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Coupling Key Block Analysis Using Stochastic-Deterministic Method in Discontinuous Rock Masses

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It is difficult to estimate the properties of rock masses before construction. This paper presents a new coupling key block analysis using stochastic-deterministic method for the prediction of the key blocks before all the discontinuity traces of the blocks have not appeared on the cutting face of the tunnel, in other words, in the moment some of the discontinuity traces of the block have appeared. The coupling key block analysis using stochastic-deterministic method developed by the authors is applied to a large tunnel excavation site where real discontinuity information was obtained. In order to illustrate the validity and applicability of this numerical analysis for stability evaluation in tunnel construction, the analytical results are compared and examined.

Key Words: stochastic-deterministic method, key block analysis, rock mass, discontinuity trace

1. Introduction

Rock masses in nature contain numerous discontinuities such as joints, faults, fractures, bedding planes, cracks, schistosities, cleavages, seams and fissures. The behavior of civil structures in hard rocks, therefore, is mainly controlled by numerous discontinuities[1-4]. In many cases, enormous cost and time are consumed to cope with the falling or sliding of rock blocks, which cannot be predicted because of the complexity of rock discontinuities. It is difficult to estimate the properties of rock masses before construction. In the design and construction of civil structures, the observational method has become increasingly important[5,6].

As for the assessment of the rock structure induced failures, the so-called block theory was suggested by Goodman and Shi[7]. Excavations in discontinuous rock masses are frequently affected by key blocks, which are critical blocks of rock bounded by discontinuities and excavation surfaces[8]. The block theory is a geometrically based set of techniques that determine where dangerous blocks can exist in a rock mass intersected by variously oriented discontinuities in three dimensions. The block theory is a useful method to determine the stability of rock blocks that were created by the intersection of discontinuities. However, a key block can be found only after all the discontinuity traces of the block have appeared on the excavation surface. In that moment, the key block is unstable and can move out of the surface.

The authors developed a coupling key block analysis using stochastic-deterministic method for the prediction of the key blocks before all the discontinuity traces of the blocks have not appeared on the cutting face of the tunnel, in other words, in the moment some of the discontinuity traces of the block have appeared. The coupling key block analysis using stochastic-deterministic method developed by the authors is applied to a large tunnel excavation site where real discontinuity information was obtained.

Using the coupling key block analysis using stochastic-deterministic method, the prediction of key block occurrences can be estimated concerning the block surrounded by some discontinuity traces and the cutting face.

2. Stochastic Method

2.1 Introduction

The size, location and frequency of key block occurrences are controlled by the number, size, location and orientation of the discontinuities which intersect the excavation. In order to provide a prediction of expected size and frequency of key block occurrences, it is necessary to consider variabilities of the orientation, size and location of discontinuities.

2.2 Discontinuity Spatial Location and Intensity

Discontinuity spatial location can be described using the coordinates of the discontinuity center point (or discontinuity trace center points). Usually, the absolute location is of less interest than the location relative to other discontinuities, particularly those within the same set. Intensity, the number of discontinuities per unit area or volume, or total joint trace length per unit area, or total joint area per unit volume is then used to describe location. Discontinuity spatial location can be a regular deterministic or stochastic process. The most often used model to define the discontinuity spatial location is Poisson model. In a Poisson model, each discontinuity is generated independently; spatial coordinates of its center are generated through a uniform distribution disregarding the locations of formerly generated discontinuities. The number of discontinuities that would be generated is controlled by only one parameter, an intensity parameter that specifies the average number of discontinuity centers within a unit volume of rock mass.

To determine how many discontinuities would be generated within a specified volume of rock mass, discontinuity intensity is a necessary parameter. In this study, there are three discontinuity intensities, one-dimensional intensity (i_1), two-dimensional intensity (i_2), and three-dimensional intensity (i_3). And,

i_1 = the number of discontinuities encountered by unit length of a borehole that is drilled along the mean direction of discontinuity unit normal.

i_2 = the number of discontinuity centers within the unit area of outcrop or excavation.

i_3 = the number of discontinuity centers within the unit volume of rock mass.

