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Inverse Analysis to determine re-bars' force from external crack widths measurement

ひび割れ開口幅計測を用いた鉄筋伝達力の逆解析

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This paper focuses on the stability of fracturing process in reinforced concrete beams and derives a fracture mechanics based transformation between crack opening displacements (COD) and re-bars force. Bridged crack model is adopted which considers distinct phenomena for damage process in concrete and bridging of cracks by re-bars. The derived transformation yields crack profiles for known re-bar force determined by either RC section analysis or computations having fracture mechanics based approach. Inverse of the integral transformation is ill-posed when experimentally collected COD's are not exact. Tikhonov method of regularization is followed to compute re-bar force from COD where the extremals of the Tikhonov functional is determined numerically. A numerical example proving the applicability of this method with different noise levels are demonstrated. It is observed that current method of numerical inverse analysis on external surface COD can satisfactorily determine location and force of re-bars in the beam cross-section within limited tolerance of noise in data. Application of this method enables the maintenance engineers to retrieve inner cross-sectional information from outer measurements in a non-destructive way.

***Key Words**: reinforced concrete, crack opening displacements, fracture mechanics, inverse analysis, crack bridging stress, re-bar force.*

1. Introduction

Determination of re-bar force from external measurement of cracks is important for existing structures when available cross-sectional information are not adequate and/or reliable. Besides, re-bar corrosion, progressive crushing or crumbling in concrete as well as retrofit works (underlay, overlay etc.) alter cross-sections from that of the detailed constructed one. System identification in health monitoring and maintenance requires determining cross-sectional information like clear cover, layers of reinforcements, reactive force in re-bars under known flexure and so on in a non-destructive way. Inverse analysis methods capable of computing inner sources from outer response measurements are right choices for such cases. In this paper, Tikhonov regularization in context of a fracture mechanics based operator equation is explained where the operator is an integral equation working out crack widths in reinforced concrete beams on flexure. Such integral equations were previously derived for continuously aligned fiber composites[1)] and for plain concrete based on cohesive force model[2)]. Extensive crack profiles' solutions for brittle-matrix ductile-fiber composites with various geometries and loading conditions are available[3), 4)] where *bridging law* was considered *a priori*. Reverse computations considering noise in crack widths encountered *Ill-posedness* while determining softening curve of cohesive crack span resembles a standard SEN fracture specimen. If beam span is large compared to the other dimensions, weight functions of SEN specimen with infinite length and finite width is applicable.

Linear elastic material response is assumed for both concrete and steel focusing on existing structures under service loading. Fracture failures under service loading are important for massive bridge girders or large dams where propagation of individual crack dominates the failure, generally below the structural capacity by limit analysis. Crumbling and crushing of concrete at the crack surfaces are taken into account by discrete crack approach i.e. all in the concrete side. It is assumed that a perfect and total stress transfer occurs between steel and concrete between two crack surfaces and there is no question of bar pull-out as adequate development length is provided in RC design. Constitutive modeling based on these assumptions for bridging effects and bond activation in RC is available including variational formulation and numerical analysis with finite elements[13].

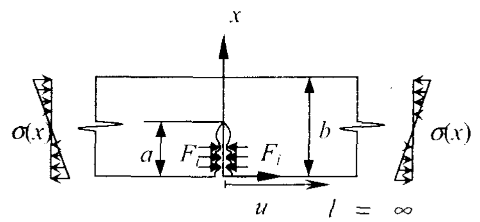

Fig. 1: Reinforced concrete beam under flexure. Re-bars are bridging the crack with forces F_i.

