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Structural matrix identification using microtremor measurements based on canonical variate analysis

微動データを利用したCVAに基づく構造行列同定手法に関する研究

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A method for structural matrix identification is presented that uses microtremor measurements based on the canonical variate analysis (CVA), one of the subspace methods that enable the simultaneous identification of parameters under the multi-input-multi-output (MIMO) system. In the case where the structural responses at all nodes can be measured as well as the vibration at the basement, the damping and stiffness matrices can be obtained using the proposed method. Compared to the methods that tries to identify directly the element stiffness and element damping based on least squares method and prediction error method, the proposed method does not need to assume any distribution of stiffness and damping matrices, and identifies directly the total stiffness and damping matrices. The method is validated with the numerical simulation using a four-story structure.

Key Words: structural identification, subspace method, microtremor measurements

1. Introduction

Presently it is predicted that a large earthquake will hit the area along the Nankai trough in Japan in near future, government, engineers, and researchers are responsible to enhance the earthquake-resistance of various types of structures. In this view of needs, the establishment of structural health monitoring system is a pressing task.

With improvements in measuring apparatuses and technology, various new structural identification and damage identification techniques have been developed through vibration measurements[1)-11)]. Artificial vibration using exciters or actuators is advantageous in that both input and output data can be used for identification, therefore identification accuracy is supposed to be high. Artificial vibration, however, is expensive and impractical, and sometimes infeasible due to the condition and scale of structures. In contrast, microtremor measurements using ground motion are freely available, and turn out to be useful alternatives to the artificial excitation. Therefore, this study proposes the structural identification method using the microtremor measurement from the ground motion.

Model-updating-based structural identification methods are based on the first order perturbation methods. They are based on the assumptions that the difference between the analytical model from the design drawing and the real structure is small. When the structural parameters change drastically between the analytical and real structures, this method fails. In such cases, we should identify directly the whole structural parameters of the structure not the change from the analytical model.

Most structural identification methods assume that the total structural matrix is the summation of the elemental structural matrices for all elements, assume the distribution of element matrices, and try to identify element stiffness and element damping. These assumptions, however, are not always true in the real situation, therefore the development of methods to identify total stiffness and damping matrices are important.

Recently, the subspace identification method attracts attentions in the field of system identification[12)-19)]. It identifies discrete-time, linear, and time invariant state space models. The advantages are as follows. It has mathematical stabilities because it adopts singular value decomposition or QR decomposition. The results will not drop into the localized optimization point. Also it enables identification problem of multi-input-multi-output (MIMO) data because it is based on the state space models, and this is the important merit over the other system identification techniques such as prediction error method. The equation of motion of structural vibration responses is converted into a state space model, and the subspace identification technique then is applied to obtain state matrices. Subspace identification does not require any nonlinear computation and this makes the calculation much faster and more robust than the prediction error method[18]. Recently several subspace-based identification techniques have been developed. Two algorithms commonly in use are eigensystem realization algorithm (ERA)[15] and canonical variate analysis (CVA)[16),17]. Norman et. al. compared two algorithms over a range of signal-to-noise ratios, and concluded that CVA technique provides estimates which are substantially more accurate than those provided by ERA[12].

In this study, we propose the structural identification method, which identify whole stiffness and damping matrices from microtremor measurements based on the CVA. We first identify the two products between the inverse of mass matrix and the damping matrix, and the inverse of mass matrix and the stiffness matrix using the CVA. We assume that the mass matrix is known priori, then the stiffness and damping matrices are obtained. The method is validated with numerical simulation using a four-story structure.

2. Proposed method for total structural matrices identification

2.1 The canonical variate analysis (CVA)

The subspace identification methods recently attract attentions in the field of system identification and are now applied for modal parameter identification as substitute for the conventional peak-picking method with visual estimation from Fourier transforms. Canonical variate analysis (CVA) is one of the subspace methods, initially applied to time series by Akaike[13), 14] and improvements for general linear systems were made by Larimore[16), 17].

A finite dimensional, discrete-time, linear, time-invariant dynamical system is modeled as the state-variable equations

$$x(t+1) = A_D x(t) + B_D u(t) + w(t) \tag{1}$$

$$y(t) = Cx(t) + v(t) \tag{2}$$

where $x(t)$ is an n-dimensional state vector, $u(t)$ is an m-dimensional control input vector, $y(t)$ is a p-dimensional output vector, and $w(t)$ and $v(t)$ are zero mean white noise vectors. A_D, B_D and C are $n\times n$-, $n\times m$- and $p\times n$-dimensional state matrices. The matrix A_D characterizes the dynamics of the system and it is a representation of mass, damping, and stiffness matrices.

The problem of system realization is to estimate state matrices $[A_D, B_D, C]$ from given input and output vectors, $u(t)$ and $y(t)$. There exist infinite number of realizations that will meet this condition. The triple $[TA_DT^{l}, TB_D, CT^{l}]$ are also the realization (T is the transfer matrix), however, the eigenvalues of the matrix A_D always are the same.

The algorithm is as follows;

Step 1: Define the two new state vector, $p(t)$ and $f(t)$; $p(t)$ represents the past information and $f(t)$ represents the future information as

$$p(t) = \begin{bmatrix} u(t-1) \\ y(t-1) \\ \vdots \\ u(t-l) \\ y(t-l) \end{bmatrix} \tag{3}$$

$$f(t) = \begin{bmatrix} y(t) \\ \vdots \\ y(t+k-1) \end{bmatrix} \tag{4}$$

where l and k are the integer indicating the considered length of time history for past and future data. The dimensions of $p(t)$ and f(t) respectively are $l(p+m)\times 1$, and $kp\times 1$. Assume that we have N data for both $u(t)$ and $y(t)$, and define $N'=N\text{-}k\text{-}l+1$, $l'=l(p+m)$, $k'=kp$.