Among the three intensities, i_1 and i_2 are in situ measured parameters. Three-dimensional intensity i_3 would be inferred from i_1 or i_2. Parameter i_3 is of special importance because the number of simulated discontinuities within modeled region is directly determined by it. Regarding the estimation of i_3 for a set of randomly oriented discontinuities, Cacas et al.[9] used a simple relationship, $SI = 2f$ (S is average discontinuity area; I is three-dimensional intensity; f is discontinuity frequency observed along a core). It is obviously an empirical equation. U. S. National Committee for Rock Mechanics, suggested a simple relationship,

$$N_L = N_V A \cos\theta \quad (1)$$

where

N_L is discontinuity frequency along sample line (or borehole); N_V is discontinuity frequency within the volume of rock mass; A is mean discontinuity area; θ is the angle between the discontinuity poles and sample line.

According to this relationship, when assuming that a discontinuity is disc-shaped, one may demonstrate that the relationship between i_1 and i_3 is,

$$i_3 = \frac{4i_1}{\pi E(I^2)} \quad (2)$$

where

$E(I^2)$ is the mean value of the squared discontinuity diameter.

Kulatilake et al.[10] have employed a little varied relationship,

$$i_3 = \frac{4i_1}{\pi E(I^2)E(|\vec{a}\cdot\vec{n}|)} \quad (3)$$

where

$\vec{a}$ = unit normal vectors of individual discontinuities (of a discontinuity set); $\vec{n}$ = the mean unit normal vector of discontinuity set n (notice above equations refer to the discontinuities among one given discontinuity set).

In a Poisson model, if the volume of rock mass of modeled region is V, the three-dimensional intensity of discontinuity set n is i_3^n, the number of generated discontinuities within the modeled region would be[5],

$$N = V\sum i_3^n \quad (4)$$

After the generation of simulated discontinuity network with equation (2) or (3), one may check the discontinuity intensity of the simulated network once again. It is not unusual that the simulated discontinuity network leads to an intensity i_1 that differs from the measured one (i.e., the one which has been used in determining the number of generated discontinuities when simulating the discontinuity network). This may be ascribed to the fact that,

(a) Equation (2) and (3) assume that discontinuities are disc-shaped, but actual discontinuities are not the case. And i_1 is an *in-situ* parameter.

(b) In the two equations, only the mean of areas and orientations are respected. Three-dimensional intensity is not only controlled by the mean of area and orientation, but also influenced by the distribution patterns of them.

To overcome this difficulty, in this study a correcting coefficient, C_n, is added to equation (2). That is, the number of generated discontinuities will be calculated by the following equation,

$$N = \sum \frac{4 V C_n \, i_1{}^n}{\pi E(I_i{}^2)} \qquad (5)$$

Coefficient C_n is one of the calibrated parameters. It will be determined through inverse method. In the inverse approach, C_n (among other calibrated parameters) will be modified in such a way that the simulated *discontinuity* network will produce the same trace length and one-dimensional intensity as the observed. I is three-dimensional intensity.

Regarding discontinuity population, an outcrop or an excavation may provide more accurate information than a borehole or a scan line. The one-dimensional intensity measured along a scan line is often biased by the scan line direction. It is almost impossible to obtain an acceptable one-dimensional intensity if the scan line is sub-parallel to the discontinuity plane; and it is not easy to prevent a scan line from being parallel to the discontinuity plane when the number of discontinuity sets is moderately large. When the discontinuities of a rock mass are mapped on some excavation surfaces (or outcrop) and some boreholes which are not sub-parallel to the excavation, this study recommends the following equation to estimate the number of discontinuities existing in a given volume of rock mass (V).

$$N = \sum C_n \, i_2 \, V f_1 \qquad (6)$$

where

C_n is a correcting parameter,

f_1 is the discontinuity frequency along the boreholes.