With the assumptions described above and applying Castigliano's theorem in **Fig. 1**, crack profiles in reinforced concrete beams are derived by computing variation of strain energy functional with an imaginary point load at any location x as[1), 4)]

$$u'(x) = \frac{4}{E'} \int_x^a \int_0^{a'} G(x', a', b)[\sigma(x') - f(x')]dx' G(x, a', b) da' \tag{2}$$

where $E' = E_c$ for plane stress and $E' = E_c/(1-\nu^2)$ for plane strain respectively , E_c being the Young's modulus of concrete and ν the Poisson ratio. $\sigma(x)$ is the stress from external loading that would exist in the x-plane if there was no crack (shown linear in **Fig. 1** at the edges) and $G(x, a, b)$ is the weight function to determine stress intensity factors. Standard forms of weight functions for a large variety of geometries are abundantly available in the stress intensity factor handbooks of fracture mechanics. The weight function for a SEN specimen of infinite length and a finite width b with a crack of length a is[14)]

$$G(x, a, b) = \frac{1}{\sqrt{\pi a}} \frac{g(x/a, a/b)}{\left(1 - x^2/a^2\right)^{1/2} (1 - a/b)^{3/2}} \tag{3}$$

where $g(x/a, a/b) = g(x/a, \xi)$ is given in the Appendix. Distribution of bridging stress by re-bars is shown in **Fig. 2** and simulated by Heaviside step functions as

$$f(x) = \sum_{i=1}^{r} f_i [H(x - h_i) - H(x - h_i - d_b)] \tag{4}$$

where $f_i = (F_i / d_b)$ are uniform re-bar (bridging) stresses within their diameters along crack length, F_i being the total force in i-th layer, r the total number of reinforcement layers, h_i the clear distance of a layer from bottom face, d_b the bar diameter. No micro-mechanical parameter in this monotonic loading case is considered for simplicity. The bar forces depend on crack openings considering relative displacements between steel and concrete governing bond activation and bonding process. Bridging stress due to post-peak strain softening of concrete is neglected for its insignificant contribution to crack closing compared to that of re-bar force.

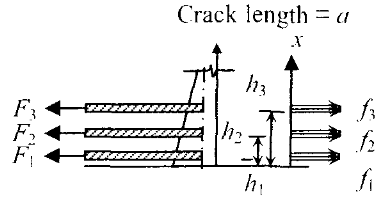

Fig. 2: Crack bridging stress by re-bars as described by Eq. (4)

3. Numerical Approximations

The net crack profile by Eq. (2) is the resultant of opening by the external load $\bar{u}(x)$ and closing by internal stress $u(x)$ which is decomposed as

$$u'(x) = \bar{u}(x) - u(x) \tag{5}$$

model for concrete[5] and crack bridging law in fiber composites[6]. Tikhonov regularization was shown successful[6] for determining monotonic "bridging stress (p)-opening displacement (u)" relationship considering p to be a continuous function of u or location along the crack x. This facilitated expansion of $p(u)$ or $p\{u(x)\}$ by orthogonal basis (e. g. Legendre polynomials) within the closed intervals of u or x. This paper embarks on inverse analysis of COD in RC beams with one or multiple layers of reinforcements which are bridged by discrete re-bar forces. Closing pressure along the crack surfaces is a step function and no analytical $p\{u(x)\}$ relation exists in the interval $[u_{min}, u_{max}]$. Although concordant results can be obtained by a mathematically compatible continuous $p(x)$, numerical analysis deduces the number and location of reinforcement layers along with re-bar forces at particular load with desirable accuracy. All these inverse analyses (analytical or numerical) need extreme noiseless data for better convergence and exact results [6), 7)]. This paper furnishes the goal of determining crack bridging stress by re-bars in reinforced concrete beams under flexure by numerical inverse analysis of Tikhonov regularization on COD data.

Fracture analysis of RC structures previously adopted standard single edge notched (SEN) fracture specimen to analyze stability of fracturing process in RC beams[8] where superposition of rotations by external and internal stress rendered statically indeterminate re-bar force. Onset of matrix cracking was demonstrated with the fracture toughness of concrete, K_{IC}, considered a material constant[9]. Applicability of a single linear elastic fracture mechanics (LEFM) parameter K_{IC} is defended if a correct stress intensity factor calibration is available for specimens large enough compared to crack length and/or aggregate size[10]. Analyses based on compliance by weight function method were extended to cyclic loading[11]. COD was never used to determine re-bar force without cross-sectional details. Cross-section was always assumed to be well-documented beforehand.

