

# EFFECT OF VERTICAL WEB STIFFENERS ON LATERAL TORSIONAL BUCKLING BEHAVIOR OF CANTILEVER STEEL I-BEAMS

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Lateral torsional buckling often governs design of I-beams. Although vertical web stiffeners are extensively used to provide internal stiffening, their effect on lateral torsional buckling behavior is totally ignored in the design codes. Here, effect of stiffeners on cantilever beams studied under static uniform moment, uniform load, and end concentrated load. Finite element buckling solver has been used and then linear regression analysis for output data of finite element was conducted to produce simplified equation for the critical moment including stiffeners effect. The results show that stiffeners cause significant magnification in the critical moment than the basic critical moment of the case of beam without stiffeners.

**Key Words:** lateral torsional buckling, cantilever beams, vertical web stiffeners, uniform torsion, warping torsion.

## 1. INTRODUCTION

It is well known that beams of thin-walled open cross sections composed of slender component plates, such as I-sections, are particularly susceptible to lateral torsional buckling. This is because the torsional rigidities of such cross sections are very low and therefore their resistance to torsional instability is low. The effects of unbraced length and end conditions on the elastic lateral torsional buckling load of the beam are rather evident. The longer the unbraced length and the less resistant the support can deliver to the beam, the lower the critical lateral buckling load will be<sup>1)2)3)</sup>. Hence, the optimum method to prevent the lateral torsional buckling is using lateral bracing to the compression flange to reduce its unsupported length<sup>4)5)</sup>. However, if the lateral bracing is not feasible the internal stiffening that provided by the vertical web stiffeners could be used to enhance beam resistance for the lateral torsional buckling<sup>6)7)</sup>.

The main idea behind the expected increase in lateral torsional buckling resistance of beams depends on connecting the behavior of the tension flange with the behavior of the compression flange by using the vertical web stiffeners. This reduces, somewhat, the tendency of the beam to buckle out of

plane in other words it will increase the warping torsion resistance of the beam<sup>8)9)10)</sup>. Some lateral torsional buckling problems have closed form solutions<sup>1)11)12)</sup> while the others have numerical solutions for different loading, and boundary conditions<sup>13)</sup>. However, on the other hand, codes specifications totally ignored the effect of vertical web stiffeners on the lateral torsional buckling.

Hereafter, cantilever beams have been studied using computer model<sup>14)</sup> that has been used for linear buckling analysis so that to match the thin-walled beam theory assumptions in buckling analysis. Moreover, different sets of static loading have been considered: Uniform Moment, Uniform Load, and End Concentrated Load. An equation is presented to take into consideration the effect of vertical web stiffeners on the lateral torsional buckling of cantilever beams for each case of loading. The equations are proposed using linear regression analysis<sup>15)</sup> on the output of finite element model buckling runs for each set of loading. The concern in this study is regarding allowable stress design methodology (ASD); consequently, studied cases have been adopted for elastic buckling. Vertical web stiffeners are commonly used at the two sides of the web to prevent web distortion through welding processes so in this study stiffeners have been used at both sides of the web.

## 2. THEORETICAL BACKGROUND

The elastic buckling moment for a cantilever beam under a uniform moment caused by an end moment  $M_0$  applied at the free end can be obtained directly from the solution of the simply supported beam by imagining the beam to be consisted of two cantilevers of equal length joined together at the fixed end. Hence, the critical moment for the cantilever beam can be obtained<sup>1)</sup> from equation (1) by replacing  $L$  by  $2L$ . Then equation (2) can represent the desired critical moment.

$$M_{ocr} = \sqrt{\left(\frac{\pi^2 EI_y GJ}{(L)^2}\right) + \left(\frac{\pi^4 E^2 I_y C_w}{(L)^4}\right)} \quad (1)$$

$$M_{cr} = \sqrt{\left(\frac{\pi^2 EI_y GJ}{(2L)^2}\right) + \left(\frac{\pi^4 E^2 I_y C_w}{(2L)^4}\right)} \quad (2)$$

For cantilever beams subjected to end concentrated load,  $Q$ , acting at distance  $y_Q$  below their centroids; approximate numerical solutions for the buckling resistance have been reported<sup>13)</sup>. Fig.1 shows these solutions, in solid lines for top flange loading, which may be approximated by:

$$\frac{M_{cr} L}{\sqrt{EI_y GJ}} = 1 \left\{ 1 + \frac{1.2\varepsilon}{\sqrt{(1+1.2^2\varepsilon^2)}} \right\} + 4(K-2) \left\{ 1 + \frac{1.2(\varepsilon-0.1)}{\sqrt{(1+1.2^2(\varepsilon-0.1)^2)}} \right\} \quad (3)$$

in which,

$$M_{cr} = Q \cdot L \quad (4)$$

$$\varepsilon = \frac{y_Q}{L} \sqrt{\left(\frac{EI_y}{GJ}\right)} = \frac{2y_Q}{h} \frac{K}{\pi} \quad (5)$$

$$K = \sqrt{(\pi^2 EC_w / GJL^2)} \quad (6)$$

In this figure  $I_w$  is equal to  $C_w$ ; however,  $\varepsilon$  is a dimensionless load height parameter. For a centroidal loading ( $\varepsilon=0$ ), the variation of the buckling resistance with the torsion parameter  $K$  is approximately linear, as it is for end moments, which do not rotate  $\phi_L$ , where  $\phi_L$  is the beam twisting angle in radians. Moreover, the effect of load height  $y_Q$  is demonstrated in Fig.1, and it can be seen that while bottom flange loading significantly increase the buckling resistance, top flange loading may reduce it substantially, especially for cantilevers with high values of  $K$ . The non-linear effect of load height is suggested by the approximate formulation of equation (3).