Naturally, in this study, either equation (5) or (6) only provides an initial estimation of discontinuity numbers (when there are mapped outcrops or excavations use (6); when there are only boreholes and scan lines use (5)). The exact number of discontinuities in a given volume of rock mass will be determined through an inverse approach.

2.3 Discontinuity Orientation

Discontinuity orientations in three-dimensional space are represented by two parameters, the strike and dip (or dip direction and dip). Discontinuity orientations are not purely random. Discontinuity orientations are usually stochastic and probability distributions are used to describe them. Usually, many of the discontinuities observed in a single outcrop are approximately parallel to three to five planes. These discontinuities, which have approximately the same orientation, constitute a discontinuity set. In most cases, various discontinuity sets are *investigated* separately. And, in most studies, if not all, discontinuities are clustered into sets with regard to their orientations. Fisher distribution, Bingham distribution, Bivariate Fisher distribution, Bivariate Bingham distribution, Bivariate Normal distribution, and Empiric distribution are the commonly employed distribution forms to represents the distribution of discontinuity poles.

Fisher distribution is the most often employed to represent the distribution of discontinuity orientations. In this section, the application of Fisher distribution will be discussed in detail. As shown in Fig.1, a discontinuity may be represented by its unit normal OP. Suppose that θ is the angle between the Z-axis and the line of OP in the clockwise direction while ϕ is the angle between the X-axis and the line OQ in the anti-clockwise direction where the point Q is the foot of perpendicular P on XOY plane.

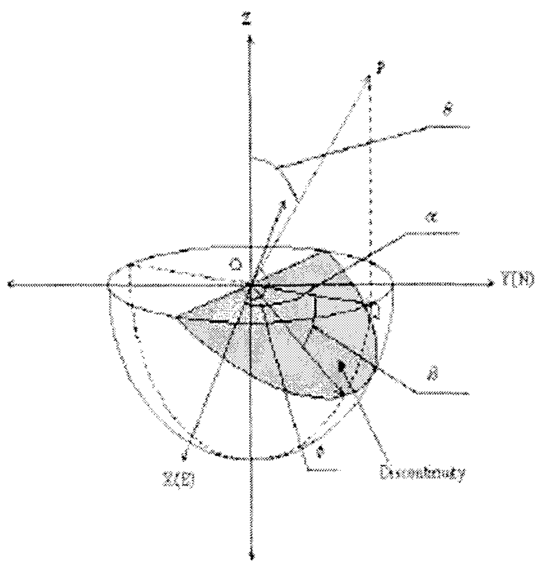

Fig.1 Coordinate System of a Discontinuity Pole

After rotating the Z-axis to the direction of (θ_{mean}, ϕ_{mean}), the density function of Fisher distribution in the new coordinate system (θ^*, ϕ^*) is[11],

$$f_\kappa(\theta^*) = \frac{\kappa}{2\sinh\kappa} e^{\kappa\cos\theta^*}\sin\theta^* \qquad 0 \le \theta^* \le \pi, \kappa \ge 0 \tag{7}$$

$$f(\phi^*) = \frac{1}{2\pi} \qquad 0 \le \phi^* \le 2\pi \tag{8}$$

The following property of Fisher distribution makes it very simple to generate Fisher distributed random numbers. And, this may be a reason that leads to Fisher distribution being widely accepted. The property is,

$$\Pr(\theta_1 < \theta < \theta_2) = (e^{\kappa\cos\theta_1} - e^{\kappa\cos\theta_2}) / (e^{\kappa} - e^{-\kappa}) \tag{9}$$

$$\Pr(\phi_1 < \phi < \phi_2) = (\phi_2 - \phi_1)/2\pi \tag{10}$$

where

$\Pr(\theta_1 < \theta < \theta_2)$ is the probability when $\theta < \theta < \theta_2$.

