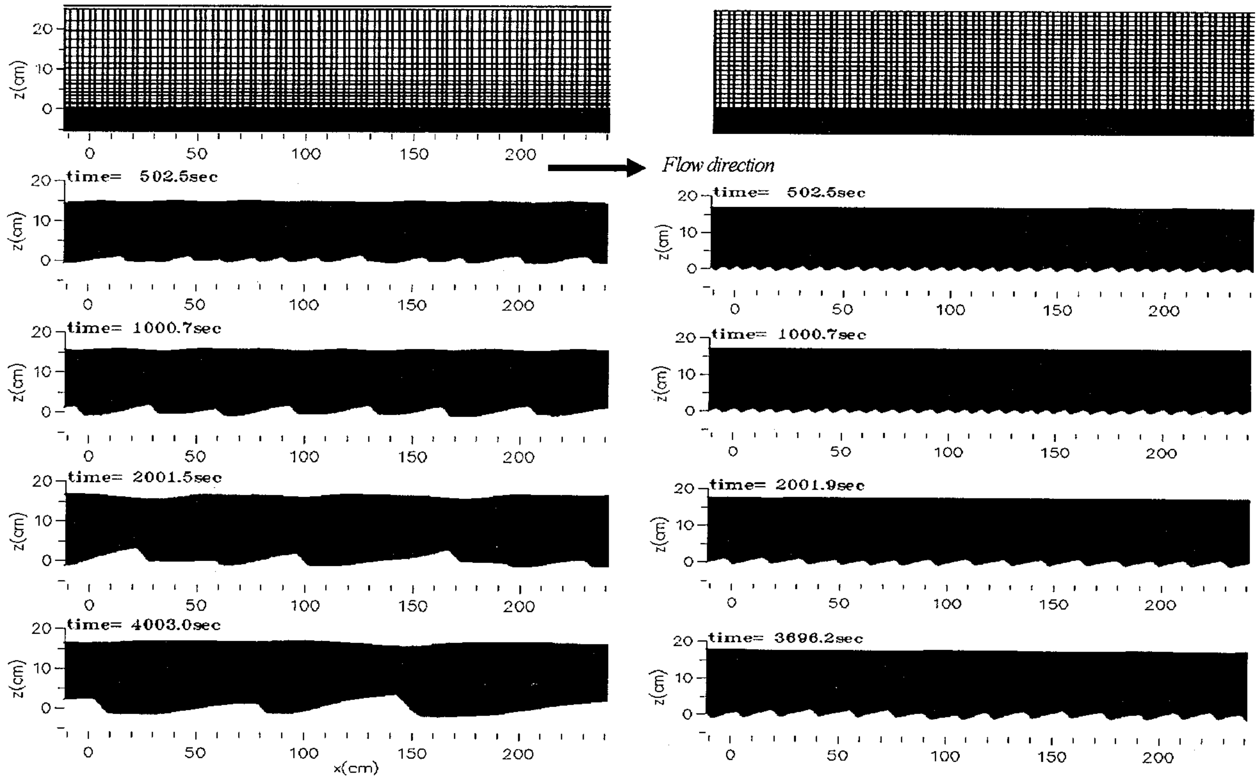


Fig. 1 Typical bed form evolution pattern with uniform (right) and non-uniform (left) vertical grid distribution (Observed dune length and height for this case after 1.5 hrs are 90cm and 4.2cm respectively)

simulation result, it can be identified that bed form celerity is higher in the beginning and decreases with the growth of bed form geometry (**Fig.6**). Most laboratory and field observations revealed similar feature of bed form celerity. For Instance, Venditti et al.[30] observed in their experiments decrement of migration rate with respect to the temporal development of bed form. They measured migration rates in initial as well as equilibrium stages of bed form development (Table 3 in Venditti et al.[30]). They reported that for their experiments A and B migration rate showed a decrease from 1.5 to 0.2 mm/s during 6000 seconds. It is also found from our computation that there is a decrease in migration rates from 1.0 to 0.3 mm/s for these experiments, which is almost of the same order.

Overall, numerical simulation of dune evolution showed fairly identical characteristic of bed forms as observed in laboratory. It can be inferred that the prediction capability of proposed computational model is acceptable considering sophisticated nature of the problem.