This paper utilizes crack widths and determines re-bar force as the bridging stress of *bridged crack model* for reinforced concrete beams under flexure. Generally, *cohesive crack model* is adopted for concrete where fracture process zone (FPZ) with extensive micro-cracking ahead of the traction free crack is considered to be under cohesive stress stretching the crack length until net stress intensity factor is made zero. Inclusion of a secondary material inside concrete shifts this comprehension to bridged crack model by the fact that intrinsic resistance of concrete to fracture is "reinforced" and *bridging law* is governed by the properties of the reinforcing phase and how it is coupled to concrete. Criterion for crack advance under monotonic loading for such model is

$$K_a - K_b = K_{IC} \tag{1}$$

where K_a and K_b are the stress intensity factors due to applied stress and reactive rebar force respectively, K_b being opposite to K_a.

It should be noted that recent constructions of smart structures with embedded sensor networks are also capable of determining real-time stresses in re-bars. This research focuses on existing infrastructure systems from a maintenance engineering point of view. Repair or retrofit works on the "sensor-less" existing infrastructures which have passed a fairly long period of service life are the aim of this paper. To furnish those, determination of physical and mechanical state of rebars is necessary and this paper describes a method to determine stress in rebars. Other non-destructive testing and evaluation (NDT&E) techniques (e.g. impulse-echo, acoustic emission and so on) have their limitations in accessibility, accuracy and economic viability[12] in determination stress in buried steel. The current method of inverse analysis on COD is versatile and applicable almost everywhere.

The objective of determining rebar force from COD can also be furnished by obtaining COD's at rebar locations only and by using steel constitutive relations. Even if it is difficult to determine COD's at rebar locations, crack mouth opening displacement can be measured and a particular shape of the crack (linear or quadratic) can be assumed to obtain COD's at those locations. Such computations inevitably needs the location of rebars known explicitly and possibly their sizes which we assume unknown in the current method. However, such a method for known cross-sections in the laboratory experiment will serve a check for the current method.

2. Bridged Crack Model of RC Beams under Flexure

Concrete fracture is characterized by extensive micro-cracking, surface roughness and three dimensional uneven apertures. Beams in flexure are idealized in **Fig. 1**. Zero or negligible shear assumption at the crack plane ensures a Mode I fracture condition. We consider a through-thickness crack to exploit 2D simulation where relevant quantities are applicable to unit thickness of the beam. A single dominant crack at the mid-

The opening of cracks by external load is deterministic which is computed exactly for any loading magnitude and distribution as

$$\bar{u}(x)=\frac{4}{E'}\int_x^a\left[\int_0^{a'}G(x',a',b)\sigma(x')dx'\right]G(x,a',b)da' \quad (6)$$

After rearranging Eq. (5), closing of cracks by the statically indeterminate reactive re-bar force can be expressed as

$$u(x)=\frac{4}{E'}\int_x^a\left[\int_0^{a'}G(x',a',b)f(x')dx'\right]G(x,a',b)da' \\ =\bar{u}(x)-u'(x) \quad (7)$$

The solutions of Eq. (2), (6) and (7) encounter singularities at $x=a$ which can be avoided analytically by substituting

$$x'=a'\sin\theta \text{ and } a'=x\cosh\theta \quad (8)$$

However, we consider Eq. (7) as a transformation between rebar force and crack closing values and approximate the transformation T by T_h, raising the upper integration limit a little bit higher. Convergence of the resulting quantities is checked for such alterations and most convergent one is considered for numerical computations. This approximation is shown in **Fig 3** using exact and approximated transformations for determination of crack profiles in the example of section 6. Such approximations in the transformation are well handled in Tikhonov method of regularization[7] as long as the corresponding approximated weight function G_h is real-valued, continuous on,

$$\Pi=\{0\le a'\le a, 0\le x<a'\} \quad (9)$$

and non-singular. The regularizing parameter α will be chosen based on this error in transformation $h>0$

$$\|T-T_h\|\le h \quad (10)$$

The values of h are calculated in numerous computations in example of section 6 with different definitions of norms in both the spaces and it was found that the order of h is always 10^{-6}-10^{-8} due to the approximation.