The relationship between the new coordinates (θ^*, ϕ^*) and the old coordinates (θ, ϕ) are,

$$l^* = (\cos\theta_{mean}\cos\phi_{mean})l + (\cos\theta_{mean}\sin\phi_{mean})m - (\sin\theta_{mean})n \tag{11}$$

$$m^* = -(\sin\phi_{mean})l + (\cos\phi_{mean})m \tag{12}$$

$$n^* = (\sin\theta_{mean}\cos\phi_{mean})l + (\sin\theta_{mean}\sin\phi_{mean})m + (\cos\theta_{mean})n \tag{13}$$

where

(l, m, n) and (l^*, m^*, n^*) are the direction cosines corresponding to (θ, ϕ) and (θ^*, ϕ^*) respectively.

As shown in Fig.1, they are defined by,

$$l = \sin\theta\cos\phi \tag{14}$$

$$m = \sin\theta\sin\phi \tag{15}$$

$$n = \cos\theta \quad (0 \le \theta \le \pi, 0 \le \phi \le 2\pi) \tag{16}$$

In geology, the data of discontinuity orientations are often given in dip direction and dip angle (α, β), not in (θ, ϕ). As shown in Fig.1, if the X-axis is oriented to the East and the Y-axis to the North, the relationship between (θ, ϕ) and (α, β) is,

$$\phi = 2\pi - (\alpha - \pi/2), \theta = \beta \tag{17}$$

Fisher distribution is specified by only one parameter κ, which is a concentration parameter: for κ = 0, the polar points are uniformly distributed; for large κ, the polar points are concentrated on a small portion of the sphere around the pole of the mean direction.

To generate a set of Fisher-distributed discontinuity orientations with specified mean dip direction and dip angle includes the following main steps,

(1) First, compute the mean pole (θ_{mean}, ϕ_{mean}) from the mean dip direction and dip angle with equation (17).
(2) Generate random-number pairs that follow the specified Fisher distribution (specified by given κ), i.e., (θ_i^*, ϕ_i^*), i=1,2,·, n (n = the number of discontinuities). The two random-number θ^* and ϕ^* are generated independently. Random variable ϕ^* is uniformly distributed from 0 to 2π.
(3) Rotate the coordinate system back to the old system to calculate (θ_i, ϕ_i) from (θ_i^*, ϕ_i^*) through equation (11) to (16). In the old coordinate system, the X-axis is oriented to the East and the Y-axis is oriented to the North, and the Z-axis is upward.
(4) Finally, transform discontinuity poles (θ_i, ϕ_i) to dip direction and dip angle (α, β) with equation (17).

3. Coupling Key Block Analysis Using Stochastic-Deterministic Method

3.1 Introduction

All discontinuity geometric characteristics may be defined either deterministically or stochastically. A stochastically modeled discontinuity network offers potential for more realistic assessment of stability status in underground excavations than predictions based entirely on deterministic discontinuities. The reliability of probabilistic models, however, depends strongly on an accurate estimation of the models variables, i.e., the discontinuity network properties from the field and laboratory observations.

In this paper, a rock discontinuity system consists of some observed deterministic discontinuities and some stochastic discontinuities. Deterministic discontinuities refer to the discontinuities whose geometric parameters (location, intensity, orientation, shape, size, etc.) have been determined through certain survey techniques.

3.2 Target Polygon for the Analysis

It is essentially to predict the existence of key blocks which would be formed by the excavation. The geometry of a key block on the tunnel wall is in the form of a finite polygon which is surrounded by some discontinuity traces. Therefore the geometry of that key block which appears partially on the wall, is in the form of a polygon which is surrounded by some discontinuity traces and a face (Fig.2).

The coupling key block analysis using stochastic-deterministic method is applied to such a polygon. The probability of a removable block occurring by the next excavation can be calculated using rock discontinuity distribution data which are obtained from discontinuity survey on the already-excavated surface.

