

New Method for Calculating Bottom Shear Stress under Skew Waves

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The bottom shear stress under skew waves can be used in a sediment transport modeling under rapid acceleration occurring in the surf zone. The bottom shear stress in a rough turbulent bottom boundary layer under sawtooth waves has been examined through two methods by Suntoyo and Tanaka¹⁾. One is to fit logarithmic velocity profile to the measured velocity in an oscillating wind tunnel, another is a calculation method incorporating acceleration effect. However, an investigation of a more reliable calculation method to estimate the time-variation of bottom shear stress has not been fully dealt with. In the present study, the bottom shear stress of experimental result will be examined with a new calculation method of bottom shear stress based on incorporating velocity and acceleration terms all at once. A new coefficient is proposed to express a wave skewness effect on bottom shear stress under skew waves.

Key Words: Bottom shear stress, skew waves, turbulent bottom boundary layer

1. Introduction

Most of flows transporting sediment are turbulent boundary layer shear flows and the forces exerted on the sea bottom are governed by turbulent characteristics. Bottom shear stress and turbulence in wave motion are key parameters for moving sediment particle and keeping it in suspension. To understand the sediment suspension in the turbulent flow is very important to analyze the influence of turbulence structure and bottom shear stress on the particle settling and pick up through turbulence. Erosion and sediment transport are initiated by cycles of downward sweeps and upward bursts resulting from turbulent behavior of the wave boundary layer.

Realistic waves in nature often have a shape of sawtooth wave or skew wave when propagating to shallow water. Their heights increase and their lengths decrease, furthermore they become remarkably nonlinear waves. Moreover, both wave velocity skewness and asymmetric increase to their maximum values at the onset of breaking. Hence, a simple harmonic variation as sinusoidal wave can not describe the boundary layer behavior occurring in the surf zone in which major part of nearshore sediment are transported.

Schäffer and Svendsen²⁾ had presented the sawtooth wave as

a method expressing the wave motion under broken waves. Asano et al.³⁾ examined the characteristics of the laminar and the turbulent boundary layer for sawtooth waves. Moreover, Samad and Tanaka⁴⁾ had analyzed the flow behavior in bottom boundary layer under sawtooth wave for both laminar and turbulent flow condition by $k-\varepsilon$ numerical model for smooth bed. Nielsen⁵⁾ proposed an incorporation of the acceleration effect to the bottom shear stress calculation under a bit of sawtooth asymmetry wave, but that method has not been applied to the sawtooth wave presented by researchers recently. Suntoyo and Tanaka¹⁾ and Suntoyo et al.⁶⁾ had investigated bottom shear stress under sawtooth waves and examined it with two simple calculation method of bottom shear stress based on the consideration of the friction coefficient for sinusoidal wave motion and that of the acceleration effect for sawtooth wave. However, an investigation of a more reliable calculation method to estimate the time-variation of bottom shear stress under skew waves has not been fully dealt with.

In the present study, we aim to examine the bottom shear stress through experiments in an oscillating wind tunnel over rough bed under skew waves by means of Laser Doppler Velocimeter (LDV) to measure velocity distribution. Furthermore, a new estimation method of the instantaneous

¶ Dedicated to the memory of Prof. Michihiro KITAHARA

bottom shear stress under skew waves based on incorporating both velocity and acceleration terms is proposed. Moreover, a new coefficient is proposed to express a wave skewness effect on bottom shear stress under skew waves.

2. Experimental Method

The experiments have been carried out in an oscillating wind tunnel connected with the piston system with air as the working fluid and smoke particles as tracer. This is intended to make an easy treatment if it is compared with water as the working fluid. The experimental system consists of the oscillatory flow generation unit and a flow-measuring unit. At first the flow rate of sharp-crested sawtooth wave was attempted as input directly to the flow generation unit, but this flow generation unit was impossible to follow the acceleration change with a rapid movement. At last the shape of sawtooth wave as presented by Schäffer and Svendsen²⁾ is done smoothing at both crest and trough part until the flow generation unit works well. The definition sketch for sawtooth wave after smoothing is further given as shown in Fig. 1.

The oscillatory flow generation unit was made up of signal control and processing components along with piston mechanism. The piston displacement signal has been fed into the instrument through a PC. Input digital signal has been converted to corresponding analog data through a digital-analog (DA) converter. A servomotor, connected through a servomotor driver, was driven by the analog signal. The piston mechanism has been mounted on a screw bar, which was connected to the servomotor. The feed-back on piston displacement, from one instant to the next, has been obtained through a potentiometer that compared the position of the piston at every instant to that of the input signal, and subsequently adjusted the servomotor driver for position at the next instant. The measured flow velocity record was collected by means of an A/D converter with 1/100s intervals, and obtained the mean velocity profile by averaging over 50 wave cycles. A schematic diagram of the experimental set-up is shown in Fig. 2.

