

## Strain Estimate Method of RC Columns Subjected to Cyclic Loading

繰返し荷重を受けたRC柱のひずみ推定方法

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A simplified finite element analysis method has been developed in estimating the strain in damaged RC column. In the ordinary step-by-step finite element analysis, starting from the initial configuration of the structural element and from initial conditions such as load and boundary conditions, the strain can be obtained for each intermediate stage. Neglecting these intermediate stages an estimate method is proposed where the crack pattern and crack width of a damaged structure are used for input data. The digital data of crack pattern and crack width are extracted by processing the image of the damaged structure using vectorization. One step analysis is done to obtain the strain in shear reinforcement that will be used in the process of damage estimation.

**Key Words:** crack, strain, shear reinforcement, finite element analysis, damage, RC column

### 1. Introduction

During reversed cyclic loading the degradation of stiffness takes place, which is the damage of the structural members. For the damage estimate of a structural member or a structure several methods such as an expert system and a direct simulation method have been developed. In the direct simulation method, the finite element analysis plays an important role in estimating the damage for each loading stage, starting from the initial configuration of the structure under the load and boundary conditions. In this study a simplified analysis method is proposed to estimate the strain in the shear reinforcement of a RC member subjected to reversed cyclic loading using the crack pattern and the crack width as input data for analysis.

The analysis method is used in particular for the case where shear cracks are observed in the damaged structure because in this case a reliable analysis result is considered to be needed to guess the remaining shear capacity of the structure for the appropriate subsequent treatment.

The strain in shear reinforcement is related to the shear crack width. Some empirical relationships have been proposed to estimate the shear crack width from the strain in shear reinforcement<sup>1),2)</sup>.

Regarding the durability of RC structures, the crack width and its control are important factors to be considered<sup>3),4)</sup>. On the other hand, if RC structures are subjected to earthquake loading and shear cracks occur, then it is also important to estimate the magnitude of loading and the remaining capacity of the structure from the observed crack data. Digital data of crack pattern and crack width can be extracted by processing the image of a damaged RC structure using vectorization. By image

processing indices for density, direction and distribution of cracks can be obtained to characterize the geometrical properties of cracks<sup>5)</sup>. The information given by these indices is used to choose the analysis method and to simplify the crack pattern by selecting the effective cracks in accordance to the analysis method.

In the method presented here, the data of crack width and crack pattern are used in finite element analysis. The crack pattern defines the orthotropic cracked concrete elements and the crack width gives the initial strain in these elements. Initial strain in the longitudinal reinforcement is also considered in the analysis to determine the strain in the shear reinforcement. Knowing the strain in the shear reinforcement an empirical formula will be used to determine the maximum shear force experienced by the reinforced concrete column<sup>6)</sup>. Then the maximum shear force is used to estimate the damage of the structure.

In the direct simulation method the load is necessary as input data to determine the state of the finite elements during loading using constitutive models derived from experimental data. In the case when the load can not be determined and there is no information on the intensity of earthquake the proposed method can be applied to estimate the maximum load using the strain in shear reinforcement.

The degradation of the finite element is given by predetermined constitutive laws in the case of direct simulation method. In the one step inverse analysis method the degradation is given by degradation factors used to decrease the stiffness of the damaged finite elements. The method is applied using different values for the degradation factors until the degradation factors give accurate results in terms of strain in shear reinforcement.

## 2. Structural Member Data for Analysis

An experiment was conducted<sup>7)</sup> where a reinforced concrete column with a square cross-section was subjected to reversed cyclic loading. The column was designed so that shear cracks occur during loading. The column has the size 300×300×1100 mm. The reinforced concrete column has 0.42% shear reinforcement ratio and 0 axial stress. The deflection of the column was measured by displacement transducers at 3 points, 300 mm, 600 mm and 900 mm height from the bottom of the column.

The strain in the reinforcement was measured by using strain gauges placed on longitudinal and transverse reinforcing bars.

The load was increased gradually with 40 kN and 20 kN load step up to yield point of longitudinal reinforcement and then by increasing the rotation angle by 1/200 rad each cycle up to 7/200 rad.

The details of the column and the positions of the strain gauges are shown in Fig. 1 and the material constants are given in Table 1.