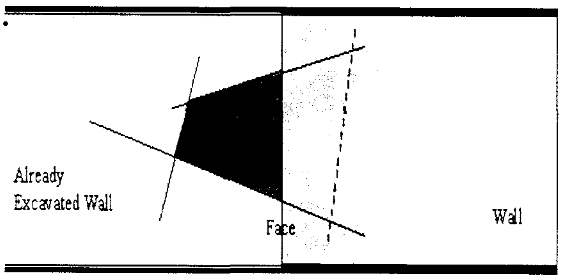

Fig.2 View of Excavation in Tunnel

3.2.1 Types of the Target Polygon for the Analysis

In general, two discontinuity traces (the upper discontinuity *A*, the lower discontinuity *B*) of the target polygon intersect the face. Two discontinuity traces are interconnected directly or connected by some discontinuity traces (linking up discontinuities *L*) on the already-excavated wall.

The target polygon for the key block analysis is divided into the divergence type and the convergence type on the basis of the directional relationship between the discontinuity *A* and the discontinuity *B* on the wall which will be exposed by the next excavation. If the objective polygon is a part of the key block, it is necessary that one of the following events, U_d and U_i, is at least occurred;

(1) Direct connection case (U_d):
Discontinuity *A* and discontinuity *B* are interconnected directly on wall.

(2) Indirect connection case (U_i):
Discontinuity *A* and discontinuity *B* are connected by hidden discontinuity *H* on wall.

3.3 Coupled Stochastic-Deterministic Simulating Procedure

The coupled stochastic-deterministic simulating procedure process consists of four main steps:

(1) Input observed deterministic discontinuities
(2) Construct random variables generator
(3) Simulate rock discontinuity system with Monte Carlo Method
(4) Improve variables generator with inverse method

When determining the criterion for the good match between simulated and observed discontinuities the following factors are respected:

(1) The mean of trace length on excavation surfaces and/or outcrops
(2) The standard deviation of trace length
(3) 1D intensity along boreholes or scanlines
(4) 2D intensity on excavation faces.

In the inverse method, the goal function[12] is designed as below.

$$F_{goal} = \sum_{i}^{N}\left\{\left(\frac{\sigma_n^c - \sigma_n^m}{\sigma_n^m}\right)^2 + \left(\frac{\mu_n^s - \mu_n^m}{\mu_n^m}\right)^2 + \left(\frac{d_1^{ns} - d_1^{nm}}{d_1^{nm}}\right)^2 + \left(\frac{d_2^{ns} - d_2^{nm}}{d_2^{nm}}\right)^2\right\} \quad (18)$$

where

N = the number of discontinuity sets; μ_n^s = mean length of simulated trace of discontinuity set n; μ_n^m = mean length of measured trace of discontinuity set n; σ_n^s = standard deviation of simulated trace of discontinuity set n; σ_n^m = standard deviation of measured trace of discontinuity set n; d_1^{ns} = simulated 1D intensity of discontinuity set n; d_1^{nm} = measured 1D intensity of discontinuity set n; d_2^{ns} = simulated 2D intensity of discontinuity set n; d_2^{nm} = measured 2D intensity of discontinuity set n.

Parameter estimation is a process to find the appropriate parameters that lead to a satisfying match between model-simulated result and observed data. This is usually carried out through modifying some initial guessed parameter values by some optimizing algorithm. In this work, this method was used. Fig.3 shows an example of the converging value of the goal function in the inverse method at a mine, which was operated as a research laboratory for the geological disposal of high-level nuclear waste (HLW)[5]. The goal function becomes from 1.9 to 1 by the iteration. Being provided the initial simplex, the downhill simplex method evaluates the objective function at all the vertices of the simplex to find the high vertices (where the objective function is minimum among all the vertices on the simplex) and the low vertices (where the objective function is maximum among all the vertices on the simplex). After this, the algorithm is then supposed to make its own way to move the simplex to the lowest point (where the minimum of the objective function exists). During the process shifting the simplex toward the lowest point, the most of the steps that the algorithm take is just to move the high point of the simplex through the opposite face of the simplex to a lower point. These steps are called reflections. To deal with some special situations, some special steps may be taken, for instance, contracting or expanding the simplex.