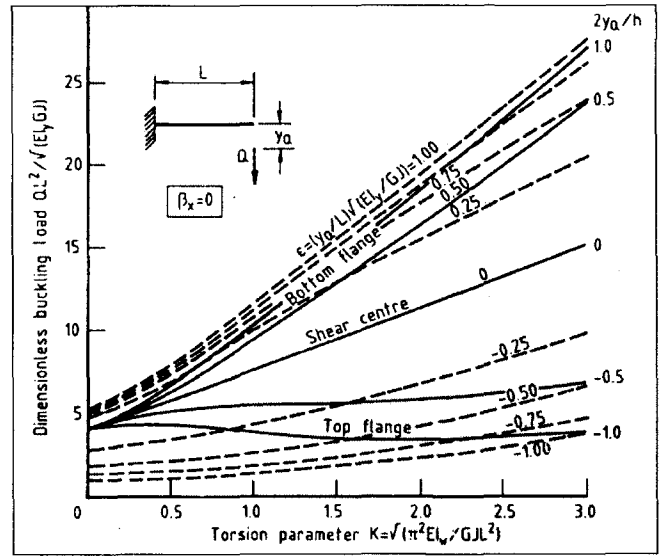


Fig.1 Buckling of cantilever beams with end loads. (N.S. TRAHAI, 2000)

For cantilever beams subjected to uniformly distributed load  $q$  acting at distances  $y_q$  from their centroids; approximated numerical solutions for the buckling resistances have been reported<sup>13)</sup>. Fig.2 shows these solutions which may be approximated by:

$$\frac{M_{cr} L}{\sqrt{EI_y GJ}} = 27 \left\{ 1 + \frac{1.4(\varepsilon-0.1)}{\sqrt{(1+1.4^2(\varepsilon-0.1)^2)}} \right\} + 10(K-2) \left\{ 1 + \frac{1.3(\varepsilon-0.1)}{\sqrt{(1+1.3^2(\varepsilon-0.1)^2)}} \right\} \quad (7)$$

in which,

$$M_{cr} = \frac{q \cdot L^2}{2} \quad (8)$$

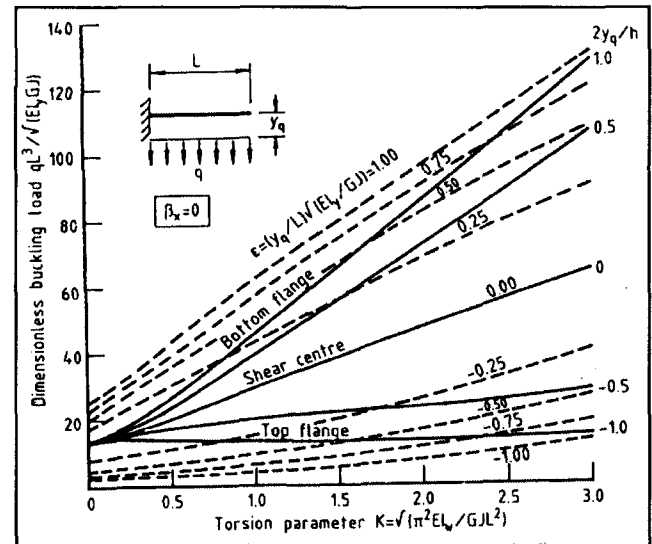


Fig.2 Buckling of cantilever beams under distributed loads. (N.S. TRAHAI, 2000)

The buckling resistance again varies almost linearly with the torsion parameter  $K$ , and is higher

than the resistance for end load as shown in Fig.1, because the bending moment is generally lower. For loading away from the centroid, the resistance changes non-linearly with the dimensionless load height  $\varepsilon$ , as suggested by the approximate formulation of equation (7).

### 3. METHODOLOGY AND NUMERICAL MODEL

The cantilever beam has been modeled using the finite elements, thin shell elements (4-nodes Quad element with 6 degrees of freedom at each node). The following are the cases of loading that have been considered in this study as follow:

**1. Uniform Moment (Unit Moment):** The uniform bending moment has been modeled as a concentrated bending moment at the free end of the cantilever beam, unit bending moment.

**2. Uniform Load (Unit Uniform Load):** The uniform load has been modeled as a pressure applied on the top flange. This pressure value times the flange width, represents the distributed load value per unit length, unit uniform load.

**3. End Concentrated Load (Unit Concentrated Load):** The end concentrated load for cantilever beam was modeled as point load applied at the end central node, unit concentrated load.

#### (1) Proposed Equation for $M_{cr}$

The proposed equation for the critical moment  $M_{cr}$ , at which lateral torsional buckling takes place, which takes into consideration the magnification factors that represent the increase in beam resistance for lateral torsional buckling due to effect of vertical stiffeners will take the following forms:

##### a) For Cantilever Beam Under Uniform Bending

According to equation (2) the magnification factors can be added in the following form:

$$M_{cr} = \sqrt{\eta(Q_1) + \alpha(Q_2)} \quad (9)$$

In which,

$$Q_1 = \left( \frac{\pi^2 EI_y GJ}{(2L)^2} \right) \quad (10)$$

$$Q_2 = \left( \frac{\pi^4 E^2 I_y C_w}{(2L)^4} \right) \quad (11)$$

$Q_1$  and  $Q_2$  are the Beam uniform torsion as well as warping torsion resistance respectively.

$\eta$  and  $\alpha$  are factors which represent the magnification in beam uniform torsion as well as warping torsion resistance respectively that resulted in using vertical web stiffeners.

##### b) For Cantilever Beam Under Uniform Load or End Concentrated Load

It was not available to classify the numerical equations, equation (3) and equation (7), into uniform torsion resistance term and warping torsion resistance term as done in case of uniform moment, equation (2) and equation (9). Hence, the proposed equation for  $M_{cr}$  including the magnification factor will take the following form:

$$M_{cr} = \beta M_{cr \text{ without stiffeners}} \quad (12)$$

Where,

$M_{cr \text{ without stiffeners}}$  is the critical moment that resulted from the approximate numerical solution for the case of beam without stiffeners.

$\beta$  is the factor which represents the magnification in cantilever beam resistance for lateral torsional buckling that resulted in using vertical web stiffeners.

#### (2) Studied Parameters

Several factors related to the vertical web stiffeners and which may affect the lateral torsional buckling behavior of the steel I-beams have been studied. These factors are the number of stiffeners, the beam aspect ratio, and the torsional rigidity as well as the bending rigidity of the used stiffeners. These factors were formulated as dimensionless normalized factors so that each of them represents effective quantity as stated, hereafter, as follow:

$$\begin{aligned} f_1 &= \frac{a}{L} \\ f_2 &= \frac{d}{L} \\ f_3 &= \left( \frac{t_{st}^3 b_f}{b_f^3 t_{st}} \right) \left( \frac{b_f}{t_f} \right) \left( \frac{t_{st}}{d} \right) \left( \frac{L}{b_f} \right) \\ &= \left( \frac{t_{st}^3}{b_f^3} \right) \left( \frac{b_f}{t_f} \right) \left( \frac{L}{d} \right) \end{aligned} \quad (13)$$

Where,

$f_1$  indicates the effect of the number of stiffeners;  $a$  is the spacing between stiffeners; and  $L$  is the beam length.

$f_2$  indicates The beam aspect ratio;  $d$  is the beam depth.

$f_3$  indicates the ratio between stiffener bending inertia, around the minor axis of the stiffener cross section, indicated in the term  $(t_{st}^3 \cdot b_f)$  and torsion rigidity indicated in the term  $(b_f^3 \cdot t_{st})$ . In addition local buckling index for flange plate has been included in the term  $(b_f/t_f)$ . Moreover rigidity index

for stiffener has been included by  $(d/t_{st})$ . Finally  $(L/b_f)$  expresses flange plate aspect ratio;  $t_{st}$  is the thickness of the vertical web stiffener;  $b_f$  is the beam flange width;  $t_f$  is the beam flange thickness.