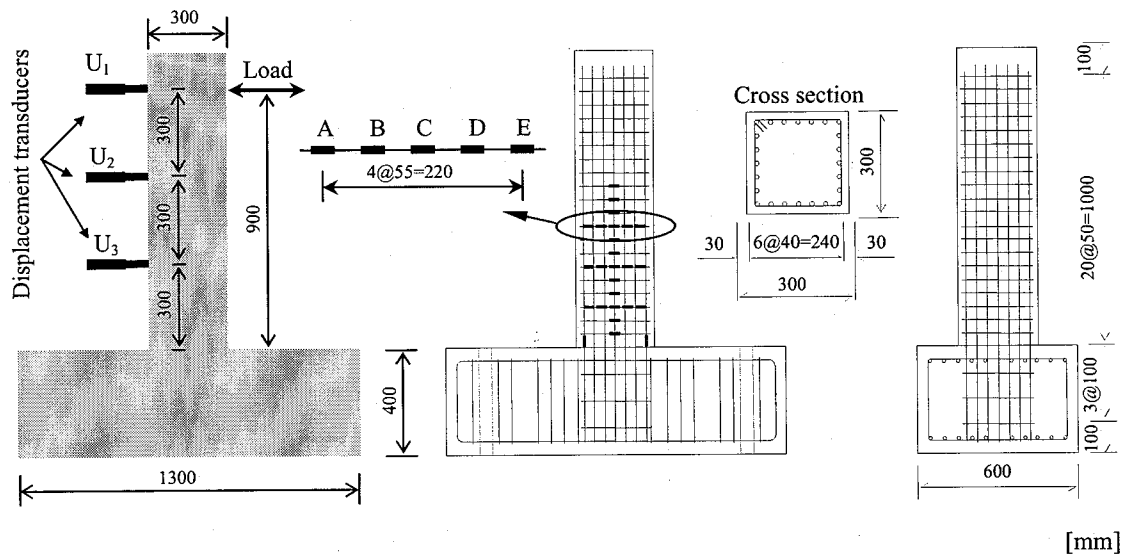


Fig. 1 RC column specimen subjected to reversed cyclic load

Table 1 Material constants

Constants Material	Young Modulus E [GPa]	PoissonRatio $\nu$	Strength		Yield Stress $f_y$ [MPa]
			Compressive $f_c$ [MPa]	Tensile $f_t$ [MPa]	
Concrete	28.1	0.18	30.9	2.57	-
Shear Reinforcement	192	-	-	-	336
Longitudinal Reinforcement	188	-	-	-	391

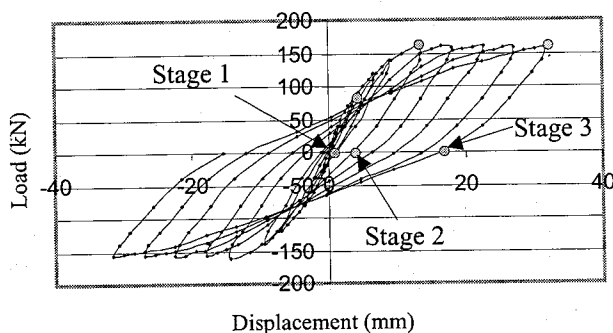


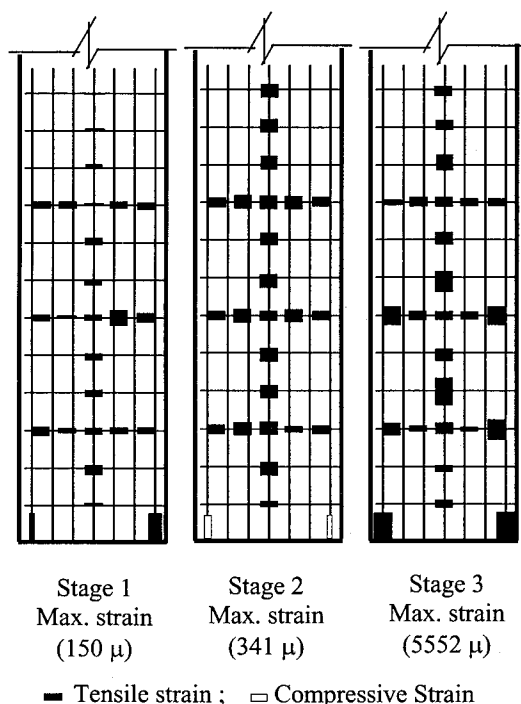
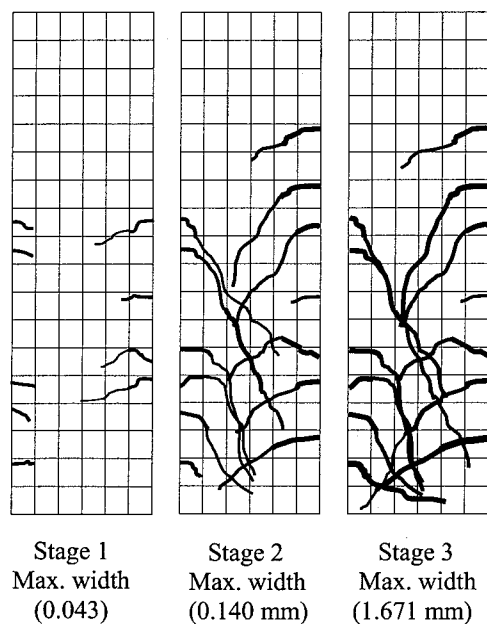
Fig. 2 Load-displacement curve