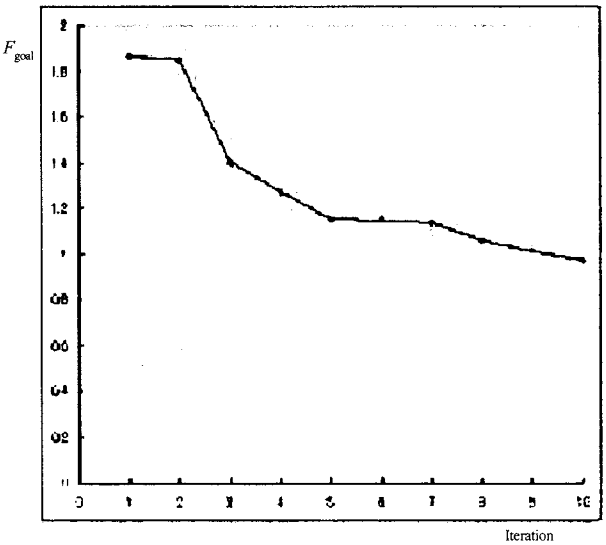

Fig.3 An Example of Converging Value of the Goal Function in the Inverse Method

4. Application to Tunnel

4.1 Construction Outline and Tunneling

The actual example site selected for this study is the large tunnel in the New Expressway between Tokyo and Kobe. The large tunnel in the New Expressway is now under construction. This tunnel construction is the world's first large and long tunnel construction which is based on block theory.

The New Expressway, which will be important to the Japanese economy in the 21st century, is under construction and has been designed to enable cars to travel safely at speeds up to 140km/h, which will make it the fastest expressway in Japan. To accommodate for high speed driving, the curvature of the expressway becomes smaller and tunnel length becomes longer. The cross-section of the tunnel is big. The new road takes three lanes in each direction.

Fig.4 shows the standard cross-section of the tunnel. The standard cross-section of the tunnel is large (200m^2) and wide (18m) compared to ordinary tunnels. The final tunnel shape has a height to width ratio of 0.65. The 5m diameter TBM pilot tunnel is at the center in the proposed tunnel. After the TBM pilot tunnel is excavated, the main tunnel is enlarged by New Austrian Tunneling Method (NATM).

4.2 Geography and Geology

The geology of the tunnel mainly consists of Tanakami granite[13-15] from the Late Cretaceous. The Tanakami granite is a massive coarse-grained biotite granite with equigranular texture. The Tanakami granite is fresh and hard. The maximum unconfined compressive strength is 100 MPa and seismic velocity (P-wave) is more than 4.7 km/s. However, a lot of small-scale faults and fractures are distributed in this area.