It is obvious that factor  $f_2$  has already been included in factor  $f_3$ ; however, this has been resulted due to indication of  $f_3$  and its normalization; on the other hand, regression analysis showed that this factor has no trivial effect.

The studied beams and stiffeners configurations were chosen to cover a wide range within the practical values; consequently, studied ranges for the three factors are considered as follows:

$$\begin{aligned} f_1 &\rightarrow \text{Range of change is } 0.025 \rightarrow 1 \\ f_2 &\rightarrow \text{Range of change is } 0.03 \rightarrow 0.121 \\ f_3 &\rightarrow \text{Range of change is } 0.0009 \rightarrow 0.35 \end{aligned}$$

The aforementioned factors  $\eta$ ,  $\alpha$ , and  $\beta$  that represent the increase in beam resistance for lateral torsional buckling are obviously expected to be greater than 1.0 since the use of the vertical web stiffeners should enhance the beam resistance for lateral torsional buckling. The relation between the three factors  $\eta$ ,  $\alpha$ , and  $\beta$  and the proposed factors  $f_1$ ,  $f_2$ , and  $f_3$  was arbitrarily chosen in accordance with the following expression:

$$\begin{aligned} \eta - 1 &= A f_1^B f_2^C f_3^D \\ \alpha - 1 &= H f_1^M f_2^N f_3^P \\ \beta - 1 &= R f_1^X f_2^Y f_3^Z \end{aligned} \quad (14)$$

Linear regression analysis was then performed to obtain all constants  $A$ ,  $B$ ,  $C$ ,  $D$ ,  $H$ ,  $M$ ...etc. The chosen cross sections were varied to cover cross-sections that have larger uniform torsion resistance and others that have larger warping torsion resistance, appendix A shows the matrix of the used cross sections and their parameters.

### (3) Linear Buckling Analysis

Linear buckling analysis was conducted using the finite element method. Four nodes thin shell element was used in the modeling. Element dimensions are with minimum of 125 mm for width and 150 mm in length varying according to the beam dimensions: beam length, flange width, and web depth. Boundary conditions have been controlled by restraining nodes degrees of freedom, translational degrees of freedom as well as rotational degrees of freedom, at the fixed end as shown in Fig.3. The cases of loadings have been modeled as follow:

1. Uniform moment modeling: the concentrated moment at both ends of the beam has been modeled by two groups of concentrated forces. One group as tension forces on the bottom flange and the other as compression forces on the top flange as shown on Fig.4.
2. End concentrated load modeling: end load has been modeled just as concentrated point load on flange mid node as shown in Fig.5.
3. Uniform load modeling: uniform load has been modeled as pressure on the shell elements; see Fig.6, so that this pressure value times the flange width result in the uniform load per unit length on the beam.

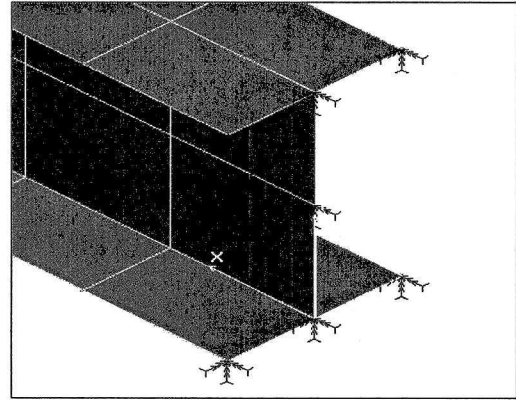


Fig.3 Modeled boundary conditions.

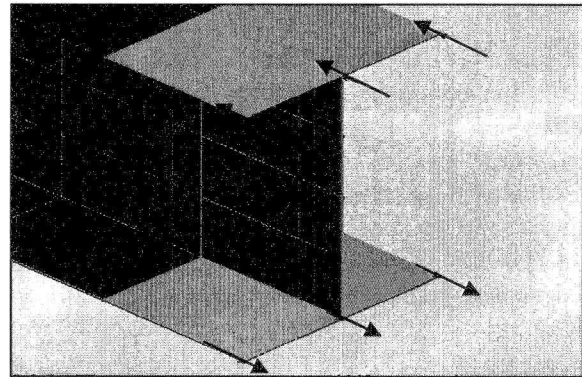


Fig.4 Modeling of concentrated moment.

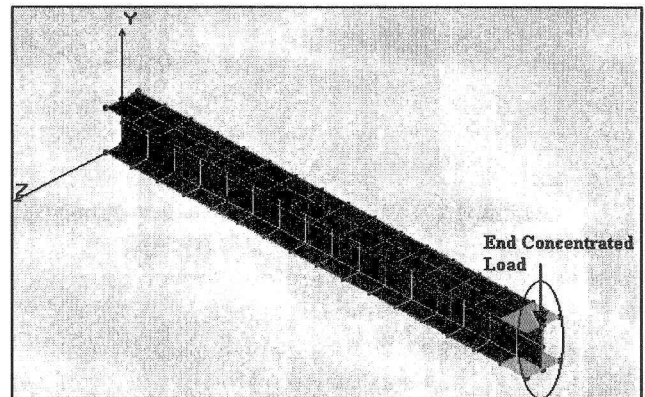


Fig.5 Modeling of concentrated load.

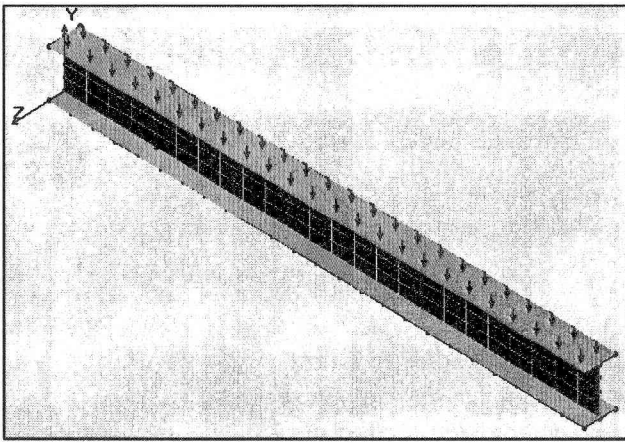


Fig.6 Modeling of uniform load.