Step 2: Calculate the covariance matrices among vectors $p(t)$ and $f(t)$, and define them as

$$\hat{\Sigma}_{pp} = \frac{1}{N'}\sum_{t=l+1}^{N-k+1} p(t)p(t)^T \tag{5}$$

$$\hat{\Sigma}_{ff} = \frac{1}{N'}\sum_{t=l+1}^{N-k+1} f(t)f(t)^T \tag{6}$$

$$\hat{\Sigma}_{pf} = \frac{1}{N'}\sum_{t=l+1}^{N-k+1} p(t)f(t)^T \tag{7}$$

Step 3: Calculate the eigenvalues and eigenvectors of $\hat{\Sigma}_{pp}$ and $\hat{\Sigma}_{ff}$ and obtain

$$\hat{\Sigma}_{pp}^{-1/2} = U_1 S_1^{-1/2} U_1^T \tag{8}$$

$$\hat{\Sigma}_{ff}^{-1/2} = U_2 S_2^{-1/2} U_2^T \tag{9}$$

where $\hat{\Sigma}_{pp}^{-1/2}$ and $\hat{\Sigma}_{ff}^{-1/2}$ respectively are the square root of inverse matrices of $\hat{\Sigma}_{pp}$ and $\hat{\Sigma}_{ff}$. U_1 and U_2 respectively are the matrix composed by eigenvectors of $\hat{\Sigma}_{pp}$ and $\hat{\Sigma}_{ff}$. $S_1^{-1/2}$ and $S_2^{-1/2}$ are the diagonal matrix composed by eigenvalues of $\hat{\Sigma}_{pp}$ and $\hat{\Sigma}_{ff}$.

Step 4: Calculate the following singular values decomposition

(SVD)

$$\hat{\Sigma}_{pp}^{-1/2}\hat{\Sigma}_{pf}\hat{\Sigma}_{ff}^{-1/2} = U_3 S_3 V_3^T \quad (10)$$

$$U_3 U_3^T = I_{l'} \quad (11)$$

$$V_3 V_3^T = I_{k'} \quad (12)$$

where $I_{l'}$ and $I_{k'}$ are respectively l'-th and k'-th order identity matrices. U_3 and V_3^T are left and right singular value vectors and S_3 is the diagonal matrix composed by singular values of $\hat{\Sigma}_{pp}^{-1/2}\hat{\Sigma}_{pf}\hat{\Sigma}_{ff}^{-1/2}$.

Then define

$$U = U_3^T \hat{\Sigma}_{pp}^{-1/2} \quad (13)$$

In CVA, U indicates the matrix which converts p(t) into the vector which has the highest correlation with f(t) we want to predict.

Step 5: Determine an optimal n-order memory function μ(t)

$$\mu(t) = [I_n \quad 0]Up(t), \quad t = l+1, \cdots, N-k+1 \quad (14)$$

where I_n is respectively n-th order identity matrices. Since μ(t) has the condensed information to predict future, it is regarded as the state vector at time t. Noting that the covariance matrix of μ(t) is identity matrix, the state vectors $\mu(t)$ and x(t) are connected with appropriate transfer matrix T as

$$x(t) = T\mu(t) \quad (15)$$

Step 6 : Substituting Eq.(15) into Eqs.(1) and (2), we obtain

$$\mu(t+1) = \begin{bmatrix} T^{-1}A_D T & T^{-1}B_D \end{bmatrix} \begin{bmatrix} \mu(t) \\ u(t) \end{bmatrix} + T^{-1}w(t) \quad (16)$$

$$y(t) = CT\mu(t) + v(t) \quad (17)$$

Matrices A_D, B_D, C depend on the transfer matrix T, but eigenvalues of matrix A_D is independent from transfer matrix T.

Step 7: The state matrices [$T^{1}A_D T$, $T^{1}B_D$, CT] are estimated using least squares method

$$\begin{bmatrix} T^{-1}A_D T & T^{-1}B_D \end{bmatrix} = \left(\frac{1}{N'} \sum_{t=l+1}^{N-k+1} [\mu(t+1)][\mu(t)^T \quad u(t)^T] \right) \left(\frac{1}{N'} \sum_{t=l+1}^{N-k+1} \begin{bmatrix} \mu(t) \\ u(t) \end{bmatrix} [\mu(t)^T \quad u(t)^T] \right)^{-1} \quad (18)$$

$$CT = \left(\frac{1}{N'} \sum_{t=l+1}^{N-k+1} [y(t)][\mu(t)^T] \right) \left(\frac{1}{N'} \sum_{t=l+1}^{N-k+1} [\mu(t)][\mu(t)^T] \right)^{-1} \quad (19)$$

where we define [$\bar{A}, \bar{B}, \bar{C}$] = [$T^{1}A_D T$, $T^{1}B_D$, CT]

The above is the algorithm when we know the dimension of n, l and k. We determined the dimension of state matrix, n, according to the number of nodes of the structural model.