3.3 Effect of turbulence models on bed form evolution

An attempt was made to assess the effect of turbulence models on bed form morphodynamic simulation. We used three types of turbulence closures in this computation, namely nonlinear k-ε, standard k-ε and zero-equation models. It was found that no noticeable distinction between simulation results with standard and nonlinear k-ε closures, in particular for morphodynamic simulation. From **Fig.7**, it can be seen that feature of bed forms differs very slightly. Moreover, physical characteristics of bed forms appear nearly same. For the sake of comparison, some simulations were conducted with zero-equation turbulence closure. In this case, model could not reproduce bed form evolution adequately (**Fig.7**). In effect, computation results pointed out towards the importance of grid spacing rather than turbulence asymmetry, particularly for morphodynamic simulation. On the other hand, simple turbulence closure, such as zero-equation model appeared to be inappropriate for the computation under similar condition. In **Fig.7**, it can be seen that computation with zero-equation model is not able to replicate flow resistance and bed form evolution adequately.

3.4 Effect of length of calculation domain

Since a periodic boundary condition has been used in present computation, it is necessary to assess the effect of length of calculation domain on simulation result. We used two different domain length for one of the experimental cases (case A), namely 4.7m and full length of flume (15.2m). The comparison is illustrated in **Fig.8**. From this comparison, it can be inferred that model is practically insensitive to the size of calculation domain.

3.5 Effect of initial perturbation

Initially, the computation of bed form evolution was performed by adding a very small random field of perturbation onto the sand bed. Venditti et al.[31] ascribed the initiation process to a Kelvin-Helmoltz instability and reintroduced a

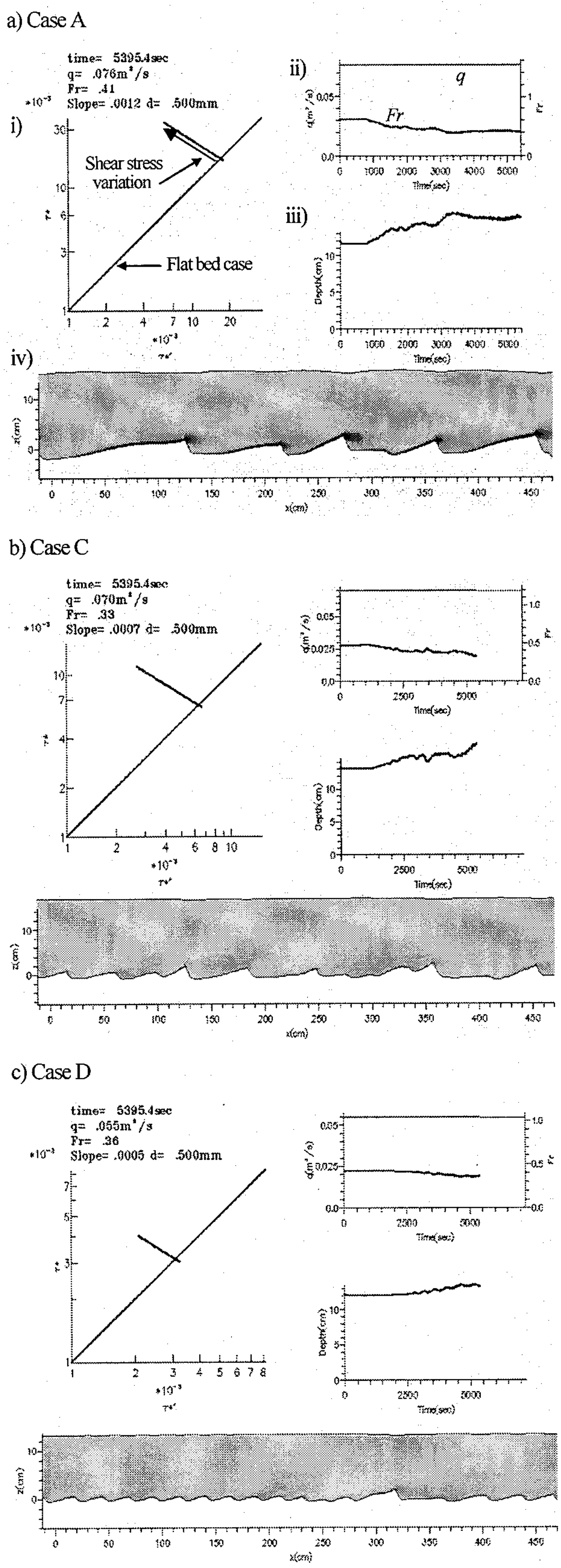


Fig.2 Numerical simulation of evolution of bed form characteristics: i) relation between total bed shear stress (τ*) and grain shear stress (τ*'), ii) discharge and Froude number variation, iii) flow-depth variation, iv) instantaneous bed form evolution