During the reversed cyclic loading the reinforced concrete column specimen passes through unloaded stages as is shown in the load-displacement diagram presented in Fig. 2. The crack pattern and strain in shear reinforcement for three unloaded stages shown in Fig. 2 are presented in Fig. 3 and Fig. 4 in order to see the connection between shear crack width and strain in shear reinforcement.

Fig. 3 represents the cracks in the three unloaded stages. Each grid represents a 5×5 cm square. In the early stage (stage 1), there are only horizontal cracks by flexure, while in the later stages, near the peak load and after the peak load, diagonal cracks become predominant.

The crack patterns of the unloaded stages are used in order to estimate the initial strains in concrete and reinforcement elements.

The crack width, represented by the thickness of the crack line, gives the information on the magnitude of the residual strain in reinforcement, which is used as the initial strains in the analysis.



**Fig. 4** Measured strains in reinforcement

In **Fig. 4** the strains in reinforcement measured by strain gauges are shown and their magnitude is given by the thickness of the lines. It is observed that the strain in the shear reinforcement increases as the number and

width of diagonal cracks increase. Regarding the longitudinal reinforcement, it is shown that the residual strain at the base of the column increases as the horizontal displacement increases.

**Table 2** Maximum displacements in unloaded stages

Loading Stages	$U_1$ (mm)	$U_2$ (mm)	$U_3$ (mm)
Stage 1	0.47 (3.7)	0.22 (2.14)	0.06 (1.06)
Stage 2	3.58 (13.21)	2.14 (8.42)	1.06 (4.28)
Stage 3	16.46 (31.59)	11.22 (21.34)	6.24 (11.54)

Note: Values in ( ) represent the maximum displacements imposed to the column specimen

The values of the residual horizontal displacements  $U_1$ ,  $U_2$ ,  $U_3$  of the column measured by three displacement transducers are shown in **Table 2**. These displacements are used as the boundary conditions in the analysis described in the following.

### 3. Modeling in Analysis

#### 3.1 2-D Modeling of RC Column

The RC column is modeled by using quadrilateral finite elements and line elements. The side length of the square element used in the analysis is 10 mm. To obtain only the strain in axial direction the shear reinforcement is modeled by using line elements having the same size as the quadrilateral elements. In **Table 3** material data as Young's modulus  $E$ , Poisson ratio  $\nu$ , thickness  $t$ , and area  $A$  are given for the quadrilateral elements and line elements used in the analysis.

The model of the column is made up of 2700 quadrilateral elements, 30 elements in width and 90 elements in height, and 396 line elements. Concrete elements and interface elements are considered isotropic in the initial stage.

**Table 3** Material data for analysis

Elements		Concrete Elements	Steel Elements
Quadrilateral Elements	$E$ [GPa]	2.81	188
	$\nu$	0.18	0.25
	$t$ [mm]	300	0.295
Line Elements	$E$ [GPa]	-	192
	$A$ [mm <sup>2</sup> ]	-	63.34

During reversed cyclic loading concrete elements corresponding to a crack become orthotropic. Interface elements will have reduced stiffness near the intersection between steel and crack by reducing the Young's modulus. Concrete-steel elements become partially orthotropic by the cracking and steel elements remain isotropic. In **Fig. 5** the type of finite elements used in the analysis are presented considering only a part of the column specimen.