It is obvious that vertical web stiffeners affect the warping torsional resistance of the beam by connecting the tension flange to the compression flange. Consequently pilot runs, almost 50 runs, were performed on the original case of loading and boundary conditions for studying the lateral torsional buckling, simply supported beam under uniform moment loading, in order to depict the effect of vertical web stiffeners on the uniform torsion resistance, St. Venant torsion. This was conducted by selecting simple beams with dimensions which have dominating uniform torsional resistance; the uniform torsional resistance was from 10 to 15 times the warping torsional resistance.

A large number of linear buckling analysis runs have been performed to get the eigen value of the lateral torsional buckling problem in the two cases: without using stiffeners and with stiffeners; that is, for each case of loading and studied parameters. The program<sup>14)</sup> was controlled to get the first 10 modes of buckling to spot on the lateral torsional buckling mode among them, see Fig.7. "Subspace iteration", as a numerical method, was used to solve the eigen value problem achieving an accuracy of 1e-5 for all the studied cases.

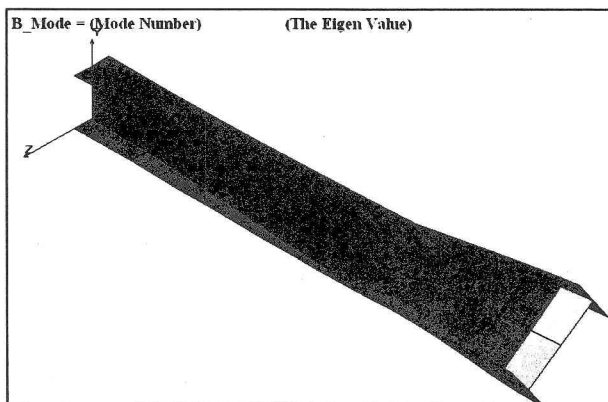


Fig.7 Lateral torsional buckling mode.

As a result of using unit value of loading for all the cases of loading, the resulted eigen value can be classified as follow:

1.  $M_{cr}$  for case of uniform moment loading.
2.  $Q$  for case of end load loading.
3.  $q$  for case of uniform load loading.

After getting the critical end load value  $Q$  and the critical uniform load value  $q$ , the critical moment for both cases can be computed according to equation (4) and equation (8) respectively. The resulted values for the critical moment for the case of without using stiffeners can be compared with the aforementioned  $M_{cr}$  that resulted from the closed form solution in equation (2) and the numerical solutions in equations (3) and (7); according to the case of loading. That comparison has been used as model verification to make sure that the finite element model is fitting well. Sample from the model verification has been introduced in Table 1. As shown in the table, the error percentage can be accepted to verify model fitting. Consequently after model verification the vertical stiffeners have been added to test their effect on the critical moment value.

Table 1 Sample for model verification of cantilever beams.

Beam Section*	$M_{cr}$ (t.m) equation (7)	$M_{cr}$ (t.m) (Finite Element)	Error %
150 x 10 / 200 x 8	10.95	10.84	0.09
175 x 10 / 250 x 8	9.84	9.63	0.22
300 x 12 / 400 x 12	42.52	42.2	0.016
250 x 8 / 300 x 10	16.48	16.4	0.03
275 x 12 / 350 x 10	48.04	47.93	0.004

\* Beam Section: flange (width x thickness) / Web (depth x thickness) all in mm.

#### (4) Regression Analysis

After performing the linear buckling analysis for the models with stiffeners, the results of the eigen values that obtained from finite element model were tabulated and all transformed to the form of  $M_{cr}$ . At this step the value of the critical moment, which include effect of the vertical stiffeners, can be used to calculate the values of magnification factors  $\eta$ ,  $\alpha$ , and  $\beta$  according to the proposed equation for  $M_{cr}$  for each case of loading.

By applying the logarithm for equation (14) the linear regression equation can be formed as follows:



$$\begin{aligned}\log(\eta-1) &= \log A + B \log f_1 + C \log f_2 + D \log f_3 \\ \log(\alpha-1) &= \log H + M \log f_1 + N \log f_2 + P \log f_3 \\ \log(\beta-1) &= \log R + X \log f_1 + Y \log f_2 + Z \log f_3\end{aligned}\quad (15)$$

By knowing the value of factors  $\eta$ ,  $\alpha$ , and  $\beta$  as a result from output observations resulted from finite element, regression analysis can be performed. Consequently, we can get the values of the constants;  $A, B, C, D, H, M \dots$ etc, from the results of the linear regression analysis. The resulted equation from regression analysis has been simplified in form of proposed equation so that can be manipulated by design engineer. Simplification for each constant has based on its sensitivity which results in regression analysis. The model of regression output as described<sup>15)</sup> was summarized in appendix B, Fig.B1. Moreover, summary of regression statistics was provided in Fig.B2.

#### 4. RESULTS AND DISCUSSION

The General trend for output that resulted from finite element analysis has been summarized in Fig.8; however, the behavior for each case of loading in accordance with the assumed parameters has been introduced hereafter.

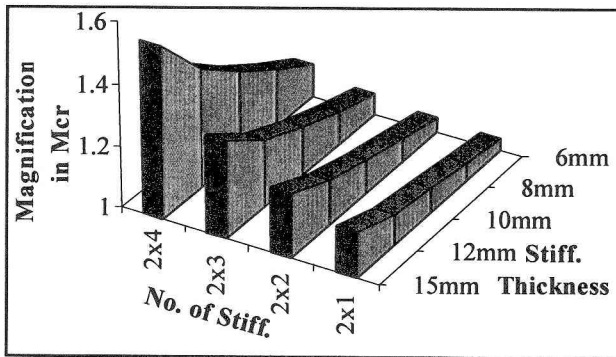


Fig.8 Trend of stiffeners effect.

Fig.8 has been drawn for sample I-beam with vertical stiffeners. Beam dimensions have been kept fixed; however, stiffeners thickness as well as stiffeners number have been changed in different values. Number of stiffeners has been indicated in multiplication of two numbers: the first number means that at one position two stiffeners have been used, one for each side of beam web; however, the second number indicates the number of positions where those two stiffeners have been located along the beam length; consequently, the total number of used stiffeners is the product of the two numbers. It is obvious from the figure that by increasing stiffeners thickness magnification factor increase and also by increasing number of used stiffeners

magnification factor increase. However, it is obvious that effect of increasing number of stiffeners is more effective than increasing stiffener thickness. This issue setback the usual nature of design processes: Optimization, to achieve high cost effectiveness.