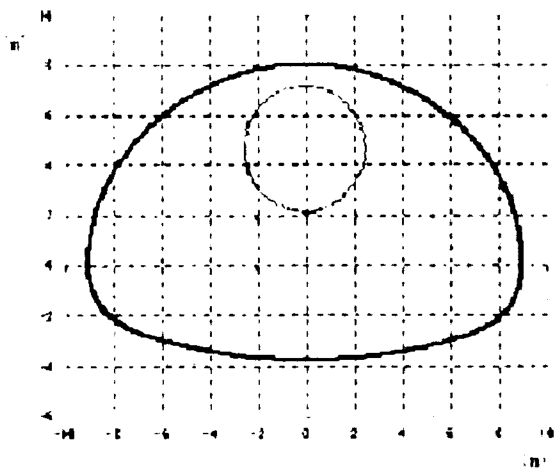

Fig.4 Cross-Section of Large Tunnel

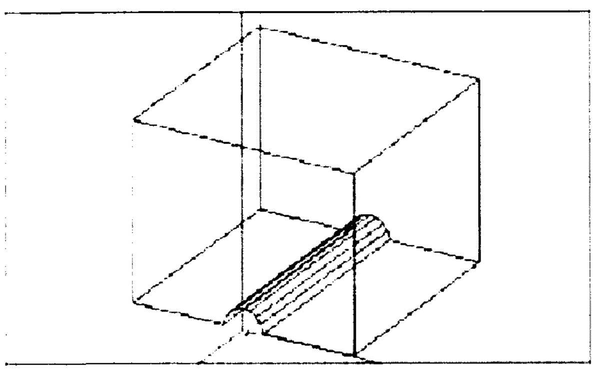

Fig.5 Modeling of Large Tunnel

4.3 Key Block Predictions

The prediction of key blocks using the proposed coupling key block analysis using stochastic-deterministic method in the large tunnel was performed before the tunnel excavation. The key blocks were predicted before all the discontinuity traces of the blocks have not appeared on the cutting face of the tunnel, in other words, in the moment some of the discontinuity traces of the block have appeared. The large tunnel was modeled as shown in Fig.5. In this section, the predicted key blocks are compared and examined with key blocks occurred actually.

Table1 Distribution of Discontinuity Data

Set No.	1	2	3
Mean dip direction	168.51	111.41	39.47
Mean dip angle	75.97	59.58	87.29
Fisher k	18.11	15.20	13.45
Spacing	27.27	37.5	25

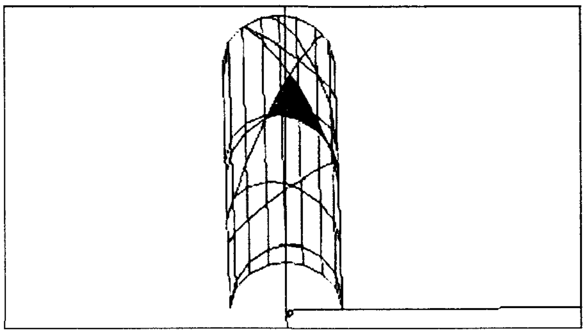

Fig.6 Key Blocks Detected by Deterministic Key Block Analysis

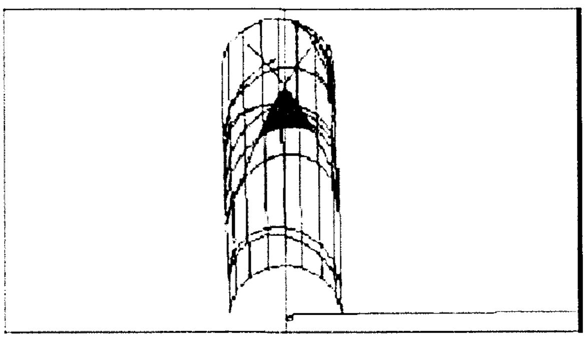

Fig.7 Key Blocks Predicted by Coupling Key Block Analysis

Table1 shows the distribution of discontinuity data acquired from the investigation of the advancing drift with TBM. Fig.6 shows key blocks detected using deterministic key block analysis. Key blocks predicted using the coupling key block analysis using stochastic-deterministic method are shown in Fig.7. The key blocks predicted using the coupling key block analysis using stochastic-deterministic method shows a good match with that detected using deterministic key block analysis. The comparisons and examinations with the analytical results have confirmed the validity and applicability of this coupling key block analysis using stochastic-deterministic method for observational design and construction method in actual tunnels.

5. Conclusions

In this paper, a new coupling key block analysis using stochastic-deterministic method has been proposed as a stability evaluation method for observational design and construction method in discontinuous rock masses. Then, the coupling key block analysis using stochastic-deterministic method proposed by the authors has been applied to the large tunnel based on actual discontinuity information observed in situ. The probability of occurring a key block by the next excavation can be calculated using rock discontinuity distribution data which are obtained from discontinuity survey on the already-excavated surface. In general, stochastic methods are often not effective to analyze local problems which are important for the active support design. In fact, this coupling key block analysis using stochastic-deterministic method is also based on stochastic theory. However the target of this analytical method is the very local problem. Furthermore it is considered that this method can be applied not only to the excavation in tunnels but also to the excavation in underground spaces, bench-cut of slope, etc.