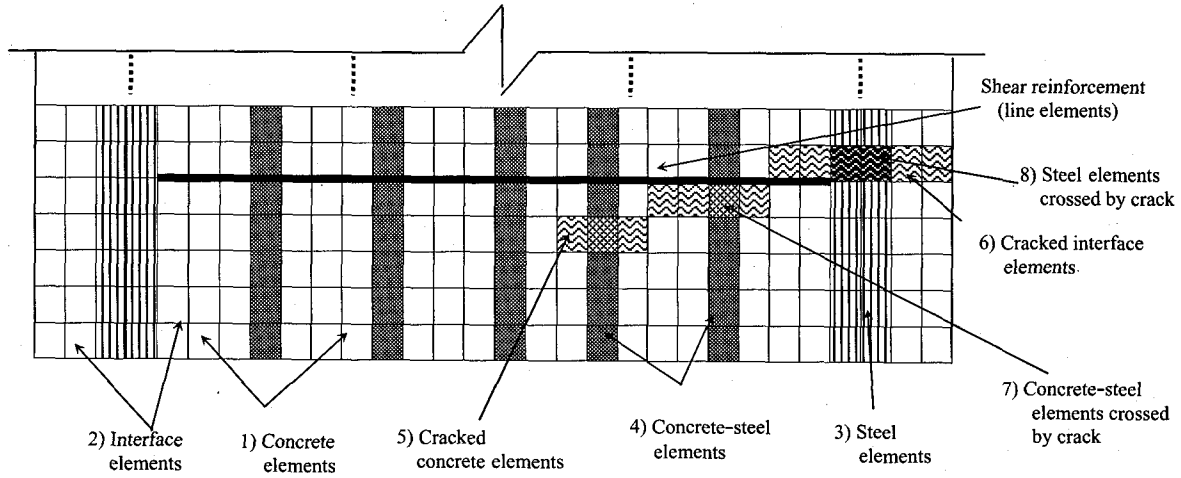


Fig. 5 RC column model

### 3.2 Crack Model

In order to express the stiffness reduction of concrete elements due to cracking, the smeared crack model<sup>8)</sup> is used and the cracked quadrilateral elements become orthotropic as it can be seen in Fig. 6.

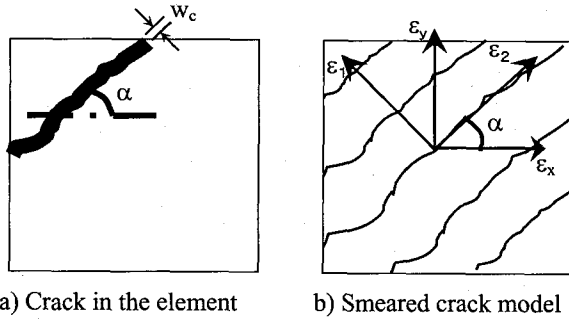


Fig. 6 Cracked element

The crack orientation gives the angle for the matrix transformation from the local to the global coordinate system.

As shown in Fig. 7 a) depending on the crack length inside the element, cracked elements are determined. In Fig. 7 b) cracks with large inclination are shown, while in Fig. 7 c), a crack with small inclination is shown.

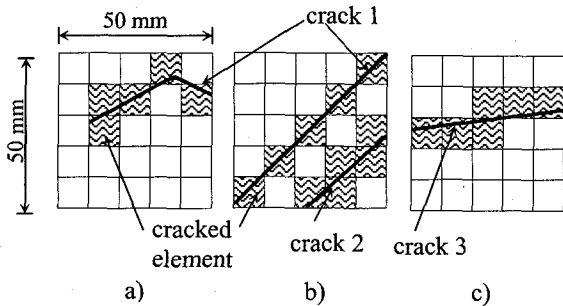


Fig. 7 Orthotropic elements defined by crack

By considering these cases, it is assumed that a square concrete element becomes cracked when the crack length inside the element,  $L_{cc}$ , is larger than  $L/2$  where  $L$  is the element side length. In this case the element side length is 10 mm. If the length of the crack is long enough to consider the concrete element as cracked element the concrete element becomes an orthotropic element and the stress-strain relationship is given as follows:

$$\sigma = D_c^{cr} \varepsilon \quad (1)$$

$$D_c^{cr} = \begin{bmatrix} \alpha_1 E & 0 & 0 \\ 0 & \alpha_2 E & 0 \\ 0 & 0 & \beta G \end{bmatrix} \quad (2)$$

where  $\alpha_1$  is equal to  $10^{-6}$ ,  $\alpha_2$  is equal to 1 and  $\beta$  is the shear transfer factor and is between 0 and 1 depending on shear crack width. For the case of steel-concrete elements the same assumption is made considering the crack length inside the element for the element to become orthotropic but the stress-strain relationship becomes as follows:

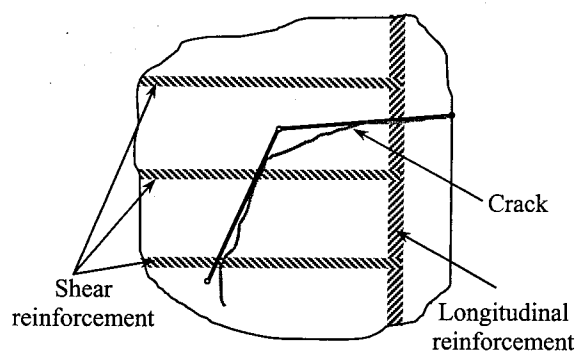
$$D^{cr} = (1 - \alpha) D_c^{cr} + \alpha D_s \quad (3)$$

where  $\alpha$  is a parameter proportional to reinforcement ratio and its value is in between 0 and 1 given by the ratio between thickness of steel  $t_s$  and thickness of concrete  $t_c$  of a finite element. In the analysis  $t_s$  is equal with 0.084 mm and  $t_c$  is 300 mm.

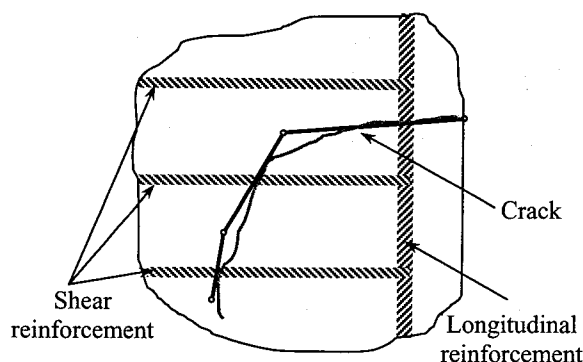
In the case when the crack crosses the longitudinal reinforcement initial strain is introduced in the steel elements and if the strains exceed the yield strain of longitudinal reinforcement the Young's modulus is decreased so that stress in longitudinal reinforcement will not exceed the yield stress. The Young's modulus of the interface element at the intersection between concrete crack elements and steel will be reduced.

### 3.3 Simplification of Cracks

Using vectorization, crack data as crack pattern and crack width are extracted by processing the image of the RC column. Best digital data are obtained by image processing if the problem to be solved is well known so that only specific and effective cracks are to be extracted and used in the analysis. In the analysis shear cracks have been considered to be related to the strain in shear reinforcement and also large flexural crack. These cracks will be selected and the digital data is introduced in the finite element analysis. The simplification of cracks is necessary to create digital data of crack pattern and crack width. The digital data consists in the coordinates of each segment and corresponding crack width.



a) Bilinear crack



b) Multi-linear crack

**Fig. 8** Simplification of cracks

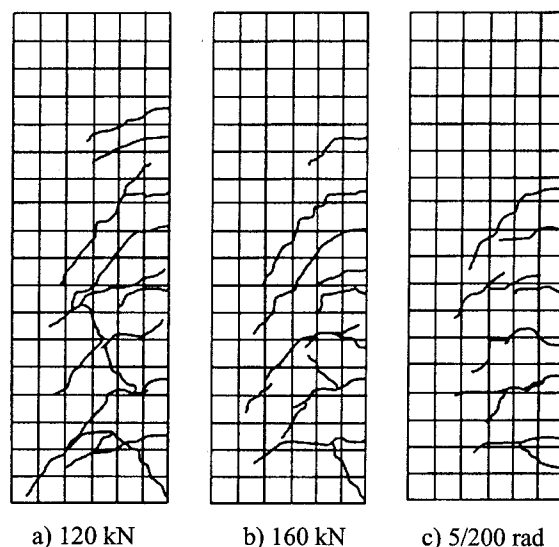
The simplification of a crack consists in the transformation of the crack from the real curved crack to bilinear or multi-linear crack as it can be seen in Fig. 8. The transformation is based on the principle that the lines that will represent the real crack must be tangent to the crack at the intersection points of the crack with the reinforcement so that the angle of the crack is kept the same at the level of the reinforcement. The angle is needed in the analysis for the calculation of strain in the reinforcement that depends on crack direction and width.