The aforementioned pilot runs, for simple beams, were developed under two cases: the first case is in presence of the two end stiffeners and the second case without the two end stiffeners. Hence, the pilot runs findings were as follow:

- In the case of the beams which have high uniform torsion resistance than the warping torsion resistance, uniform torsion is dominant; the effect of vertical stiffeners is very small, not more than 3%. That is because in this case the dependency on the warping torsion resistance to resist the lateral torsional buckling is small with respect to the dependency on the uniform torsion part. As a result of that vertical stiffener main idea is enhancing the warping torsion resistance; stiffeners effect in resisting lateral torsional buckling became small.
- In the case of using only the two end stiffeners the magnification in  $M_{cr}$  is with percentage not more than 3%; that is the aforementioned percentage where the two end stiffeners cause the total effect on the warping torsion resistance for such beams with dominant uniform torsion resistance.
- Consequently the effect of vertical web stiffeners on the beam uniform torsion resistance can be neglected ( $\eta=1$ ); however, effect of vertical web stiffeners on the beam warping torsion resistance is significant especially if the warping torsion resistance was dominating.

By returning back to cantilever beams under uniform moment, equation (9) can be modified to the following form:

$$M_{cr} = \sqrt{(Q_1) + \alpha(Q_2)} \quad (16)$$

As mentioned before that wide range of studied parameters  $f_1$ ,  $f_2$ , and  $f_3$  has been covered in this study; however, sample has been introduced for each case of loading. Sample values of the studied parameters have been intended to be the same for all cases of loading to make it comparable. Figures have been drawn for certain value of factor  $f_2$ , which means certain beam depth and length (beam aspect ratio), with changing the both factors  $f_1$  and  $f_3$ . The three factors have affected the value of

magnification in the critical moment. Each case of loading has been investigated as follow:

### (1) Cantilever beam under uniform moment

Fig.9 and Fig.12 show the change in value of the magnification factor with respect to the change in factors  $f_1$  and  $f_3$  for certain case of  $f_2$  for case of uniform moment loading. For sample cases see Table 2.

Table 2 Sample points from Fig.9

Case	No. of Stiff.	Stiff. Thickness (mm)	S (%)
1	6 ( $f_1 = 0.33$ )	12 ( $f_3 = 0.0731$ )	20
2	4 ( $f_1 = 0.5$ )	15 ( $f_3 = 0.1429$ )	22

The regression analysis yielded an equation in the following form:

$$\alpha - 1 = 23.605 f_1^{-0.7944} f_2^{1.009} f_3^{0.715} \quad (17)$$

Then the resulted equation from the regression analysis has been simplified to be the proposed equation in the following form:

$$\alpha - 1 = 23.6 f_1^{-0.8} f_2 f_3^{0.7} \quad (18)$$

### (2) Cantilever beam under uniform load

Fig.10 and Fig.13 show the change in value of the magnification factor with respect to the change in factors  $f_1$  and  $f_3$  for certain case of  $f_2$  for case of uniform load loading. For sample cases see Table 3.

Table 3 Sample points from Fig.10

Case	No. of Stiff.	Stiff. Thickness (mm)	S (%)
1	6 ( $f_1 = 0.33$ )	12 ( $f_3 = 0.0731$ )	21
2	4 ( $f_1 = 0.5$ )	15 ( $f_3 = 0.1429$ )	17.5

The regression analysis yielded an equation in the following form:

$$\beta - 1 = 156.7789 f_1^{-0.9018} f_2^{2.9441} f_3^{0.4431} \quad (19)$$

Then the resulted equation from the regression analysis has been simplified to be the proposed equation in the following form:

$$\beta - 1 = 150 f_1^{-0.9} f_2^{2.9} f_3^{0.44} \quad (20)$$

### (3) Cantilever beam under end concentrated load

Fig.11 and Fig.14 show the change in value of the magnification factor with respect to the change in factors  $f_1$  and  $f_3$  for certain case of  $f_2$  for case of end concentrated load. For sample cases see Table 4.

Table 4 Sample points from Fig.11

Case	No. of Stiff.	Stiff. Thickness (mm)	S (%)
1	6 ( $f_1 = 0.33$ )	12 ( $f_3 = 0.0731$ )	26
2	4 ( $f_1 = 0.5$ )	15 ( $f_3 = 0.1429$ )	24

The regression analysis yielded an equation in the following form:

$$\beta - 1 = 72.3232 f_1^{-0.6811} f_2^{2.3525} f_3^{0.4439} \quad (21)$$

Then the resulted equation from the regression analysis has been simplified to be the proposed equation in the following form:

$$\beta - 1 = 77 f_1^{-0.7} f_2^{2.4} f_3^{0.44} \quad (22)$$

By comparing Fig.9, 10, and 11 or on the other hand by comparing the values of magnification factor in Table 2, 3, and 4 it can be concluded that effectiveness of using vertical web stiffeners is higher in case of concentrated end load than other studied cases of loading. Moreover effectiveness of vertical web stiffeners is nearly the same for the two other cases of loading, uniform moment and uniform load. Also from figures it can be inferred that optimization between the effective parameters  $f_1$  and  $f_3$  for certain beam dimension;  $f_2$ , is the key point to achieve high cost effectiveness in beam design.

The aforementioned proposed Simplification for each constant has based on its sensitivity which results in regression analysis and it can be proposed in any other forms with respect to its error percentage. Regarding power constants, they have been rounded to the proper digits; however, the common constant has been justified so that to make the error in equation within 7% from the regression value.