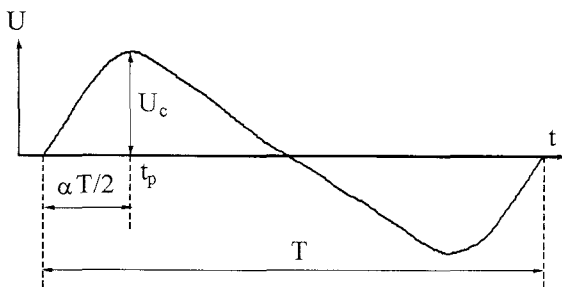


Fig. 1 Definition sketch for sawtooth wave

The flow-measuring unit comprised of a wind tunnel and one component Laser Doppler Velocimeter (LDV) for flow

measurement. Velocity measurements were carried out at 20 points in the vertical direction at the center part of wind tunnel by means of Laser Doppler Velocimeter. The wind tunnel has a length of 5 m and the height and width of the cross-section are 20 cm and 10 cm, respectively. The dimension of this cross-section of wind tunnel has been considered in order to the flow velocity was not influenced by the sidewalls effect. The triangular roughness having a height of 5 mm and 10 mm width was pasted over the bottom surface of the wind tunnel at spacing of 12 mm along the wind tunnel, as shown in Fig. 3. This geometry of roughness elements is chosen in order to the roughness elements protrude out of the viscous sub-layer. This causes a wake behind each roughness element, and the shear stress is transmitted to the bottom by the pressure drag on the roughness elements. Viscosity becomes irrelevant for determining either the velocity distribution or the overall drag on the surface. Thus, the velocity distribution near a rough surface is logarithmic. It can be therefore assumed that the logarithmic law as shown in Eq. (1) can be used to estimate the bottom shear stress $\tau_o(t)$ over rough bed^{7,8)}.

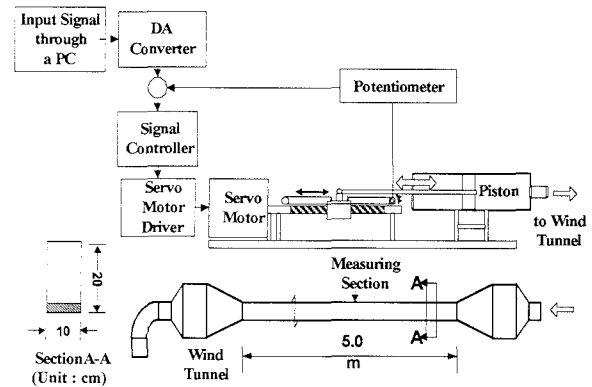


Fig. 2 Schematic diagram of experimental set-up

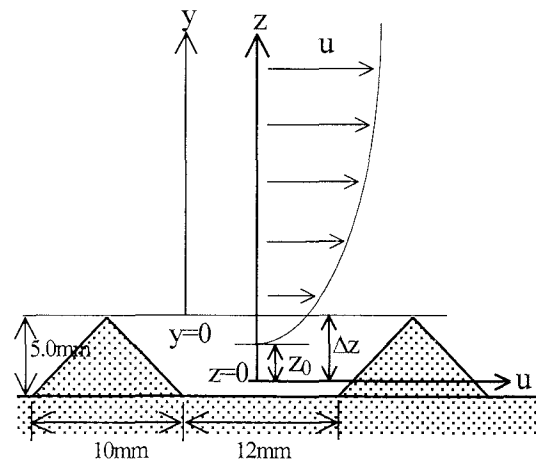


Fig. 3 Definition sketch for roughness

Experiments have been carried out for four cases under sawtooth waves. The experimental conditions are given in **Table 1**. The peak flow rate was kept almost 400 cm/s for all cases. The Reynolds number magnitude defined for each case has sufficed to locate these cases in the rough turbulent regime.

Here, a_m/k_s : the roughness parameter, k_s : the Nikuradse's roughness equivalent and $a_m \cdot U_c / \omega \lambda$, where, U_c : the velocity at wave crest, ω the angular frequency, T : wave period, t_p : time interval measured from the zero-up cross point to wave crest in the time variation of free stream velocity, while α shows the wave skewness parameter, as shown in **Fig.1**. The smaller α indicates more remarkable wave skewness, while for $\alpha=0.500$ corresponds to the symmetric wave without skewness. Furthermore, the shape of waves at the free stream velocity, U in these cases is shown in **Fig. 4**. **Fig. 5** shows the time-variation of acceleration for all cases. It can be seen that smaller value of α gives higher acceleration, otherwise higher value of α gives lower acceleration as seen in Case 4 for $\alpha = 0.500$.

In addition, when the boundary layer thickness was estimated with the existing formula for rough turbulent as proposed by Jonsson and Carlsen⁷, and was compared with the experimental data show apparently in a good agreement, namely about 7.5 cm. Therefore, it is judged that boundary layer develops enough in the test section, as shown in **Fig. 3**. Moreover, it was confirmed that the velocity measurement at the center of the roughness and at the flaking off region around the roughness has shown a similar flow distribution⁷.