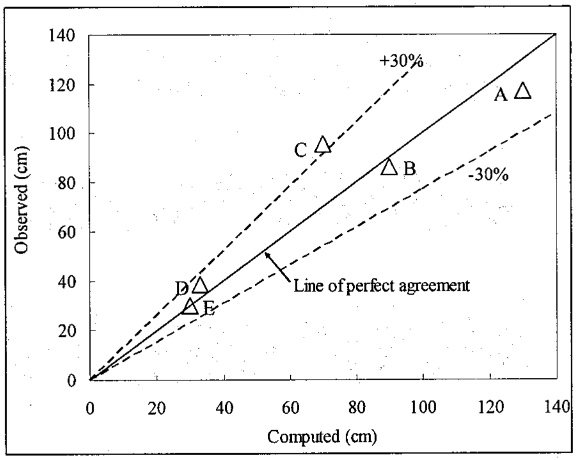


Fig.3 Comparison between simulation results and observed values on bed form length (five plotted points indicate five experimental cases)

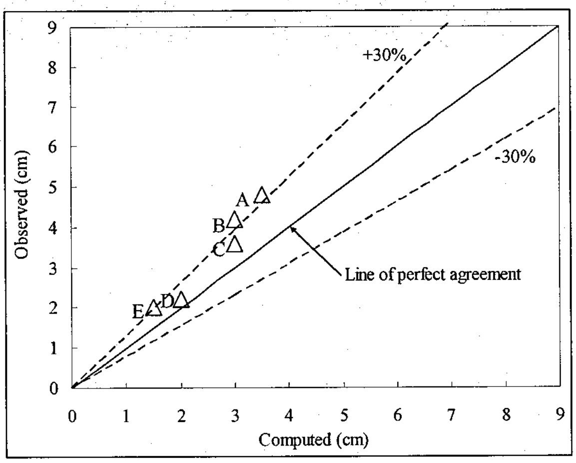


Fig.4 Comparison between simulation results and observed values on bed form height

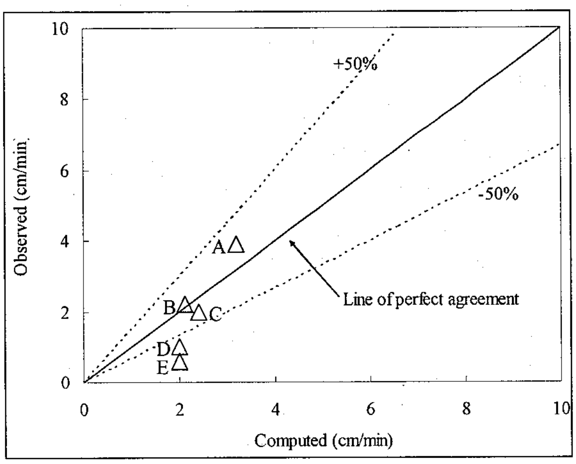


Fig.5 Comparison between simulation results and observed values on bed form migration rate (five plotted points indicate five experimental cases)

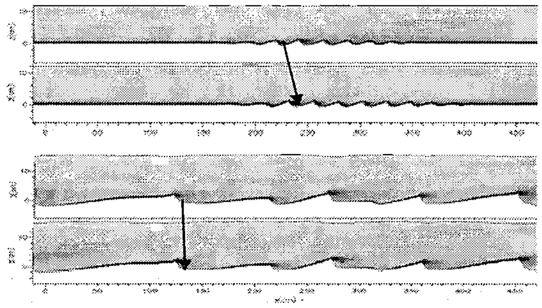
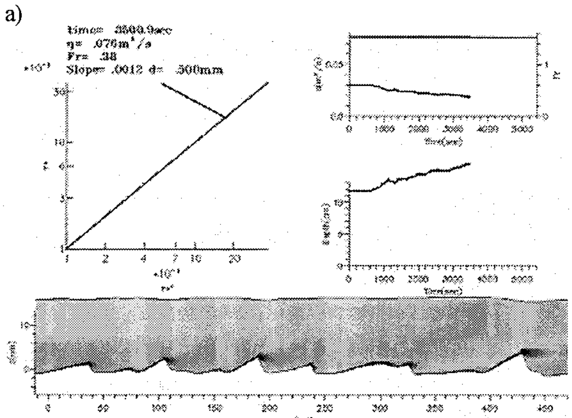
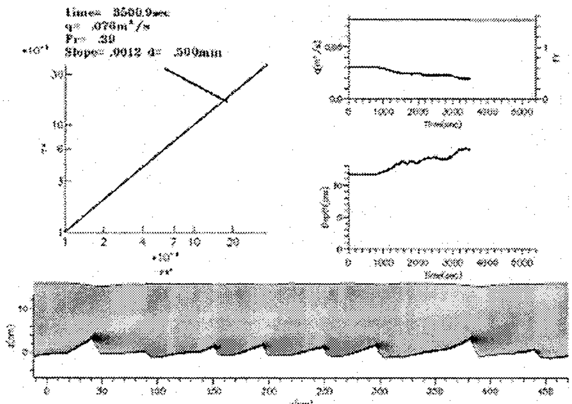