Further approximations include discretization of the weight function into its finite difference equivalent. A suitable grid is chosen as

$$\{a_k, k=1,\ldots\ldots,p\} \text{ and } \{x_k, k=1,\ldots\ldots,p\} \quad (11)$$

where $x_i\le a_j$, for $i\le j$ with equal step interval h_x. Consequently, the weight function is approximated with the matrix $d_{ij}\in \mathbf{D}\ (\cong G_h)$ for constant width of specimen as

$$\left.\begin{aligned} d_{ij}&=G_h(x_i,a_j),\quad i=2,\ldots\ldots,p-1\\ d_{ij}&=\frac{G_h(x_i,a_j)}{2},\ i=1,p\end{aligned}\right\}\ j=1,\ldots,p \quad (12)$$

The matrix D is a lower triangular matrix for the choice of grids stated above. The elements of the domain of T are the discrete bridging stresses, either computed by section analysis for an "forward problem" or to be determined by inverse analysis

$$f:=\{f_k, k=1,\ldots\ldots,p\} \quad (13)$$

where f_k for cracked parts without re-bars are almost zero. Experimental COD data should be collected following the same grid as

$$u':=\{u'_k, k=1,\ldots\ldots,p\} \quad (14)$$

Same discretization is utilized to determine crack opening by external load from Eq. (6)

$$\bar{u}:=\{\bar{u}_k, k=1,\ldots\ldots,p\} \quad (15)$$

Probable elements of the range of T are the crack closings in the grid deduced as

$$u=u'-\bar{u}:=\{u_k, k=1,\ldots\ldots,p\} \quad (16)$$

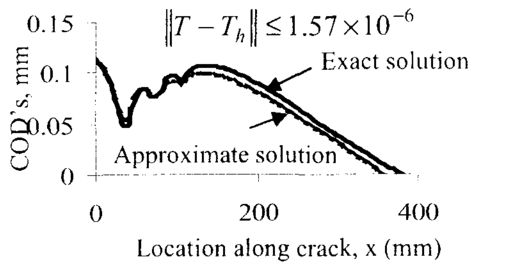

Fig. 3: Approximation in the transformation

4. Tikhonov Regularization

We consider Eq. (7) as a linear ill-posed problem

$$Tf=u, \text{ with } f\in F \text{ and } u\in U \quad (17)$$

with $T:D(T)\subset F\rightarrow U$ being a linear operator with closed range. F and U are infinite dimensional real Hilbert spaces with corresponding inner products and norms. Linearity of the transformation can easily be checked by proving

$$T(\alpha_1 f_1 + \alpha_2 f_2) = \alpha_1 T(f_1) + \alpha_2 T(f_2) \quad (18)$$

We are interested in the approximate solution f' giving the extremum of the following Tikhonov functional

$$M^{\alpha}[f] = \|T_h f - u_\delta\|^2_{L_2} + \alpha \|f\|^2_{E^2} \quad (19)$$

where we have $u_\delta \in U$, the available noisy data as

$$\|u - u_\delta\|_{L_2} \leq \delta \quad (20)$$

with known noise level, δ in the space $U = L_2[u_{\min}, u_{\max}]$. The p-dimensional Euclidean space F has the following definition of norm for its elements $f \in E^2$, with q=2

$$\|f\| = \left[\sum_{k=1}^{p} f_k^{\,q}\right]^{\frac{1}{q}}, \quad 1 < q < \infty \quad (21)$$