Table 1 Experimental conditions

Exp.	U_c (cm/s)	T (s)	Re	a_m/k_s	α
Case 1	398	4.00	6.96×10^5	35.1	0.314
Case 2	399	4.00	6.89×10^5	35.1	0.363
Case 3	400	4.00	6.93×10^5	35.2	0.406
Case 4	400	4.00	6.75×10^5	35.2	0.500

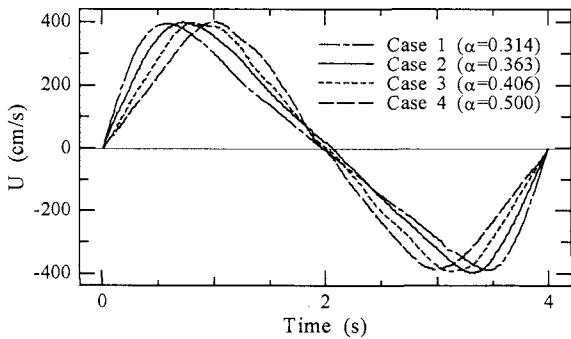


Fig. 4 Time-variation of free stream velocity for all cases

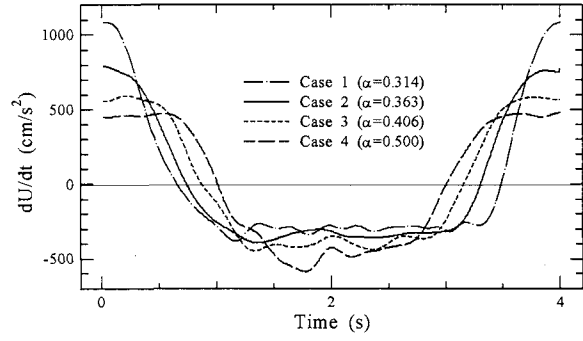


Fig. 5 Time - variation of acceleration for all cases

3. Results and Discussions

3.1 Bottom shear stress of experimental results

Bottom shear stress is estimated by fitting the logarithmic velocity distribution to the measured velocity data, which is given in Eq. (1),

$$u = \frac{U_*}{\kappa} \ln \left(\frac{z}{z_0} \right) \quad (1)$$

where, u : the flow velocity in the boundary layer, κ : the von Karman's constant (≈ 0.4), z : the cross-stream distance from theoretical bed level ($z = y + \Delta z$) as previously shown in **Fig. 3**. Eq.(1) denotes the logarithmic velocity profile expressed in term of z_0 denoting the value of z at which the logarithmic velocity profile predicts a velocity of zero. For a smooth bottom $z_0=0$, but for rough bottom, the elevation of theoretical bed level is not a single value above the actual bed surface. The value of z_0 for the fully rough turbulent flow is obtained by extrapolation of the logarithmic velocity distribution above the bed to the value $z = z_0$ where u vanishes. The value variation of Δz and z_0 are obtained from the extrapolation results of the logarithmic velocity distribution on the fitting a straight line of the logarithmic distribution through a set of velocity profile data at the selected phases angle for each case. These obtained values of Δz and z_0 are further averaged to get $z_0 = 0.241$ cm for all cases and $\Delta z = 0.254$ cm, $\Delta z = 0.171$ cm, $\Delta z = 0.180$ cm and $\Delta z = 0.010$ cm, for Case 1, Case 2, Case 3 and Case 4, respectively in the present study. Furthermore, the bottom roughness, k_s can be obtained by applying the Nikuradse's equivalent roughness in which $z_0 = k_s/30$. By plotting u against $\ln(z/z_0)$, a straight line is drawn through the experimental data, the value of friction velocity, U_* can be obtained from the slope of this line and bottom shear stress, τ_0 can then be obtained from Eq. (2).

$$U_* = \sqrt{\tau_0 / \rho} \quad (2)$$

The obtained value of Δz and z_0 as the above mentioned has a sufficient accuracy for application of logarithmic law in a wide range of velocity profile near bottom region. Moreover, in the previous study, Suzuki et al.⁹⁾ has presented the method to obtain these value in detail and shown in a good accuracy.

Fig. 6 shows the time-variation of bottom shear stress under skew waves with variation in the wave skewness parameter, α . It can be seen that the bottom shear stress under skew waves have an asymmetric shape on both crest and trough. The asymmetric of bottom shear stress are caused by wave skewness effect corresponding with acceleration effect. The increasing of wave skewness is followed by increasingly the asymmetric of bottom shear stress, otherwise for the wave without skewness is close to a symmetric shape, as seen in Case 4 for $\alpha = 0.500$ in Fig. 6.