References

1) Ohnishi, Y., Survey, Analysis and Evaluation for Discontinuous Rock Mass. *Soil and Foundation*, JGS, Vol. 47-12, pp. 61-62, 1999.
2) Hwang, J.-Y., Ohnishi, Y. and Nishiyama, S., Key Block Analysis in Tunnel Construction for the Geological Disposal of High-Level Radioactive Waste, *Proc. 2nd Japan-Korea Joint Seminar on Geoenv. Engrg.*, Japan, pp. 61-68, 2002.
3) Ohnishi, Y., Keynote Lecture: Numerical Methods and Tunneling, *Proc. 4th Intl Summer Symp.*, JSCE, Kyoto, Japan, pp. 1-21, 2002.
4) Hwang, J.-Y., Ohnishi, Y. and Nishiyama, S., Removability and Stability Analysis Method of Rock Blocks Considering Discontinuity Persistence in Tunnel Construction, *Jour. of the KGS*, KGS, Vol. 19, pp. 39-48, 2003.
5) Hwang, J.-Y., *Stability Evaluation of Rock Blocks in Tunnels for Observational Method*. Ph.D. Dissertation, Kyoto University, Kyoto, Japan, p. 289, 2003.
6) Hwang, J.-Y. and Sato, M., New Practical Rock Block Analysis for Observational Design and Construction Method in Large Tunnels, *Jour. Tunnel Engrg.*, JSCE, Vol. 14, pp.

17-26, 2004.
7) Goodman, R. E. and Shi, G. H., *Block Theory and Its Application to Rock Engrg.*, Prentice-Hall, 1985.
8) Ohnishi, Y., Nagano, K. and Fujikawa, T., Evaluation of Stability of Excavated Jointed Rock Mass by Block Theory, *Jour. Geotech. Engrg.*, JSCE, No. 364 / III-4, pp. 209-218, 1985.
9) Cacas, M. C., E. Ledoux, G. Marsily, B. Tillie, A. Barbreau, E. Durand, B. Feuga, and P. Peaudecerf, Modeling Fracture Flow with a Stochastic Discrete Fracture Network: Calibration and Validation, 1. The Flow Model, *Water Resources Research*, Vol. 26, pp. 479-489, 1990.
10) Kulatilake, P. H. S., Wathugala, D. N. M. and Stephansson, O., Joint Network Modeling with a Validation Exercise in Stripa Mine, Sweden. *Int. Jour. Rock Mech. Min. Sci. & Geomech. Abstr.*, Vol. 30: 5, pp. 503526, 1993.
11) Mardia, K. V., *Statistics of Directional Data*, Academic Press Inc., London, 1972.
12) Press, W. H., B. P. Flannery, S. A. Teukolsky, and W. T. Vetterling, *Numerical Recipes, the Art of Scientific Computing (FORTRAN version)*, Cambridge University Press, 1989.
13) Miyamura, M., Yoahida, F., Yamada, N., Sato, T. and Sangawa, A., *Geology of the Kameyama district*, Quadrangle Series, Scale 1:50,000, Geol. Surv. Japan, 1981.
14) Collaborative Research Group for the Granites around Lake Biwa, Granitic masses around Lake Biwa, Southwest Japan - On the granites in the Koga area -, *Jour. the Geol. Soc. of Japan*, Vol. 88, No. 4, pp. 289-298, 1982.
15) Kimura, K., Yoshioka, T., Imoto, N., Tanaka, S. Musashino, M. and Takahashi, Y., *Geology of the Kyoto-Tohokubu district. With Geological Sheet Map at 1:50,000*, Geol. Surv. Japan, 1998.

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