Numerical solution of ill-posed problems approximates the initial infinite-dimensional problem to a finite dimensional one, for which numerical algorithm and computer programs can be developed[7]. To ensure better convergence of the extremals of Tikhonov functional with that of finite dimensional approximation, the dimension of the finite dimensional approximation should increase unboundedly. We choose a sufficiently large dimension so that the error in the approximation is substantially small. Thus the following Tikhonov functional set by using Eq. (7) and (19)

$$M[f] = \int_0^{a}\left[\left\{\int_0^{a'} G_h(x', a') f(x') dx'\right\} \times G_h(x, a) da' - u_\delta(x)\right]^2 + \alpha \sum_{k=1}^{p} f_k^{\,2} \quad (22)$$

is approximated as

$$\hat{M}^{\alpha}[f] = \sum_{k=1}^{p}\left[\sum_{j=1}^{p} d_{kj}\left(\sum_{i=1}^{p} d_{ij} f_i\right) h_x - u_k\right]^2 h_x + \alpha \sum_{k=1}^{p} f_k^{\,2} \quad (23)$$

Variation of Eq. (23) with respect to f_k lead to the following p number of equations

$$h_x^2 \sum_{j=1}^{p}\sum_{k=1}^{p}\left(\sum_{i=1}^{p} d_{li} d_{ki}\right)\left(\sum_{i=1}^{p} d_{ji} d_{ki}\right) f_j + \alpha h_x f_l = h_x \sum_{k=1}^{p}\left(\sum_{i=1}^{p} d_{li} d_{ki}\right) u_k \quad (24)$$

for $l = 1, \ldots\ldots, p$. We solve this as the following system of equations

$$\mathbf{B}\, f + \alpha \mathbf{C}\, f = v \quad (25)$$

where

$$b_{lj} = h_x \sum_{k=1}^{p}\left(\sum_{i=1}^{p} d_{li} d_{ki}\right)\left(\sum_{i=1}^{p} d_{ji} d_{ki}\right) \in \mathbf{B} \quad (26)$$

$$v_l = \sum_{k=1}^{p}\left(\sum_{i=1}^{p} d_{li} d_{ki}\right) u_k \in \mathbf{v} \quad (27)$$

For the current choice of Euclidean norm shown in Eq. (21), the matrix $\mathbf{C}$ is a $p \times p$ identity matrix. The choice of the regularization parameter $\alpha > 0$ should be such that $\alpha \to 0$ in such a way that

$$\frac{(h+\delta)^2}{\alpha} \to 0. \quad (28)$$

5. Solution Procedure

Direct solution of Eq. (2) yields crack profiles where total rebar force, F under bending moment M is determined by a different RC section analysis as

$$F = \frac{M}{jd} \quad (29)$$

where j is the well-known section parameter. This computation usually under-estimates the rebar force for which COD profiles have small values at rebar location. Exact computation of F would have yielded zero COD at rebar locations. However, the inverse analysis will determine this rebar force from COD's without any cross-section details as required by other methods.

Small errors in computed bridging stress do not lead to large variations in COD as the transformation is defined by integration. The inverse mapping is the opposite of integration where a small error in COD can lead to errors greater than the *worst case error*[15].

The stringent 2D assumptions under estimate the COD's in direct analysis due to slip at rebar-concrete interface and practical COD profiles would be of higher values due to this. However, we assume that the gradient of a COD profile does not change for slip and so, there is no effect on inverse analysis as the governing equation Eq. (1) is an integral transform. The results of the inverse problem depend on the gradient of the COD profile, not on the absolute values.

In addition, tension in surrounding concrete leads a reverse channel shaped 3D profile for which COD's at rebar surface are almost zero but the surface COD's are higher. This affects the results of inverse analysis for which the inverse analysis will underestimate the rebar force in practical cases which needs to be verified by experiment.

Experimental COD's $[u'(x)]$ should be deducted from crack openings in Eq. (15) to get crack closings by bridging stress for inverse analysis of the operator equation Eq. (7). For analytical checking, experimental noise can be simulated by adding random numbers of particular mean (e.g. zero) and width ω of Gaussian distribution to the COD's in Eq. (14).