We also have to determine the value of l and k, the length of time history of past and future. In this study, we decided the l and k in order that they minimize the residual error covariance matrix R.

$$R = \frac{1}{N'} \sum_{t=l+1}^{N-k+1} (y(t) - CT\mu(t))(y(t) - CT\mu(t))^T \quad (20)$$

2.2 Vibration response of a structure

The vibration response is obtained by solving the equation of motion;

$$M\ddot{z}(t) + D\dot{z}(t) + Kz(t) = -Mz_g(t) \quad (21)$$

where M, D, and K respectively are the mass, damping, and stiffness matrices of the baseline model. $z(t)$ is the displacement response vector, and $z_g(t)$ is the time history of input ground motion.

The equation of motion can be rewritten in the following continuous-time system

$$\dot{x}(t) = A_C x(t) + B_C u(t) \quad (22)$$

where A_C and B_C are state matrices in continuous-time system. Followings are one representation. Here we omitted the noise term.

$$x(t) = \{z(t) \quad \dot{z}(t)\}^T \quad (23)$$

$$A_C = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}D \end{bmatrix} \quad (24)$$

$$B_C = I \quad (25)$$

$$u(t) = \{0 \quad -z_g(t)\}^T \quad (26)$$

The continuous-time system can be converted into discrete-time system as

$$A_D = \exp(A_C \Delta) \quad (27)$$

$$B_D = \int_0^{\Delta} \exp(A_C \tau) d\tau B_C \quad (28)$$

where Δ is the time interval.

Eq.(1) corresponds to Eq.(22). In the structural vibration responses, the counterpart of y(t) in Eq.(2) is x(t) in Eq.(23) and that of C in Eq.(2) is the identity matrix.

2.3 Identification of continuous and discontinuous-time system

The proposed method identifies not A_D but $T^{1}A_D T$. But because the eigenvalues of matrix A_D are independent from transfer matrix T, we can obtain the eigenvalues and eigenvectors of A_D

$$A_D \Psi = \mu \Psi \quad (29)$$

where μ denotes the diagonal matrix whose i-th component, μ_i , is the i-th order eigenvalue. Ψ denotes the matrix whose i-th column is the i-th order eigenvector ϕ_i. I-th natural frequency f_i and damping ratio h_i are obtained as

$$f_i = \frac{1}{2\pi\Delta}\sqrt{\mathrm{Re}(\ln\mu_i)^2 + \mathrm{Im}(\ln\mu v)^2} \quad (30)$$

$$h_i = \frac{-\mathrm{Re}(\ln\mu_i)}{\sqrt{\mathrm{Re}(\ln\mu_i)^2 + \mathrm{Im}(\ln\mu_i)^2}} \quad (31)$$

Mode shapes are obtained as $C\Psi$. We regard CT obtained from Eq.(19) as C.

Here, the mode shapes of A_C, and those of A_D is the same. The eigenvalues of A_C, λ, are related to those of A_D, μ, as

$$\mathrm{Re}\{\lambda\} = \frac{1}{2\Delta}\log\{\mathrm{Re}(\mu)^2 + \mathrm{Im}(\mu)^2\} \quad (32)$$

$$\mathrm{Im}\{\lambda\} = \frac{1}{\Delta}\arctan\left\{\frac{\mathrm{Im}(\mu)}{\mathrm{Re}(\mu)}\right\} \quad (33)$$

where λ and μ are the eigenvalue of A_C and A_D.

Then the matrix A_C can be obtained as

$$A_c = C\Psi\Lambda(C\Psi)^{-1} \quad (34)$$

where Λ is the diagonal matrix composed by eigenvalues λ.

2.4 Structural matrices identification method

Two matrices $M^{-1}D$ and $M^{-1}K$ are obtained simultaneously from Ac according to Eq.(24). Assuming that the mass matrix M can be calculated from the design document of the structure, stiffness and damping matrices are obtained by multiplying M to $M^{-1}C$ and $M^{-1}K$ from the left side.

3. Numerical Verification

3.1 Analytical model

The proposed method is applied to a four-story structure modeled by a four degree of freedom mass-damping-stiffness system as shown in Fig.1. The structural parameters for each floor are shown in Table.1. The mass, damping, and stiffness matrices respectively are as follows.

$$M = \begin{bmatrix} 0.50\mathrm{E}+07 & 0 & 0 & 0 \\ 0 & 0.10\mathrm{E}+08 & 0 & 0 \\ 0 & 0 & 0.13\mathrm{E}+08 & 0 \\ 0 & 0 & 0 & 0.11\mathrm{E}+08 \end{bmatrix} \quad (35)$$

$$D = \begin{bmatrix} 0.1194\mathrm{E}+09 & -0.753\mathrm{E}+08 & 0 & 0 \\ -0.753\mathrm{E}+08 & 0.1257\mathrm{E}+09 & -0.504\mathrm{E}+08 & 0 \\ 0 & -0.504\mathrm{E}+08 & 0.1194\mathrm{E}+09 & -0.69\mathrm{E}+08 \\ 0 & 0 & -0.69\mathrm{E}+08 & 0.69\mathrm{E}+08 \end{bmatrix} \quad (36)$$

$$K = \begin{bmatrix} 0.315\mathrm{E}+11 & -0.175\mathrm{E}+11 & 0 & 0 \\ -0.175\mathrm{E}+11 & 0.386\mathrm{E}+11 & -0.211\mathrm{E}+11 & 0 \\ 0 & -0.211\mathrm{E}+11 & 0.351\mathrm{E}+11 & -0.14\mathrm{E}+11 \\ 0 & 0 & -0.14\mathrm{E}+11 & 0.14\mathrm{E}+11 \end{bmatrix} \quad (37)$$

The natural frequencies and damping ratios are shown in Table.2.