Fig.6 Evaluation of bed form migration during 100sec for initial stage (upper) and latter stage (lower)

a)



b)



c)

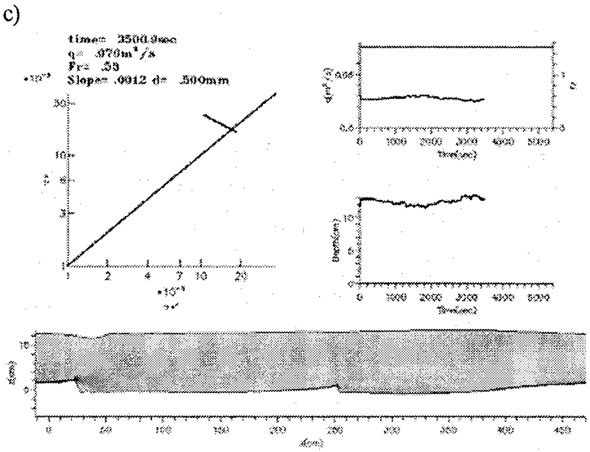
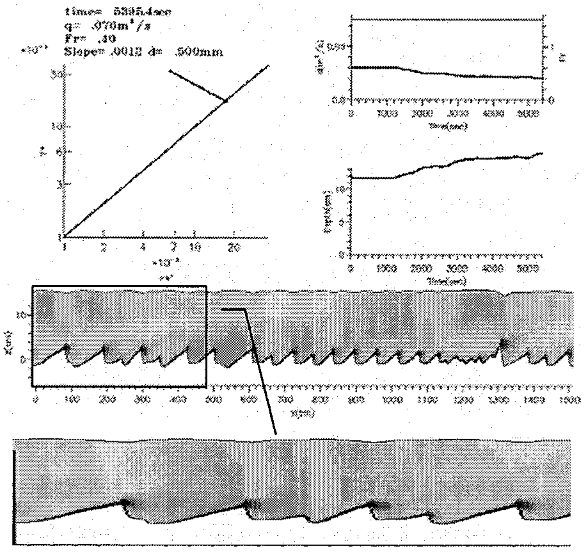


Fig.7 Simulation of bed form characteristics with a) linear k-e model b) nonlinear k-e model and, c) zero-equation model

a)



b)

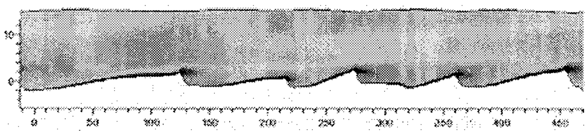


Fig.8 Effect of calculation domain on simulation result: a) simulation with domain length 15.2m and, b) 4.7m