In the following example, re-bar forces in a cracked beam are estimated as bridging stress per unit length by RC cracked section analysis for a particular external load (**Fig. 4**). The crack length is computed by fracture condition at the crack tip for monotonic loading. Crack profiles are estimated by Eq. (2). The COD's are made noisy by adding computer generated random numbers of Gaussian distribution with them (**Fig. 5**). The noisy crack profiles are input in Eq. (27) for inverse analysis to retrieve the bridging stress distribution in **Fig. 4**. No cross-sectional information is necessary here. The retrieved bridging stress with different percentages of error levels are shown in **Fig. 6**. It is observed that larger error level leads to larger variations in the results.

6. Numerical Example

A 250 mm x 554 mm concrete beam is reinforced with three layers of re-bars with steel areas 900 mm^2, 400 mm^2 and 200 mm^2 arranged at 25 mm, 65 mm and 101 mm clear distances from the bottom face. A section is cracked under a bending moment 176 KN-m where shear stress is zero. Fracture condition in Eq. (1) shows that the crack length is 380 mm for $K_{IC} = 15$ N/mm$^{3/2}$. The steel stresses found by cracked transformed section analysis are 278 MPa, 258 MPa and 240 MPa respectively from the bottom. Using a concrete of cylinder strength, $f_c' = 20\,\text{MPa}$, steel Young's modulus, $E_s = 200\,\text{GPa}$ and a plain strain condition, the crack profile is determined by solving Eq. (2) which is called forward analysis here.

6.1 Forward Analysis

The distribution of re-bar stress at different layers is appropriately simulated as bridging stress per unit length of the crack following Eq. (4) and is shown in **Fig. 4**. The step function has values only at the re-bar locations and zero elsewhere.

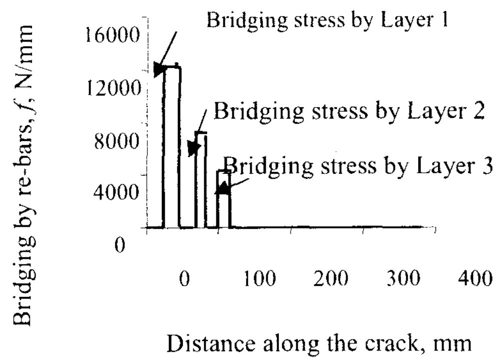

Fig. 4: Simulation of bridging stress by re-bars with Heaviside step function in Eq. (4).

The only forward solution for this problem by Eq. (2) with linear bending stress is the crack profile (firm line) in **Fig. 5**. The profile shows depressions at the re-bar locations. One hundred data points (COD's) are picked up from this curve. Random numbers of Gaussian distribution with zero mean and widths 0.001 and 0.005 are added to make them noisy (dotted lines) which created errors of 1.32 % and 7.25 % in COD data. It is advantageous to have more points near the re-bar locations where the crack widths vary rapidly and has profound effect on the resulting re-bar stress. But experimentally collected surface COD's lack reliability of the information about re-bars' location. So, it is better to choose as fine grid as permitted by experimental equipment and computation time.

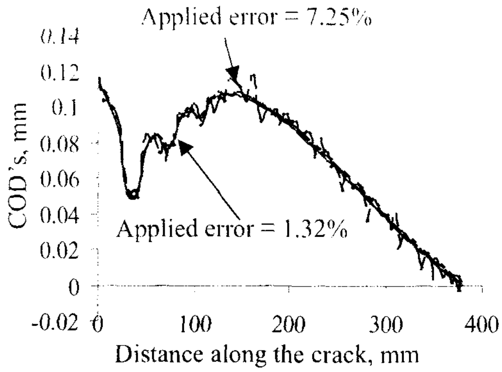

Fig. 5: Exact and noisy crack profiles

6.2 Inverse Analysis

Noisy crack profiles yielded the bridging stress distributions in **Fig. 6** by Tikhonov regularization method i.e. by solving of Eq. (25) for *f*.