3.2 Analytical conditions

We assumed that the structure is affected by the ground motion with very small amplitudes at the basement. The time history of the input ground motion, which was generated using random numbers between –5 to 5 cm/sec^2, as well as its Fourier amplitude are shown in Fig.2. We assumed that the displacement and velocity response at all floors are measured, as well as the input ground motion. The noise u(t) in Eq.(1) was neglected, and the noise v(t) in Eq.(2) was only considered.

The noise ratio was defined as,

$$v = \frac{\sigma_{noise}}{\sigma_{resp}} \times 100(\%) \quad (38)$$

where σ_{resp} is the standard deviation of the true responses without measurement noise, and σ_{noise} is that of measurement noise. In this study we considered 0%, 1%, 3% and 5% measurement noise cases.

The time history of calculated displacement and velocity responses are shown in Figs. 3 and 4. The difference in responses of each case is very small. We use these responses of 20 minutes time duration as the input of the proposed method. We applied l=4 and k =1 because they minimized the residual error covariance matrix R in Eq. (20).

3.3 Identified results

(1) Identified natural frequency and damping ratio

The Fourier amplitudes of velocity and transfer functions for noise free case are shown in Figs. 5 and 6. In this study, the transfer function is obtained by dividing the Fourier spectrum of response at each floor by that of the input ground motion. Most popular method that identifies natural frequencies is the peak-picking method from the transfer function by the eye inspection. As for identifying damping ratios, the methods commonly in use are half power method and curve fitting method done by eye inspection. These methods regard the frequencies whose transfer function takes the peak values, and estimate damping from the shapes of the transfer function around the natural frequencies. In this case, however, it seems very difficult to estimate them from the transfer function shown in Fig.6 because there are many frequencies that take peaks corresponding to the peaks of input ground motion.

In this situation where traditional eye inspection method are infeasible, the subspace-based technique

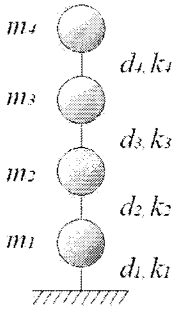


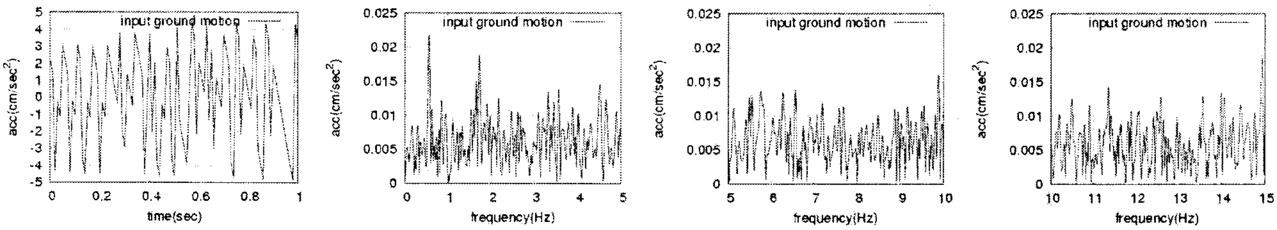
Fig.1 Analytical model

Table 1 Structural parameter

	unit	1st floor	2nd floor	3rd floor	4th floor
mass	kg	5.00E+06	1.00E+07	1.30E+07	1.10E+07
stiffness coefficient	N	1.40E+10	1.75E+10	2.11E+10	1.40E+10
damping coefficient	N/m	4.41E+07	7.53E+07	5.04E+07	6.90E+07

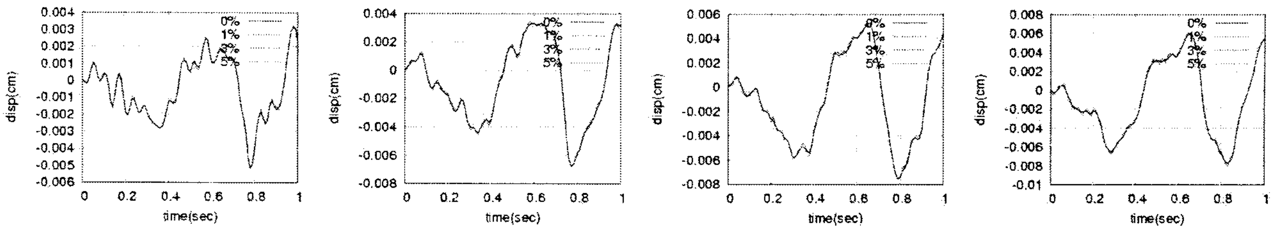
Table 2 Natural frequencies and damping ratio

	1st mode	2nd mode	3rd mode	4th mode
natural frequencies (Hz)	2.11E+00	6.58E+00	1.03E+01	1.43E+01
damping ratio	2.32E-02	8.73E-02	1.06E-01	1.69E-01



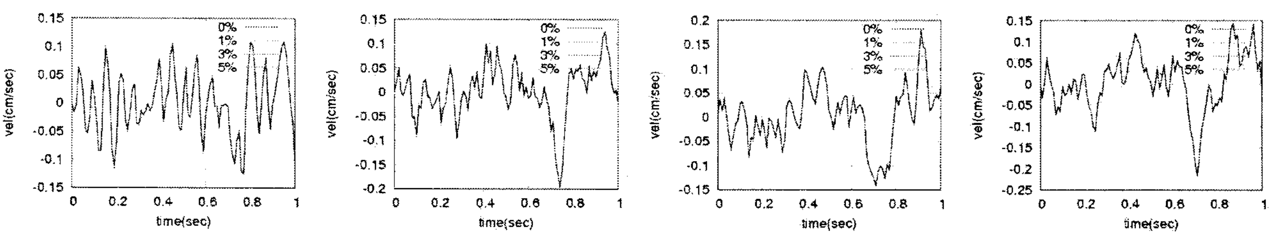
(a) time history of acceleration (b) Fourier amplitude (0-5Hz) (c) Fourier amplitude (5-10Hz) (d) Fourier amplitude (10-15Hz)