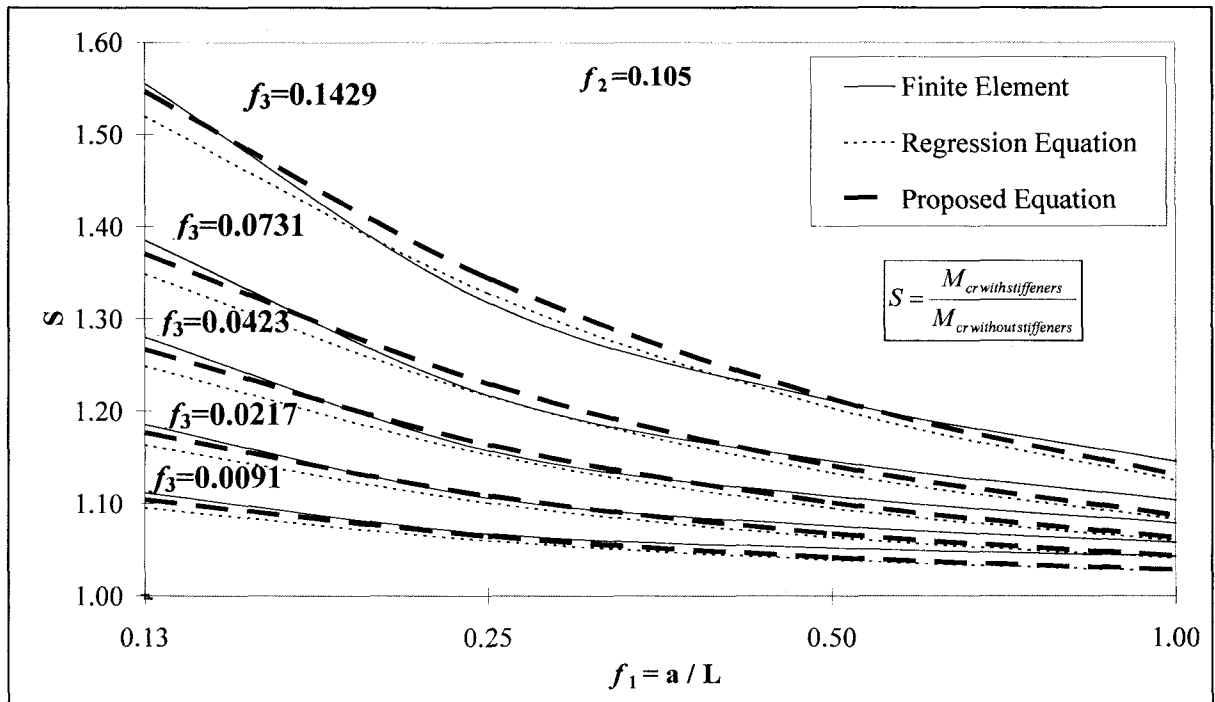


Fig.9 Percentage increase of critical moment,  $M_{cr}$ , for  $f_2=0.105$ . (Uniform moment)

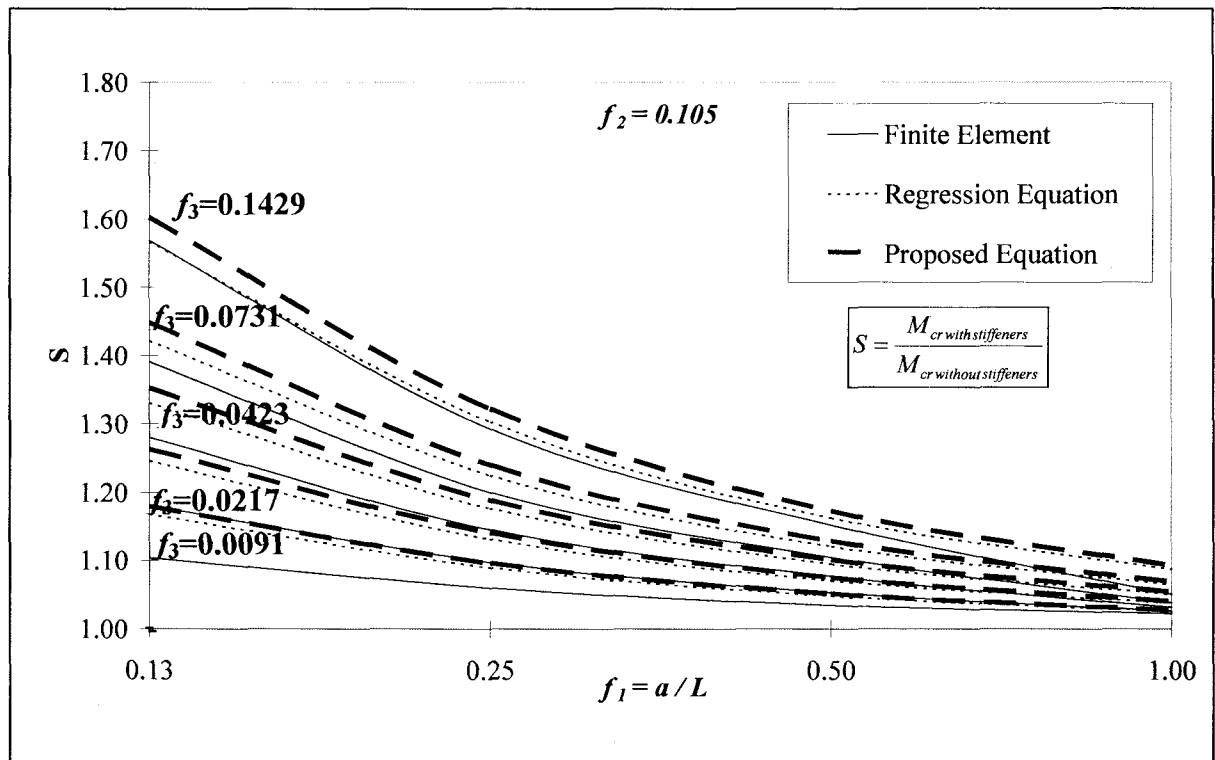


Fig.10 Percentage increase of critical moment,  $M_{cr}$ , for  $f_2=0.105$ . (Uniform load)