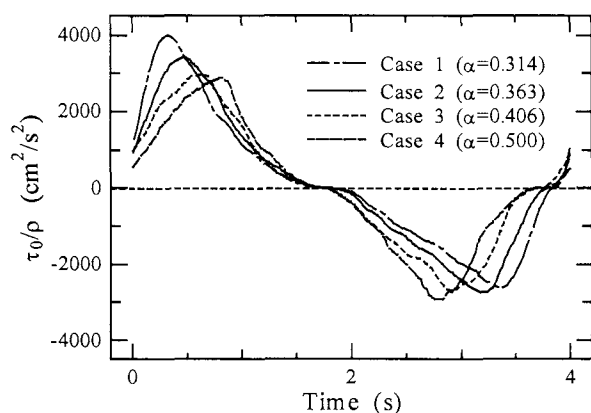


Fig. 6 Time - variation of bottom shear stress of experimental results.

3.2 Calculation method of bottom shear stress under sawtooth waves

Suntoyo and Tanaka¹⁾ and Suntoyo et al.⁶⁾ had examined the bottom shear stress under sawtooth waves from experimental result with two existing calculation methods to estimate bottom shear stress, namely based on consideration of friction coefficient for sinusoidal wave as proposed by Tanaka et al.¹⁰⁾ and based on incorporating the acceleration effect under a bit of sawtooth asymmetry wave as proposed by Nielsen⁵⁾. It had been shown that both existing methods could not cover the characteristic of bottom shear stress under sawtooth waves in which the crest part was larger than the trough part as expressed in the experimental results^{1),6)}. In this paper, we will present a new calculation method to compute the bottom shear stress under skew waves and the existing calculation method as proposed by Nielsen⁵⁾ and Suntoyo and Tanaka¹⁾ are also given.

(1) A new calculation method of bottom shear stress under skew waves

A new calculation method of bottom shear stress under skew waves is based on incorporating velocity and acceleration terms all at once that is given through the instantaneous friction velocity, $U_*(t)$ as proposed by authors in Eq. (3). Both velocity and acceleration terms are adopted from a calculation method proposed by Nielsen⁵⁾, but that method could not give a good agreement with experimental data, so in the new calculation method is proposed a new acceleration coefficient expressing the wave skewness effect on the bottom shear stress under skew waves. The instantaneous bottom shear stress can be calculated proportional to the square of the proposed instantaneous friction velocity, as shown in Eq. (4),

$$U_*(t) = \sqrt{f_w/2} \left\{ U \left(t + \frac{\varphi}{\omega} \right) + \frac{a_c}{\omega} \frac{\partial U(t)}{\partial t} \right\} \quad (3)$$

$$\tau_0(t) = \rho U_*(t) |U_*(t)| \quad (4)$$

$$f_w = \exp \left\{ -7.53 + 8.07 \left(\frac{a_m}{z_0} \right)^{-0.100} \right\} \quad (5)$$

The value of acceleration coefficient, a_c is obtained from average value of the time variation of acceleration coefficient, $a_c(t)$ as expressed in the following Eq. (6),

$$a_c(t) = \frac{U_*(t) - \sqrt{f_w/2} U \left(t + \frac{\varphi}{\omega} \right)}{\frac{\sqrt{f_w/2}}{\omega} \frac{\partial U(t)}{\partial t}} \quad (6)$$

where, f_w : the wave friction coefficient. The friction coefficient proposed by Tanaka and Thu¹¹⁾ as given in Eq. (5) can be used for evaluating in Eq. (3). $\tau_0(t)$: the instantaneous bottom shear stress, φ : the phase difference between free stream velocity and bottom shear stress and a_c : the acceleration coefficient.

Fig. 7 shows the phase difference obtained from measured data under skew waves, as well as from theory as proposed by Tanaka and Thu¹¹⁾ in Eq. (7). This formula was reasonable to estimate the phase difference for sinusoidal wave cases, as shown by Tanaka and Thu¹¹⁾. Fig. 7 shows that the phase difference at the crest (O) is more close to theory for all cases, otherwise at the trough (Δ) is lower than theory, namely about 10° to 19° and the average values between crest and trough (\blacktriangle) is about 18° . For the case of

$\alpha = 0.500$ the phase difference at the crest catch to this theory, namely about 23° . It is then proposed to modify this theory with consider the wave skewness effect under skew wave as shown in Eq. (9). Thus, there are three approximation of phase difference that will be used to obtain the value of a_c , namely (i) phase difference calculated based on an equation proposed by Tanaka and Thu⁽¹¹⁾, as given in Eq. (7); (ii) phase difference obtained from the average value between crest and trough part of measured data under skew waves and (iii) phase difference calculated from Eq. (9) based on Eq. (7) with consider the wave skewness effect under skew waves.