Fig.2 Input ground motion



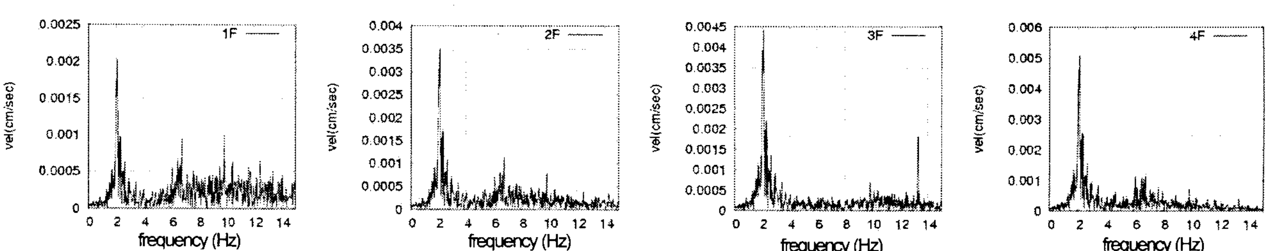
(a) 1st floor (b) 2nd floor (c) 3rd floor (d) 4th floor

Fig.3 Displacement responses



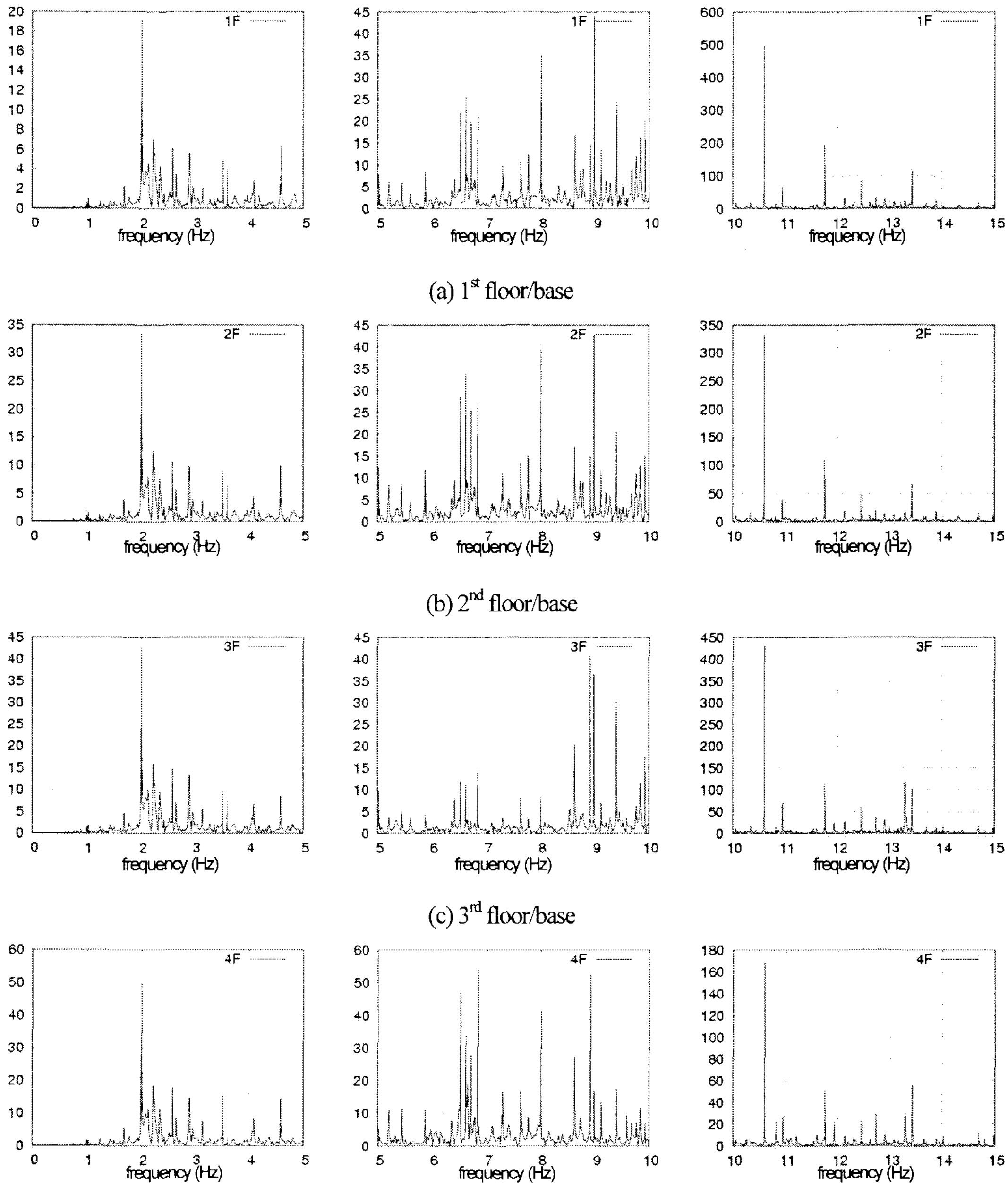
(a) 1st floor (b) 2nd floor (c) 3rd floor (d) 4th floor