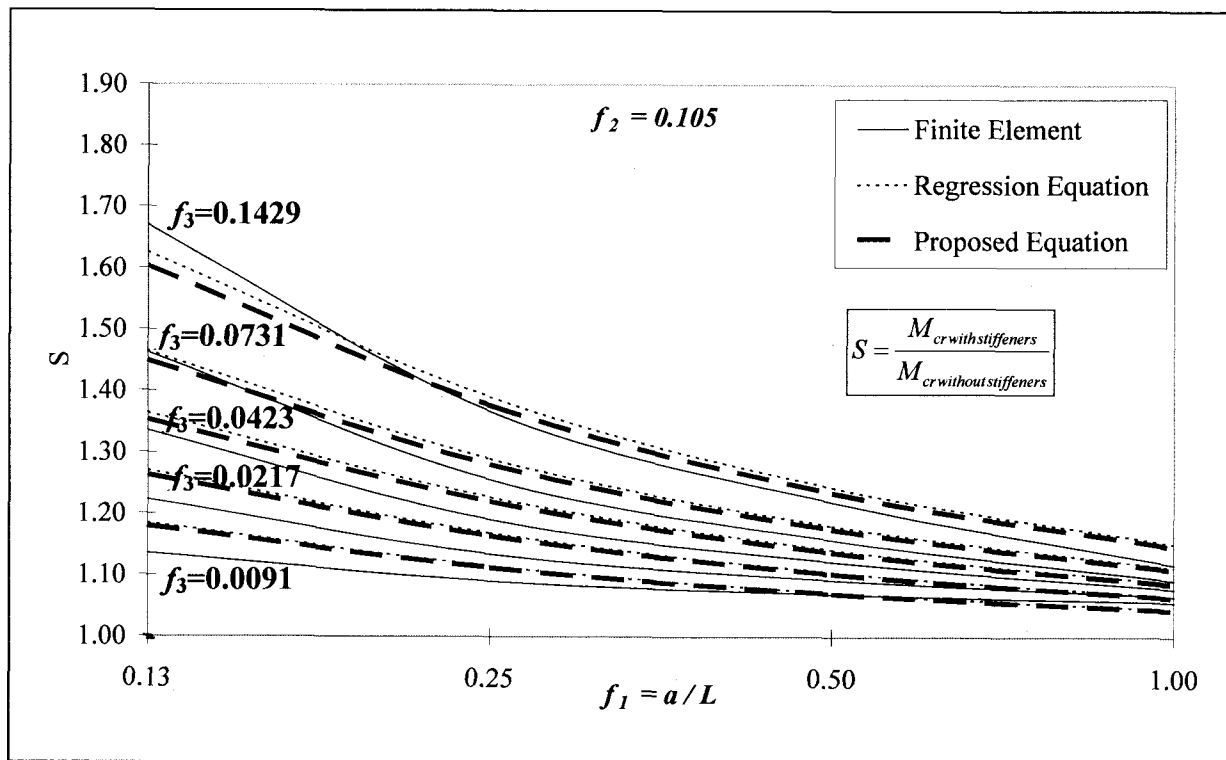


Fig. 11 Percentage increase of critical moment,  $M_{cr}$ , for  $f_2=0.105$ . (End concentrated load)

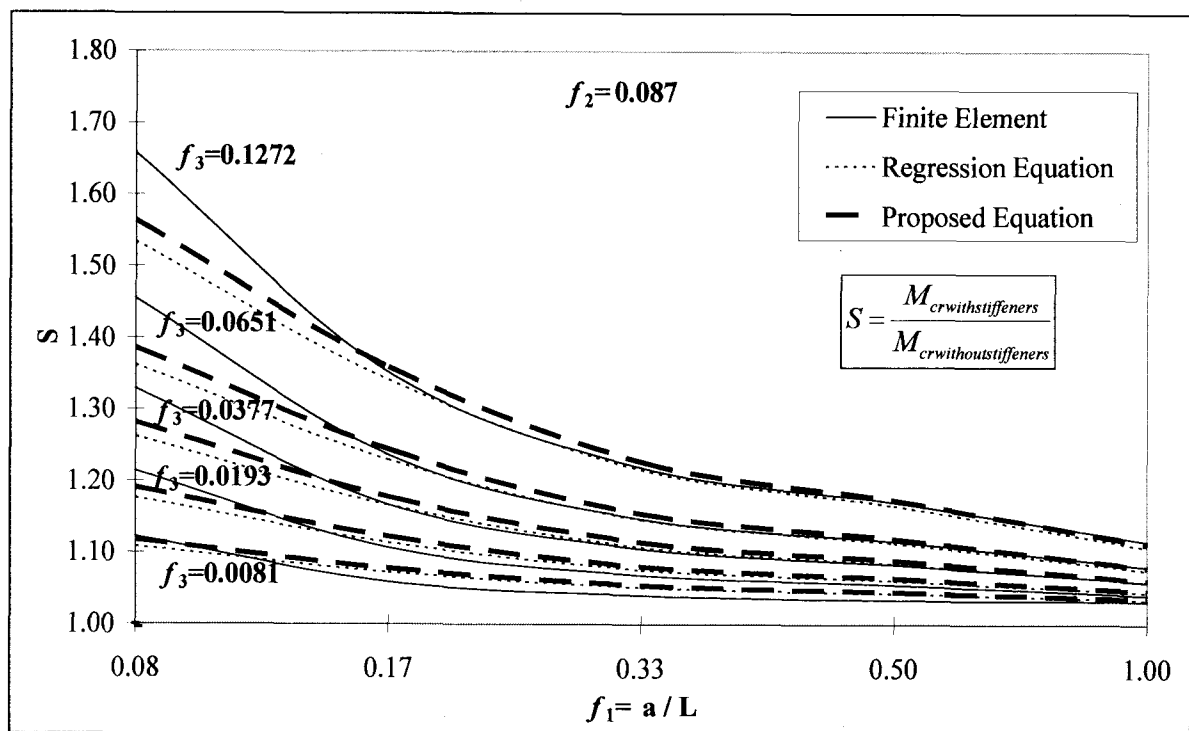


Fig. 12 Percentage increase of critical moment,  $M_{cr}$ , for  $f_2=0.087$ . (Uniform Moment)

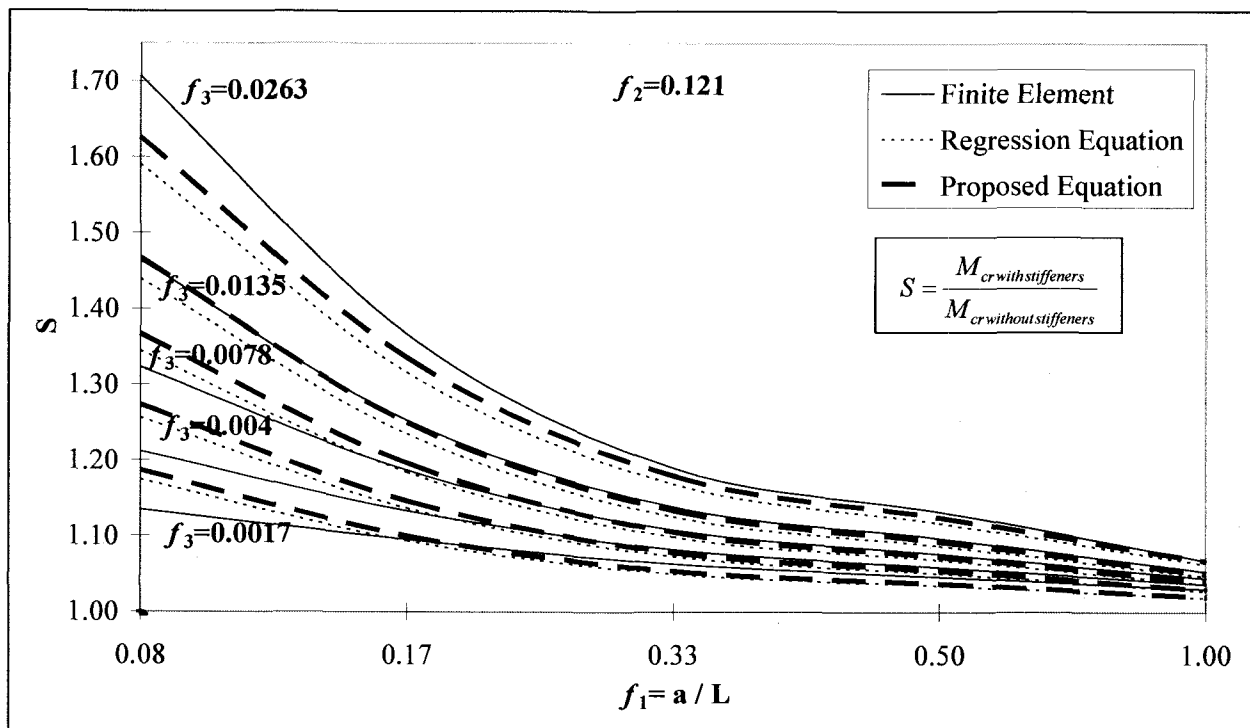


Fig. 13 Percentage increase of critical moment,  $M_{cr}$ , for  $f_2=0.121$ . (Uniform Load)