$$\varphi = 42.4C^{0.153} \frac{1 + 0.00279C^{-0.357}}{1 + 0.127C^{0.563}} \text{ (degree)} \quad (7)$$

$$C = \frac{1}{\kappa \sqrt{\frac{f_w}{2} \frac{a_m}{z_0}}} \quad (8)$$

$$\varphi_1 = \frac{\alpha T}{2} \frac{4\varphi}{T} \text{ (degree)} \quad (9)$$

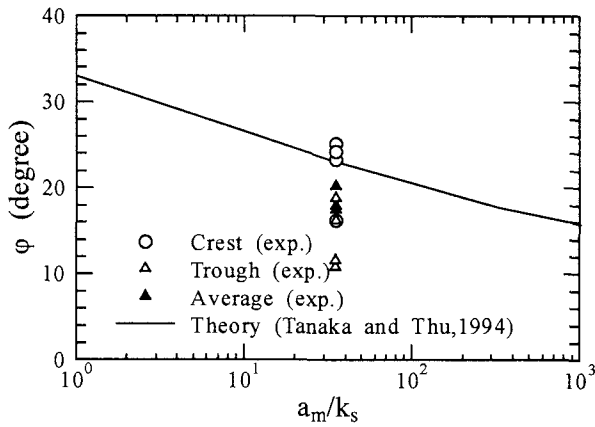


Fig. 7 Phase difference

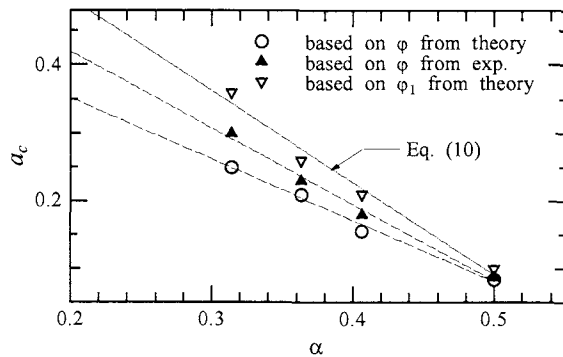


Fig. 8 Acceleration coefficient, a_c as function of α

Hereafter, the values of a_c as function of α from three approximation of phase difference are plotted in Fig. 8. The value of a_c based on phase difference from Eq. (9) gives the best result than others as shown on correlation result of friction velocity, U_* between experiment and calculation results as shown in Fig. 9, 10, 11 and 12 for Case 1, Case 2, Case 3 and Case 4, respectively. It can be concluded that the phase difference as proposed in Eq. (9) is reasonable to be used to evaluate the bottom shear stress under skew waves. Furthermore, an equation based on regression line to estimate the acceleration coefficient, a_c is proposed as given in Eq. (10), as follows

$$a_c = -1.355\alpha + 0.769 \quad (10)$$

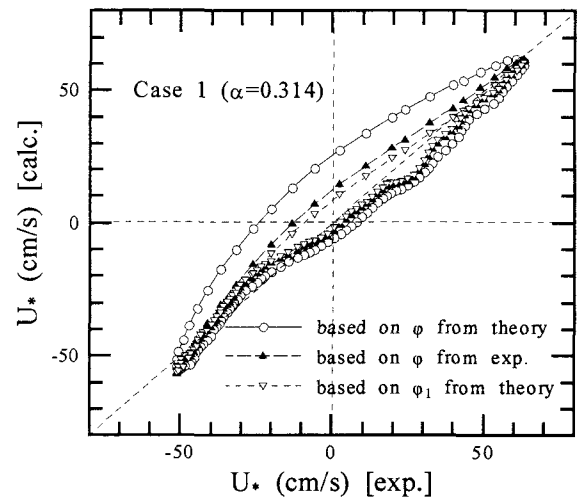


Fig. 9 Correlation of friction velocity between experiment and calculation result, Case 1

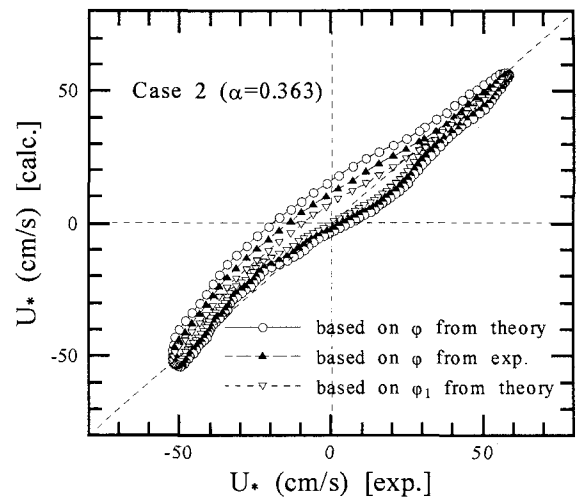


Fig. 10 Correlation of friction velocity between experiment and calculation result, Case 2