Fig.4 Velocity responses



(a) 1st floor (b) 2nd floor (c) 3rd floor (d) 4th floor

Fig.5 Fourier amplitude of velocity



(a) 1[st] floor/base

(b) 2[nd] floor/base

(c) 3[rd] floor/base

(d) 4[th] floor/base

Fig.6 Transfer functions

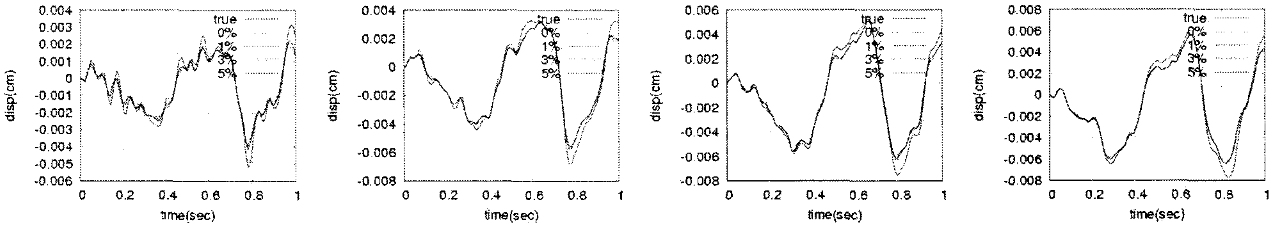
y works efficiently. We obtained natural frequencies and damping ratios by solving the eigenvalue problem of identified A_D. Results are shown in Table 3. Comparing the true values shown in Table 2, natural frequencies are identified very well especially in lower modes. Damping ratios are overestimated and do not have good agreement compared with natural frequencies. Lower modes damping ratios have the better accuracy than higher modes. Even though damping estimation is not with high accuracy, the subspace-based technique made estimation possible.

(2) Identified stiffness and damping matrix

The identified damping and stiffness matrices are follows.

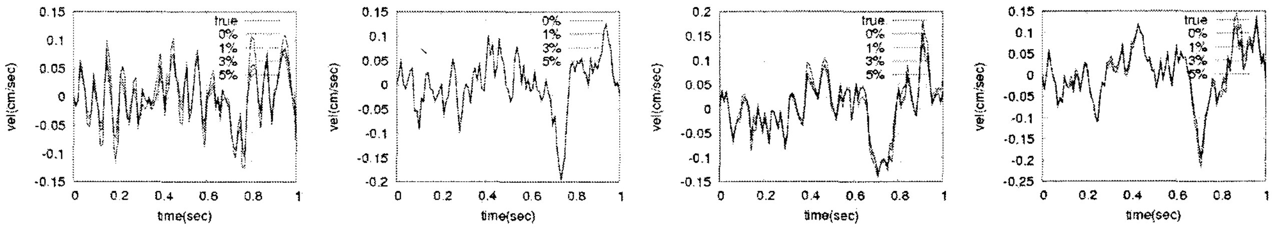
$$D(0\%) = \begin{bmatrix} 1.17E+08 & -6.76E+07 & 6.33E+06 & -3.58E+06 \\ -6.77E+07 & 1.46E+08 & -7.07E+07 & -6.19E+06 \\ 6.27E+06 & -7.11E+07 & 1.20E+08 & -4.92E+07 \\ -3.55E+06 & -6.07E+06 & -4.91E+07 & 5.61E+07 \end{bmatrix} \quad (39)$$

$$D(1\%) = \begin{bmatrix} 1.20E+08 & -6.97E+07 & 6.76E+06 & -3.62E+06 \\ -6.97E+07 & 1.50E+08 & -7.36E+07 & -5.68E+06 \\ 5.81E+06 & -7.29E+07 & 1.23E+08 & -5.07E+07 \\ -3.14E+06 & -6.17E+06 & -5.05E+07 & 5.72E+07 \end{bmatrix} \quad (40)$$



(a) 1st floor (b) 2nd floor (c) 3rd floor (d) 4th floor

Fig.7 Reconstructed displacement responses



(a) 1st floor (b) 2nd floor (c) 3rd floor (d) 4th floor

Fig.8 Reconstructed velocity responses

$$D(3\%)=\begin{bmatrix} 1.43E+08 & -8.70E+07 & 1.05E+07 & -4.05E+06 \\ -8.48E+07 & 1.82E+08 & -9.77E+07 & -9.69E+05 \\ 3.26E+06 & -8.99E+07 & 1.53E+08 & -6.34E+07 \\ 9.19E+04 & -6.33E+06 & -6.12E+07 & 6.53E+07 \end{bmatrix} \quad (41)$$

$$D(5\%)=\begin{bmatrix} 1.88E+08 & -1.24E+08 & 2.24E+07 & -6.27E+06 \\ -1.17E+08 & 2.47E+08 & -1.49E+08 & 1.06E+07 \\ 8.50E+05 & -1.23E+08 & 2.10E+08 & -8.93E+07 \\ 6.53E+06 & -7.74E+06 & -8.11E+07 & 8.09E+07 \end{bmatrix} \quad (42)$$