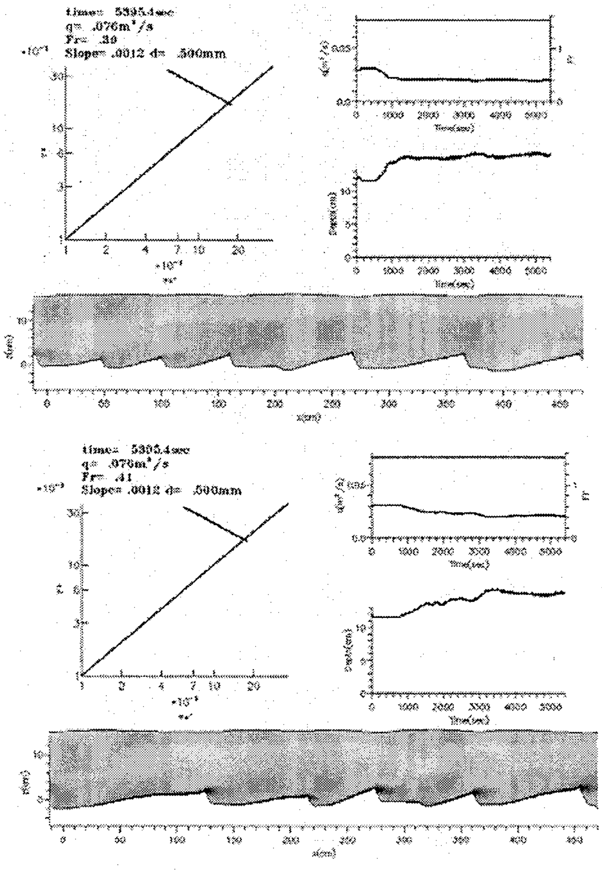


Fig. 9 Simulation with initial perturbation based on K-H instability (upper) and random field of perturbation (lower)

simple quantitative model for hydrodynamic instability to explain bed form initiation and validated with the results of their experiments. They proposed a relationship for interfacial instability to determine the initial wavelength at which the interface becomes unstable. We used the K-H model proposed by them to set the initial wavelength as an initial sinusoidal perturbation and then allow it to grow using our simulation technique. We tested this idea for one of the experimental cases (case A). The result is depicted in **Fig.9**. Some small distinction can be seen in pattern of depth variation, final flow-depth and characteristics in equilibrium condition are quantitatively identical for both cases of initial perturbation.

3.6 Simulation of bedform-induced flow resistance and flow-depth

Flow resistance is associated with skin friction of sediment particles and form drag exerted by bed form. In case of flat bed, effective shear stress is equivalent to the grain shear stress. Contribution of form drag becomes significant in the presence of bed form due to the flow separation and pressure variation. An adequate determination of bed form-induced resistance to flow that accompanies flow-depth variation is essential from practical engineering point of view. Proposed model is able to replicate bed shear stress variation associated with form drag exerted by temporal growth of bed form. Almost all figures with simulation results depicted in previous sections represent the feature of simulated evolution of effective shear stress, in other words, relationship between effective shear stress and grain shear stress. In the same figures, Froude number and depth variations associated with temporal bed form growth are also depicted (see **Fig.2*a*** as an example).

A quantitative comparison was also made to evaluate the model simulation on bed form-induced flow-depth. Computed flow-depth after dune formation was found to be in good agreement with observed values (**Fig.10**). It should be emphasized that the bed form-induced flow-depth prediction can be improved using vertically stretched grid spacing. The most likely reason is the improvement in prediction of bed form geometry in case of non-uniform vertical grid.

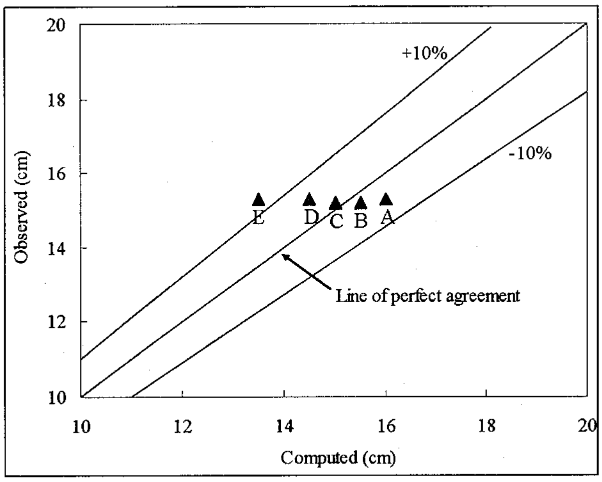


Fig.10 Comparison between simulation results and observed values of flow-depth (five plotted points indicate five experimental cases)

4. Conclusion

A vertical two-dimensional morphodynamic computation model was proposed and developed that can capture most of the flow characteristics as well as morphodynamic features of bed form in physically based manner. Model successfully simulated geometric characteristics as well as evolution mechanism of bed form. It was found that grid spacing pattern in vertical direction produces significant impact on model performance for morphodynamic simulation. Computation with exponentially stretched grid in vertical direction showed significantly improved results. Model with non-uniform grid spacing was able to reproduce some important features of bed form evolution, namely asymmetry shape of bed form, amalgamation and variation in celerity according as bed form growth. It is to be noted that the number of grid was kept to be same as in case of uniform grid spacing so as to retain the calculation time.