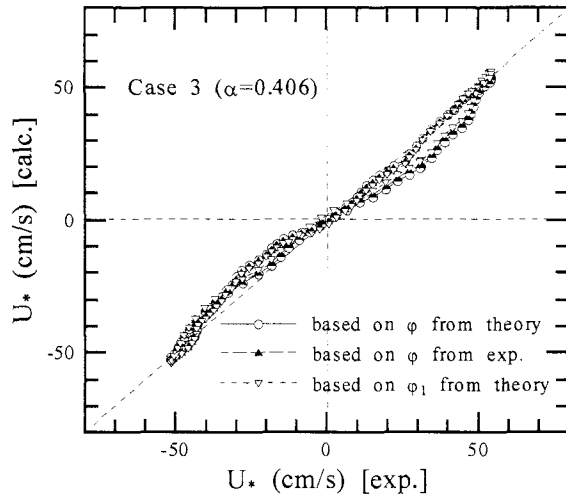


Fig. 11 Correlation of friction velocity between experiment and calculation result, Case 3

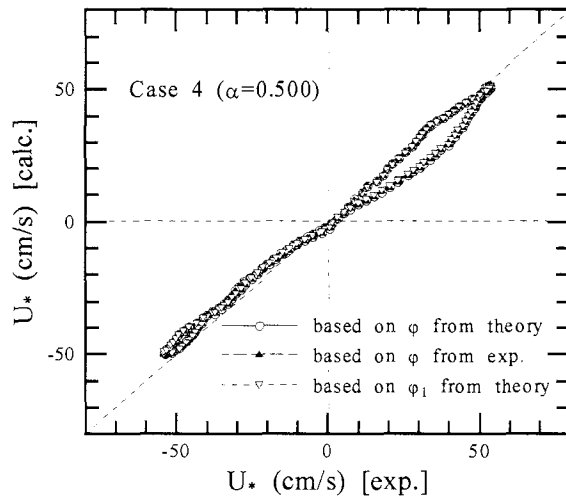


Fig. 12 Correlation of friction velocity between experiment and calculation result, Case 4

Figs. 13, 14, 15 and 16 show the time variation of friction velocity from experimental and the new calculation method results incorporating velocity and acceleration terms as expressed in Eq. (3) for Case 1, Case 2, Case 3 and Case 4, respectively. Acceleration term gives a significant contribution for smaller α as shown in **Fig. 13**, while for $\alpha=0.500$ the contribution of acceleration term almost does not appear as shown in **Fig. 16**.

It can be concluded that the contribution of acceleration term decreases with increasing the value of α or with decreasing the wave skewness parameter. And the acceleration term is unnecessary to be considered significantly in calculating the bottom shear stress for the symmetric waves without skewness at $\alpha = 0.500$.