$$K(0\%)=\begin{bmatrix} 3.16E+10 & -1.75E+10 & -3.73E+07 & 1.75E+07 \\ -1.75E+10 & 3.85E+10 & -2.10E+10 & -3.58E+07 \\ -1.78E+08 & -2.10E+10 & 3.51E+10 & -1.40E+10 \\ 1.27E+07 & 5.06E+07 & -1.41E+10 & 1.40E+10 \end{bmatrix} \quad (43)$$

$$K(1\%)=\begin{bmatrix} 3.15E+10 & -1.75E+10 & 5.24E+06 & -1.28E+06 \\ -1.75E+10 & 3.86E+10 & -2.10E+10 & -1.50E+07 \\ -2.39E+08 & -2.09E+10 & 3.50E+10 & -1.40E+10 \\ 3.82E+07 & 2.75E+07 & -1.41E+10 & 1.41E+10 \end{bmatrix} \quad (44)$$

$$K(3\%)=\begin{bmatrix} 3.14E+10 & -1.76E+10 & 2.55E+08 & -1.02E+08 \\ -1.74E+10 & 3.85E+10 & -2.13E+10 & 1.65E+08 \\ -5.86E+08 & 2.05E+10 & 3.50E+10 & 1.41E+10 \\ 2.66E+08 & -2.65E+08 & -1.40E+10 & 1.41E+10 \end{bmatrix} \quad (45)$$

$$K(5\%)=\begin{bmatrix} 3.10E+10 & -1.77E+10 & 7.03E+08 & -2.81E+08 \\ -1.71E+10 & 3.84E+10 & -2.18E+10 & 5.78E+08 \\ -1.12E+09 & -1.98E+10 & 3.50E+10 & -1.44E+10 \\ 6.34E+08 & -7.98E+08 & -1.38E+10 & 1.41E+10 \end{bmatrix} \quad (46)$$

where $D(0\%)$, $D(1\%)$, $D(3\%)$ and $D(5\%)$ are the identified damping matrices with 0%, 1%, 3% and 5% noise. Identification for stiffness was described in the same way.

The identification error (ERR) was estimated in the following equation.

Table 3 Identified natural frequencies and damping ratio by CVA

(a) natural frequencies (Hz)

noise	1st mode	2nd mode	3rd mode	4th mode
0%	2.11E+00	6.59E+00	1.03E+01	1.43E+01
1%	2.11E+00	6.60E+00	1.04E+01	1.43E+01
3%	2.11E+00	6.63E+00	1.05E+01	1.47E+01
5%	2.11E+00	6.70E+00	1.09E+01	1.54E+01

(b) damping ratio

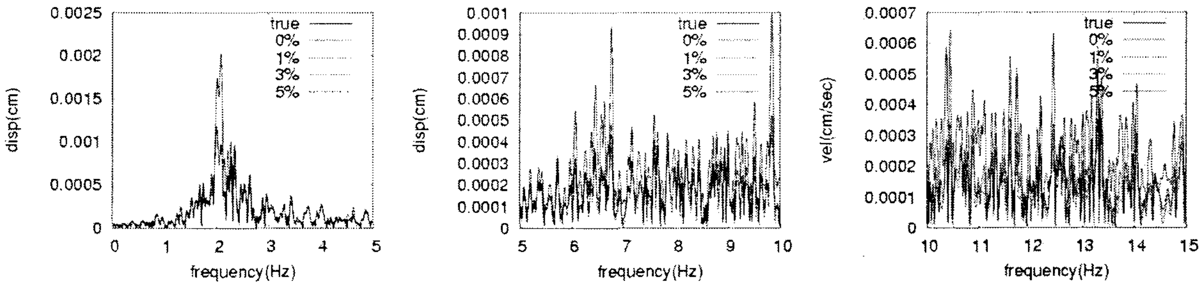
noise	1st mode	2nd mode	3rd mode	4th mode
0%	2.86E-02	8.90E-02	1.07E-01	1.73E-01
1%	2.86E-02	9.02E-02	1.10E-01	1.73E-01
3%	2.89E-02	9.94E-02	1.34E-01	2.05E-01
5%	2.94E-02	1.17E-01	1.76E-01	2.64E-01

Table 4 Identification error (%)

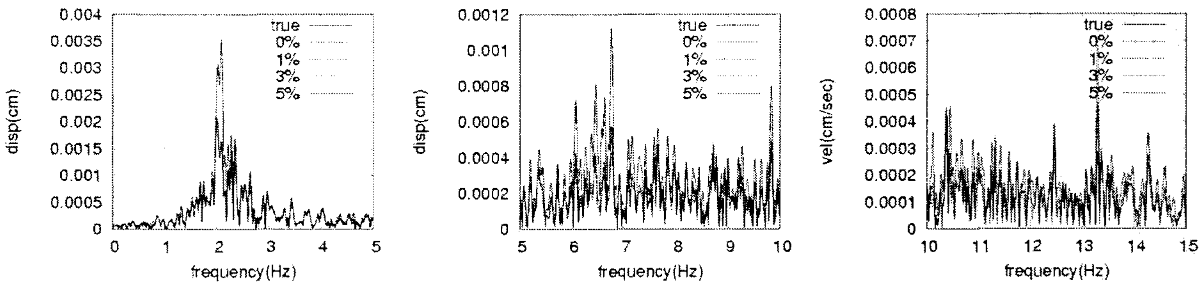
noise	stiffess	damping
0%	4.10E-01	1.82E+01
1%	4.63E-01	1.89E+01
3%	1.35E+00	3.48E+01
5%	2.11E+00	7.99E+01

$$ERR=\sqrt{\sum_{i=1}^{4}\sum_{j=1}^{4}(a_{ij}^{identified}-a_{ij}^{true})^2}\Bigg/\sqrt{\sum_{i=1}^{4}\sum_{j=1}^{4}(a_{ij}^{true})^2} \quad (47)$$