## 4. Analysis Results

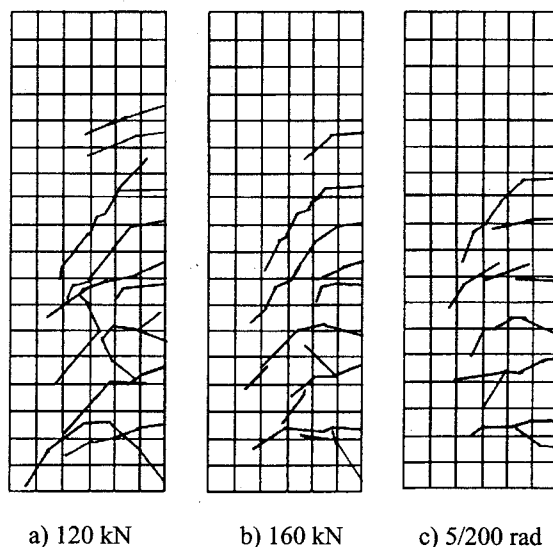
### 4.1 Crack Data Used in Analysis

The analysis has been made considering the column specimen having 0.42% shear reinforcement ratio and no axial load. Three stages have been considered in the analysis: at 120 kN when shear cracks occurred, at 160 kN which is the maximum load, and at 5/200 rad which corresponds with the displacement after the peak load.

In Fig. 9 crack patterns of these three stages are shown and in Figs. 10-12 the simplified crack pattern of these stages are shown in



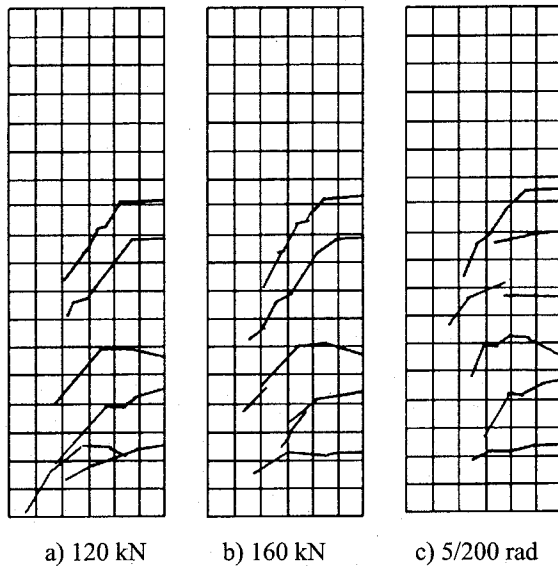
**Fig. 9** Crack patterns during loading



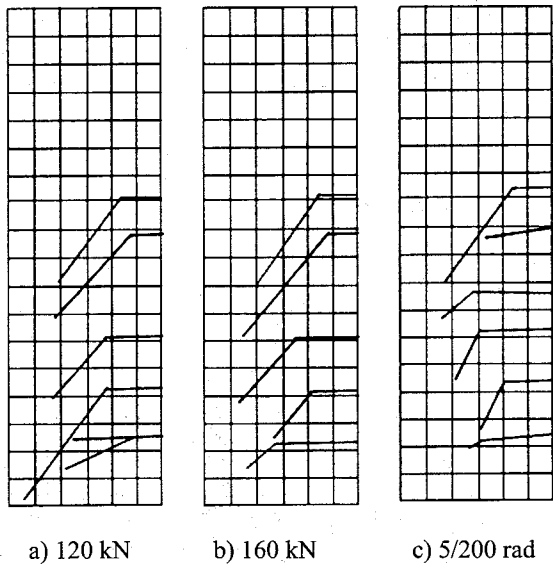
**Fig. 10** Multi-linear representation of all cracks

Fig. 10 presents the multi-linear crack pattern considering all the cracks as in the original crack pattern. In Fig. 11 the cracks are multi-linear but only the cracks considered effective are taken in the analysis. The selected cracks are the shear cracks and the flexural

cracks with large crack width that can have an effect on strain in shear reinforcement.