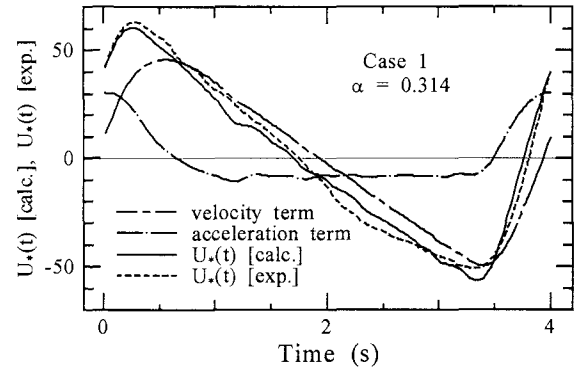


Fig. 13 Time - variation of friction velocity, for Case 1

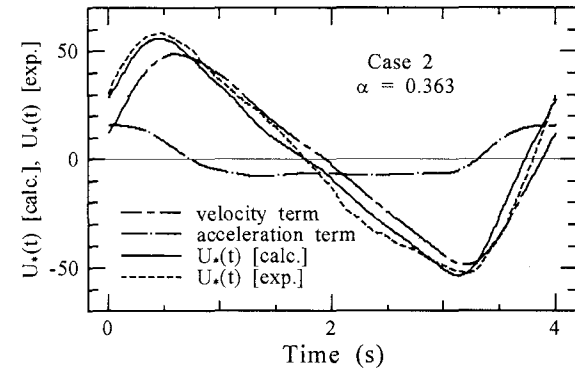


Fig. 14 Time - variation of friction velocity, for Case 2

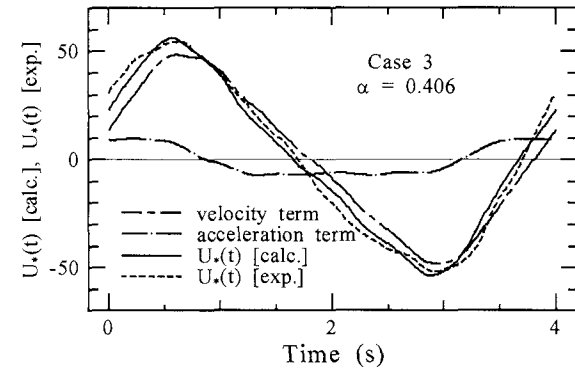


Fig. 15 Time - variation of friction velocity, for Case 3

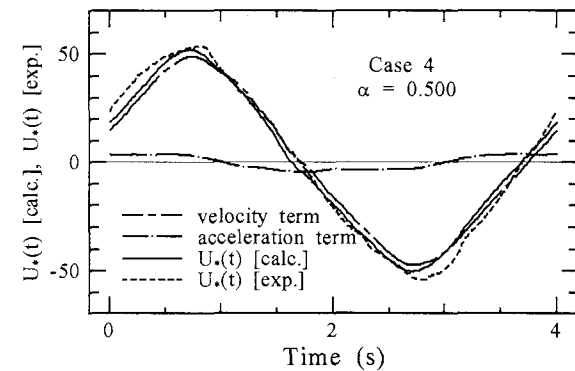


Fig. 16 Time - variation of friction velocity, for Case 4

(2) Comparison with existing calculation methods

The new calculation method of bottom shear stress under skew waves is examined by the existing calculation methods that had been used to examine experimental results as used by Suntoyo and Tanaka¹⁾ and Suntoyo et al.⁶⁾. Method 1 is proportional to the square of the time variation of $U(t)$, as proposed by Tanaka et al.⁸⁾ in Eq. (11), where f_w is calculated from Eq. (5);

$$\tau_0 \left(t - \frac{\varphi}{\omega} \right) = \frac{1}{2} \rho f_w U(t) |U(t)| \quad (11)$$

Method 2 is proportional to the square of the instantaneous friction velocity, $U_*(t)$ incorporating the acceleration effect under a bit of sawtooth asymmetric wave as proposed by Nielsen⁷⁾ in Eq. (12) and (13), as follows

$$U_*(t) = \sqrt{\frac{f_w}{2}} \left\{ \cos \varphi U(t) + \sin \varphi \frac{\partial U(t)}{\partial t} \right\} \quad (12)$$

$$\tau_0(t) = \rho U_*(t) |U_*(t)| \quad (13)$$

Phase difference equation given in Eq. (9) is used for calculating in Method 1 and Method 2. Friction coefficient used in Method 2 is calculated from an equation in Eq. (14) as proposed by Nielsen¹²⁾, as follows

$$f_w = \exp \left\{ 5.5 \left(\frac{a_m}{k_s} \right)^{-0.2} - 6.3 \right\} \quad (14)$$

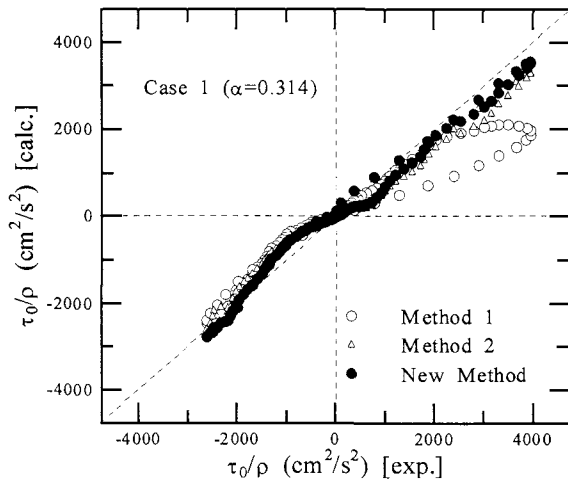


Fig. 17 Correlation between experimental and calculation results of bottom shear stress, for Case 1

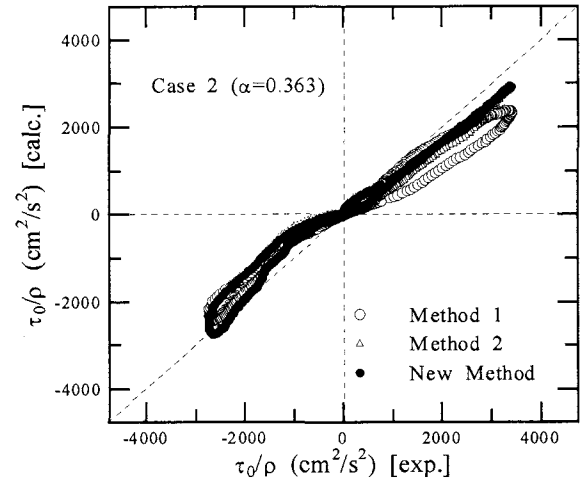


Fig. 18 Correlation between experimental and calculation results of bottom shear stress, for Case 2