Model performance on bed form migration simulation showed satisfactory results. Bed form celerity in both numerical simulation and physical observations was found to be varied according as the growth of bed forms geometry. The model performance was evaluated for different length of calculation domain and found it insensitive. Likewise, bed form evolution process was simulated with two kind of initial perturbations on sand bed- a random field of perturbation and other one is consistent with K-H instability model.

With regard to the effect of turbulence models, no noticeable distinction was found for morphodynamic simulation, particularly when a non-linear k-ε model was compared with a standard k-ε closure. On the other hand, computation with zero-equation model showed poor results for bed forms morphodynamic simulation. This leads to the fact that significance of turbulence model in the context of bed form morphodynamics computation seems to be ambiguous and, thus, needs to be explored more comprehensively.

Simulation result of water surface oscillation due to form drag exerted by migrating bed form was found to be realistic. Bed form-induced flow-depth was predicted with good accuracy for most laboratory observations. Variation of effective bed shear stress as well as flow-depth according as bed form evolution can be assessed using proposed model. These results suggest that free surface condition is a prerequisite that enables water surface oscillation over migrating bed form to be reproduced realistically.

Additionally, a module to compute suspended sediment flux, incorporated in proposed model, might be relevant to consider the bed form instability during upper flow regime.

However, in present study, model was not verified for upper flow regime when suspended load is dominant. This condition will be explored in future investigations.

This study has demonstrated the scope and usability of a numerical simulation technique that could be a supplementary tool to be applied to river engineering practices.

Acknowledgement

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Appendix

(1) Flow equations

The governing equations for vertical two-dimensional flow in cartesian coordinate system reads as:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (4)$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\frac{\partial}{\partial x}\left(-\overline{u'u'}\right)+\frac{\partial}{\partial y}\left(-\overline{u'v'}\right) \qquad (5)$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\frac{\partial}{\partial x}\left(-\overline{u'v'}\right)+\frac{\partial}{\partial y}\left(-\overline{v'v'}\right)-g \qquad (6)$$

where x and y = coordinates in horizontal and vertical direction respectively; u and v = components of velocity in horizontal and vertical direction respectively, $-\overline{u'u'}, -\overline{u'v'}$ and $-\overline{v'v'}$ = Reynolds stress tensors, ρ= fluid density, g = gravitational acceleration, p = pressure.

Eq. (4)-(6) are transformed from x, y, t cartesian coordinate system to a moving boundary fitted ξ, η and τ coordinate system[4].

Pressure term in momentum equations can be computed considering non-hydrostatic component as follows:

$$p=p_0+p'=g\int_y^H \rho\, dy+p' \qquad (7)$$

Eq.(4) can be rewritten as:

$$p=\rho g(H-y)+p' \qquad (8)$$

in which, p'= non-hydrostatic component of pressure, H = the location of free surface. Calculation is performed substituting pressure term (p) by Eq.(8) in momentum equations, consequently, computing the non-hydrostatic component.

In present study, the kinematic condition is imposed along the free surface (at y = H) to compute the temporal water surface elevation. The kinematic condition is expressed as follows:

$$\frac{\partial H}{\partial t}+u\frac{\partial H}{\partial x}=v \qquad (9)$$

$$H=y_b+h \qquad (10)$$

where y_b = bed elevation and h = local flow depth.

(2) Sediment transport equation

$$p_s\sqrt{d/(\rho_s/\rho-1)g}=0.03\tau_*(1-0.035/\tau_*)^3 \qquad (11)$$

Where p_s =sediment pick up rate, ρ and ρ_s = fluid and sediment density respectively and τ_*= dimensionless local bed shear stress.

The sediment deposition rate reads as:

$$p_d=p_s f_s(s) \qquad (12)$$

where p_d = sediment deposition rate and $f_s(s)$ = distribution function of step length.

Distribution function of step length is found to be exponential as follows:

$$f_s(s)=\frac{1}{\Lambda}\exp\left(-\frac{s}{\Lambda}\right) \qquad (13)$$

where Λ = the mean step length and s = the distance of sediment motion from pick up point.

The mean step length can be calculated as $\Lambda = \alpha d$, in which α is an empirical constant and proposed to be 100.

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