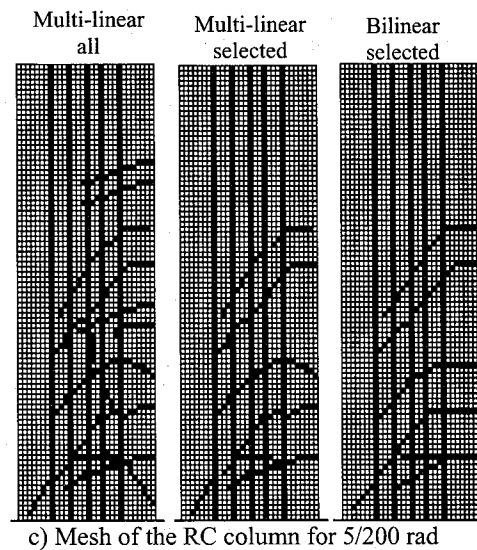
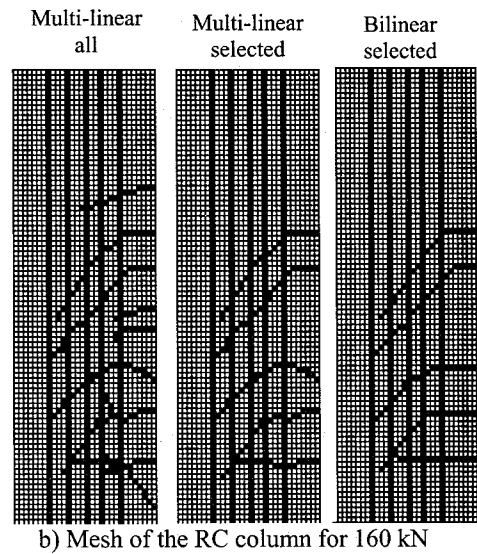
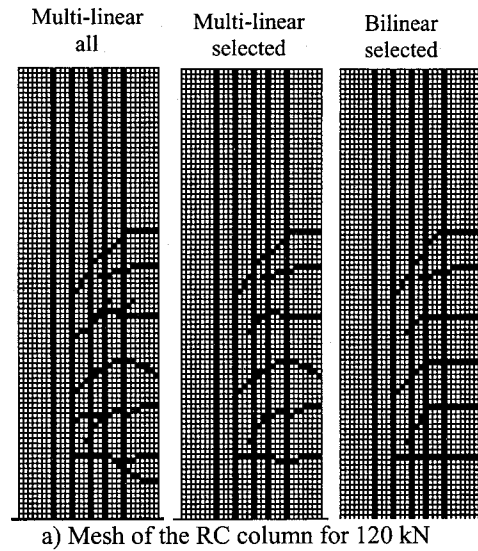


**Fig. 11** Multi-linear representation of selected cracks



**Fig. 12** Bilinear representation of selected cracks

The bilinear cracks are easy to be transformed into digital data and the crack patterns are shown in Fig. 12. Considering the transformed crack patterns the digital data of these cracks are obtained by introducing the coordinates of each segment line and the corresponding crack width. Automatically, the crack angle is determined and initial strain in orthotropic elements depending on crack width can be calculated. The digital data are used to generate the mesh of the finite elements considering the damaged elements given by crack. The meshes for the all three simplified crack patterns are shown in Fig. 13 for the considered stages.



**Fig. 13** Mesh for finite element analysis

## 4.2 Comparison

In the case of one step inverse analysis method the result is strictly related to the degradation rules applied to finite elements.

In this method degradation is represented by the reduction of Young's modulus  $E$  which is multiplied by a degradation factor  $\alpha_d$  and reduction of the shear transfer factor  $\beta$ . To obtain a good agreement between experimental results and analysis results the degradation factor  $\alpha_d$  for interface elements, orthotropic elements, steel-concrete elements and steel elements and shear transfer factor  $\beta$  have been determined using the bilinear crack assumption. These factors represent the degradation rule of the finite elements and Table 4 shows these factors as a result of the analysis. The degradation factor for steel elements is function of initial strain in  $y$  direction  $\epsilon_{iy}$ , modulus of elasticity  $E_s$  and the yield stress  $f_y$ .