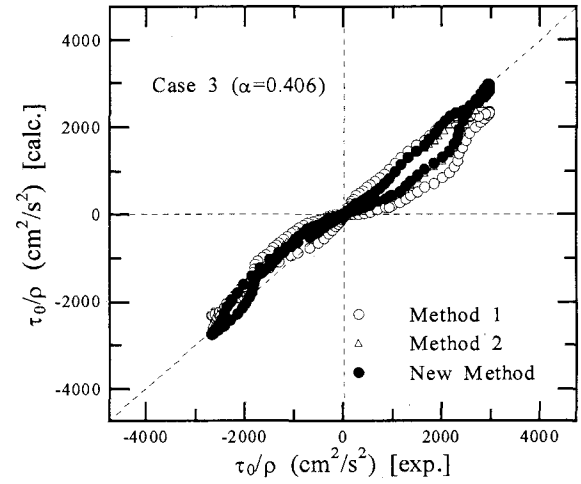


Fig. 19 Correlation between experimental and calculation results of bottom shear stress, for Case 3

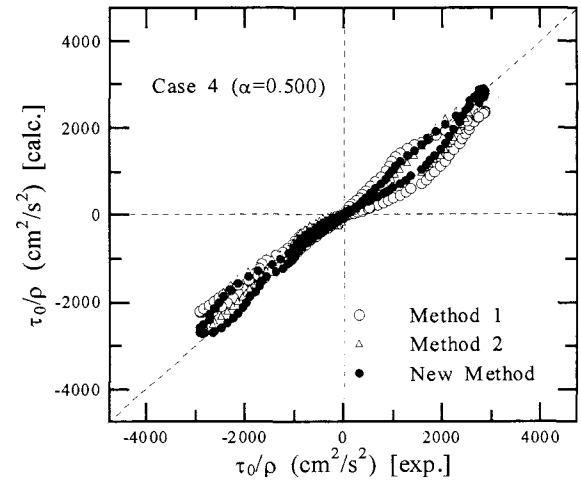


Fig. 20 Correlation between experimental and calculation results of bottom shear stress, for Case 4

Correlation between the bottom shear stress of experimental result and the calculation results from three calculation methods for all cases are shown in Figs. 17, 18, 19 and 20. The new

method gives the best agreement with the bottom shear stress under skew waves from experimental results than others method. While, Method 1 and Method 2 gave underestimated value at crest part of bottom shear stress from experimental results, nevertheless, at trough part of bottom shear stress from three methods gave almost the same estimated value, as shown in Figs. 17, 18 and 19. Moreover, for the symmetric wave case without skewness at $\alpha = 0.500$ as shown in Fig. 20 for Case 4, the new method still gave the best agreement, though the difference from others methods was very small.

3.3 Performance of calculation methods of bottom shear stress

The calculation method performance of bottom shear stress can be evaluated by the root-mean-square error (*RMSE*), as follow

$$RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^N (U_{*cal,i} - U_{*exp,i})^2} \quad (15)$$

where, U_{*cal} : the friction velocity from calculation methods, U_{*exp} : the friction velocity from experimental results, N : the total number of data and i : index. If the calculation method is perfect, it can be indicated that the *RMSE* should be zero. It can be concluded that the smaller *RMSE* is better the performance of the calculation methods. The summary of calculation method performance of bottom shear stress is shown in Table 2.

Table 2 The summary of calculation method performance of bottom shear stress

Exp.	The Root-Mean-Square Error (<i>RMSE</i>) (cm/s)		
	Method 1	Method 2	New Method
Case 1	8.33	5.94	4.48
Case 2	8.22	5.61	4.38
Case 3	8.11	5.44	4.33
Case 4	5.52	4.95	4.30

As shown in Table 2 that the new method has highest performance than others methods with *RMSE* = 4.48 for Case 1, *RMSE* = 4.38 for Case 2, *RMSE* = 4.33 for Case 3 and *RMSE* = 4.30 for Case 4. For all the cases, Method 2 is better than Method 1. While, for Case 4 where the wave skewness is small, at $\alpha = 0.500$ the difference of *RMSE* value for all methods is very small, but the new method still gave the smallest the *RMSE* value indicating that the new method has the best agreement with the bottom shear stress of experimental results. It can be concluded that the new method can be used to estimate the bottom shear stress under skew waves for higher wave skewness

up to the symmetric wave without skewness for $\alpha = 0.500$ and also the phase difference and acceleration coefficient, a_c that have been defined in Eq. (9) and Eq. (10) were sufficient for this calculation. Therefore, the new method can be used to calculate the bottom shear stress under skew waves that can be further used to an input sediment transport model under rapid acceleration in practical application.

4. Conclusions

In this study, the bottom shear stress under skew waves have been examined and a number of conclusions can be drawn, as follows

1. Bottom shear stress under skew waves has the asymmetric shape caused by wave skewness effect corresponding with acceleration effect. The increasing of wave skewness is followed by increasingly the asymmetric of bottom shear stress on the crest and trough.
2. The new calculation method gave the best agreement with bottom shear stress under skew waves from experimental result. Furthermore, both the phase difference and the acceleration coefficient defined in the new method were sufficient for this calculation. It can be concluded that the new method for calculating the instantaneous bottom shear stress under skew waves proposed in this study has a sufficient accuracy. Therefore, this method can be used to an input sediment transport model under rapid acceleration in a practical application.

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