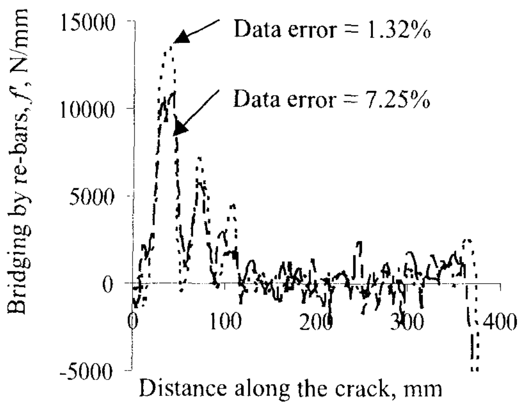

Fig. 6: Retrieval of bridging stress profile of Fig. 4 by inverse analysis on noisy COD data.

It is observed that data with very low level of noise retrieves exact stress pattern in **Fig. 4** and re-bar force becomes more and more approximate with the increase of noise levels. A comparative assessment of the errors in data with the errors in results is shown in **Table 1**. It is noticed that data errors increase errors in results but lacks consistency due to randomness. Data errors are calculated in terms on its L_2 norm as

$$\text{error} = \frac{\|u - u_\delta\|}{\|u\|} \times 100 \qquad (30)$$

Errors in results are calculated as percent errors in total force of re-bars from the exact value of section analysis. Small fluctuations in choosing the regularizing parameter are not significant as long as it is chosen according to Eq. (28). Further improvement of the results can be achieved by another forward analysis with the current re-bar forces from inverse analysis. But, results changed very insignificantly in this case and it became stable after one iteration.

Table 1: Demonstration of errors in inverse analysis results with errors in input COD data.

Gaussian distribution width of random numbers ω	Errors of COD data in L_2-norm δ Eq. (20)	Regularizing parameter α	Percent error of COD in terms of L_2-norm Eq. (29)	Percent error in total re-bars force
0.002	0.0196	0.2	2.75	4.68
0.004	0.0368	0.3	4.84	4.63
0.006	0.0601	0.6	7.91	5.38
0.008	0.0768	0.7	10.11	4.21
0.010	0.1000	0.8	13.22	7.34

Very sophisticated technique should be followed to collect COD's from field to minimize noise as much as possible. Instrumental and computational errors should be defined as accurately as possible for correct choice of the regularizing parameter. A technique involving image analysis on microscopic pictures is being developed under this research.

7. Conclusions

A straightforward method has been presented to compute inner re-bar location and force from outer surface crack measurements. The method is suitable for decaying infrastructures which need urgent maintenance and require their system identification by non-destructive methods. Current method requiring only crack opening data has huge potential for industrial use in maintenance engineering. Although it is explained for an ideal case of single edge cracked specimen under monotonic loading, it can be extended to multiple cracking if appropriate weight function is derived. It can also be extended to repeated loading cases by including variable micro-structural parameters in the model.

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APPENDIX

Weight function for single edged cracked specimen is expressed with the help of following $h_1(x/a, a/w)$ function as

$$h_1(x/a, a/w) = \frac{g(x/a, a/w)}{(1-a/w)^{3/2}} \quad \text{(A-1)}$$

where $g(x/a, a/w) = g(r,s)$ is given by

$$g(r,s) = g_1(s) + rg_2(s) + r^2 g_3(s) + r^3 g_4(s) \quad \text{(A-2)}$$

$$g_1(s) = 0.46 + 3.06s + 0.84(1-s)^5 + 0.66s^2(1-s)^2 \quad \text{(A-3)}$$

$$g_2(s) = -3.52s^2 \quad \text{(A-4)}$$

$$g_3(s) = 6.17 - 28.22s + 34.54s^2 - 14.39s^3 - (1-s)^{3/2} - 5.88(1-s)^5 - 2.64s^2(1-s)^2 \quad \text{(A-5)}$$

$$g_4(s) = -6.63 + 25.16s - 31.04s^2 + 14.41s^3 + 2(1-s)^{3/2} - 5.04(1-s)^5 + 1.98s^2(1-s)^2 \quad \text{(A-6)}$$

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