**Table 4** Degradation factors

Quadrilateral Elements	Degradation factor	
	$\alpha_d$	$\beta$
Concrete Elements	$10^{-6}$	0.9
Steel Elements	$E_s f_y / \epsilon_{iy}$	-

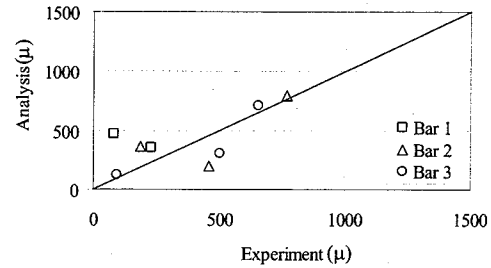
Digital data as crack coordinates and crack width of the simplified crack patterns have been extracted for the all three stages presented before. Considering the degradation factors presented before the analysis has been made for each case and the results are presented in Fig. 14.

The average strain in shear reinforcement from analysis has been compare with the average strain from the experiment. Three shear reinforcement bars with strain gauges at five positions have been considered for comparison. The numbering of the shear reinforcement bars starts from the bottom to the top of the RC column. A perfect bond between shear reinforcement and concrete has been assumed in the analysis.

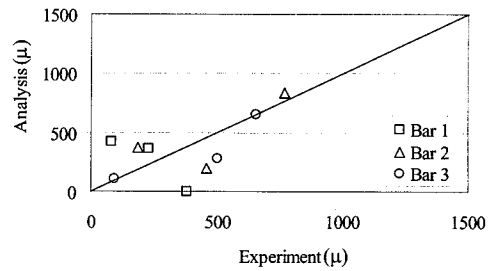
In the case of the selected bilinear cracks the results of the analysis are in good agreement with the data from the experiment. One reason is that in the analysis the selected bilinear cracks were considered to determine the degradation factors due to the simplicity in obtaining the digital input data. Another reason is the methodology used to determine the orthotropic elements and their initial strain. The procedure to determine the orthotropic elements and their initial strain is simple and suitable for the case of selected bilinear crack. The crack line segments are considered one by one and the finite elements crossed by the crack line segments turn into orthotropic elements with initial strain if the condition of crack length inside element is satisfied. If two crack line segments cross the same finite element then only the characteristics of the last line segment are considered in the analysis. Thus, the finite element which contains the link point between two adjacent crack line segments may not turn into orthotropic element because the last crack

line segment considered may not satisfy the length condition.

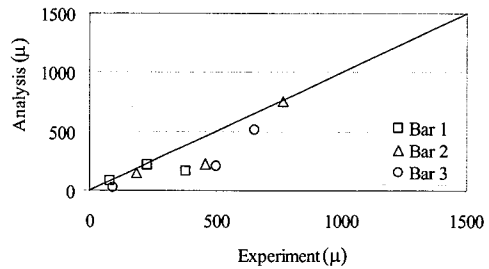
For the case of multi-linear linear cracks the number of finite elements containing link points is proportional with the number of crack line segments and it is obviously larger comparing to the case of bilinear cracks. This may explain why selected bilinear cracks gave better results.



a) All multi-linear cracks



b) Selected multi-linear cracks



c) Selected bilinear cracks

**Fig. 14** Comparison between experimental results and analysis results

## 5. Conclusions

In order to be used as the basic information on the damage of RC column subjected to reversed cyclic loading such as earthquake loading, the strain in shear reinforcement is tried to be obtained by a simplified finite element analysis method. The results of applying the present method to the RC column specimen are summarized as follows.

- (1) By imposing the initial strains corresponding to the crack width, the strain in shear reinforcement can be obtained in agreement with the experimental data.

- (2) The configuration of the crack can be simplified to a bilinear shape so that the crack angle is correctly set at the intersection with the reinforcement.
- (3) For the case of multi-linear cracks the summation of the crack line segments crossing a finite element and summation of initial strain given by each crack line segment must be considered to obtain the proper input data for the analysis.
- (4) Using a simplified one step finite analysis method and assuming a simplified bilinear crack pattern as input data the degradation factors governing the analysis are determined as being constant for finite concrete elements and function of crack width for finite steel elements.

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