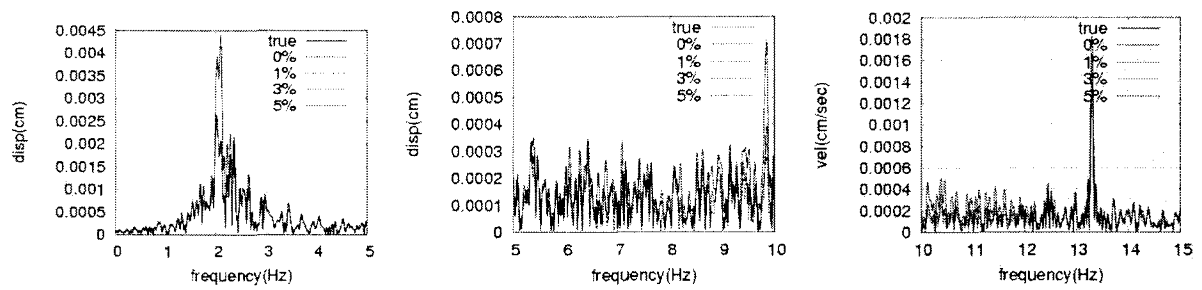
where a_{ij}^{true} and $a_{ij}^{identified}$ are true and identified value of the (i,j) component of matrix A. The ERR of identified stiffness and damping matrices for each noise level are shown in Table 4.



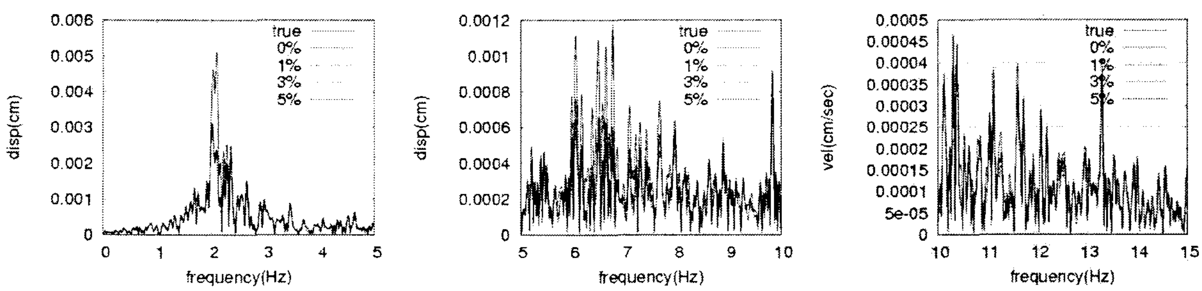
(a) 1st floor



(b) 2nd floor



(c) 3rd floor



(d) 4th floor

Fig.9 Reconstructed Fourier amplitude of velocity

Compared to the true values in Eqs.(35) and (36), the results for stiffness have good agreement in the case of 5% case, and the results for damping are not good. This is corresponding to the accuracy of natural frequencies and damping ratio stated in the previous section.

The structural matrices of real structure are not always the symmetric and cannot be separated into each element matrices. The proposed method directly identifies the total stiffness and damping matrices simultaneously that represent the relationship between input and output best.

(3) Reconstruction of responses

We recalculated the time histories of displacement and velocity at each floor using the identified stiffness and damping matrices. Comparisons with the true responses are shown in Figs. 7 and 8. The reconstructed response of each case is very close to the true responses. For cases where 0% and 1% noise are exist, the reconstructed responses show a good agreement with the true responses. As noise ratio becomes larger, the accuracy becomes worse, but even the case with 5% noise can trace the responses very well. The Fourier amplitudes of the recalculated velocity are shown in Fig. 9. It is shown that the frequency characteristics are also traced very well. All prominent frequencies are estimated correctly, and this corresponds to the fact that the natural frequencies are identified with high degree of accuracy. The values at the natural frequencies are underestimated, and this corresponds to the fact that the damping ratios were overestimated.

4. Conclusions

A vibration-based structural identification method is proposed that evaluates the total structural matrices by use of microtremor measurements. It is based on the CVA, one of the subspace identification methods that identify the discrete-time, linear, and time invariant state space models. We first identify the state matrix of the discrete-time system, and obtain eigenvalues and eigenvectors of the state matrix. Next, we obtain the mode shapes and convert the eigenvalues of the state matrix of discrete-time system to those of continuous-time system. At this point, natural frequencies and damping ratios can be obtained from the eigenvalues. Then, we can identify the state matrix of the continuous-time system using the converted eigenvalues and mode shapes. Finally, stiffness and damping matrices can be obtained by multiplying the mass matrix with the continuous-time state matrix.

We conducted the numerical simulation using the 4 degrees of freedom mass-damping- stiffness system. The CVA-based technique could identify natural frequencies and damping ratios with higher accuracy than visual estimation method. Identified natural frequencies have a high degree of accuracy. All damping ratios were overestimated. The identified stiffness matrix also had good accuracy in the case of 5% noise. The accuracy of damping matrices is inferior to that of stiffness and this is because of the identification error in damping ratios. The reconstructed responses had a good agreement with the true values both in the time and the frequency domain.

The proposed method assumed the linear and time-invariant system. For the future work, we want to develop the method to identify a nonlinear system.

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