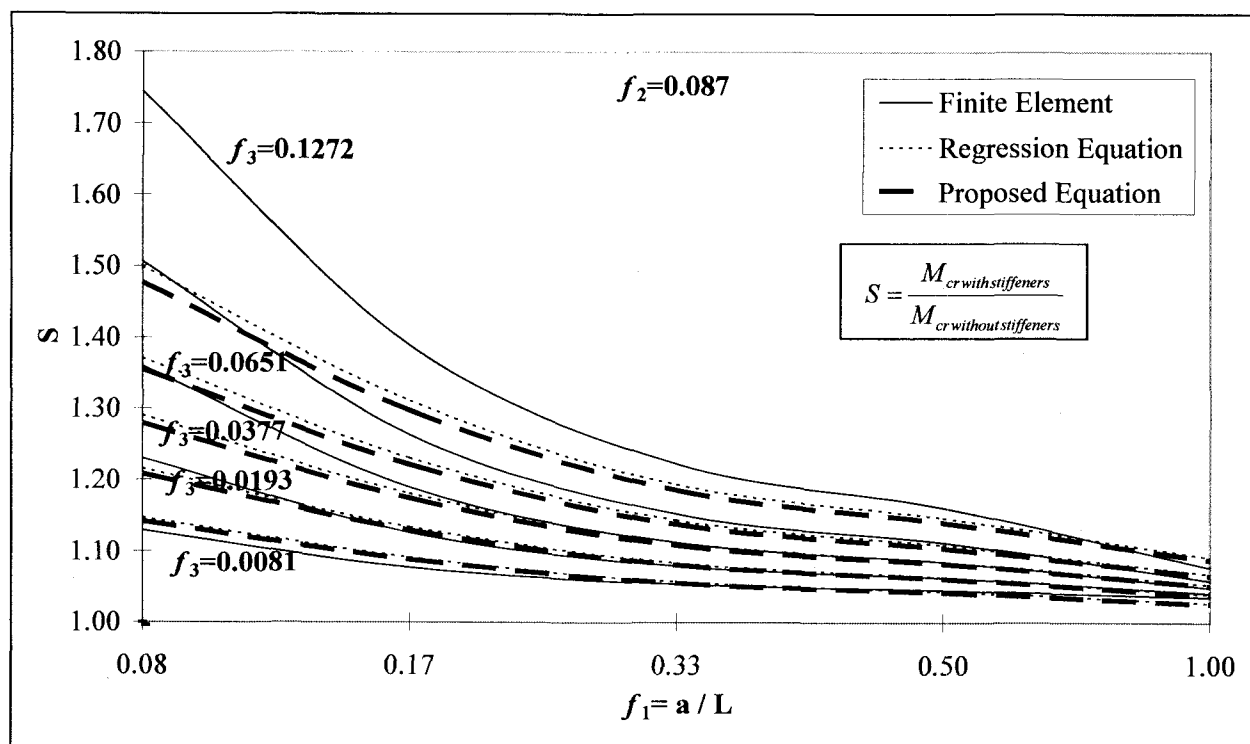


Fig. 14 Percentage increase of critical moment,  $M_{cr}$ , for  $f_2=0.087$ . (End concentrated Load)

## 5. CONCLUSION

The following paragraphs highlight the main conclusions of this study:

1. Effect of vertical web stiffeners on the beam uniform torsion resistance is nearly negligible as resulted from the pilot run; because the basic idea for effect of vertical stiffeners depend on connecting the tension and the compression flange which deeply affect the warping torsion resistance.
2. The vertical web stiffeners were found to have significant effect in the cantilever beams resistance to the lateral torsional buckling by the method of connecting the tension flange to the compression flange.
3. The proposed equation for  $M_{cr}$  that takes into consideration effect of vertical stiffeners for the case of cantilever beam subjected to uniform bending can take the following form using beam length equal to double the actual beam length:

$$M_{cr} = \sqrt{Q_1 + \alpha(Q_2)} \quad (23)$$

Where,  $Q_1$  and  $Q_2$  are beam uniform torsion resistance and warping torsion resistance respectively;  $\alpha$  is the warping torsion resistance magnification factor.

4. The proposed equation for  $M_{cr}$  that takes into consideration effect of vertical stiffeners for the case of cantilever beam subjected to uniform load or end concentrated load can take the following form:

$$M_{cr} = \beta(M_{cr})_{without\ stiffeners} \quad (24)$$

Where  $\beta$  is the moment magnification factor

5. The resulted equations for magnification factors for the studied cases of cantilever beams are summarized in **Table 5**.
6. Effectiveness of using vertical web stiffeners is higher in case of concentrated end load than other studied cases of loading. Moreover effectiveness of vertical web stiffeners is nearly the same for the two other cases of loading, uniform moment and uniform load.

7. The over all, average, magnification percentage in the critical moment  $M_{cr}$  for the case of cantilever beams is about 25%. This value may increase up to 60% according to the number of stiffeners and stiffeners torsional rigidity, stiffener thickness.

**Table 5** Magnification factors equations.

Case of Loading	Magnification Factor Proposed Equation
Uniform bending	$\alpha - 1 = 23.6 f_1^{-0.8} f_2 f_3^{0.7}$
Uniform Load	$\beta - 1 = 150 f_1^{-0.9} f_2^{2.9} f_3^{0.44}$
End Load	$\beta - 1 = 77 f_1^{-0.7} f_2^{2.4} f_3^{0.44}$

8. By increasing stiffener thickness magnification factor increase and also by increasing number of used stiffeners magnification factor increase. However, it is obvious that effect of increasing number of stiffeners is more effective than increasing stiffener thickness.
9. The use of only two end stiffener increases the lateral torsional buckling resistance by about 6%.
10. Optimization between the effective parameters  $f_1$  and  $f_3$  for certain beam dimension;  $f_2$ , is the key point to achieve high cost effectiveness in beam design.

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## APPENDIX A

**Table A.1** Cross-Sections data for cantilever beams under uniform bending.

Section	$b_f$	$t_f$	$h$	$t_w$	$J$	$C_w$	$I_y$	$Lu$	$L_b$	$W$
<b>150x10/200x8</b>	15.0	1.0	20.0	0.8	1.3 E+1	0.6 E+5	5.6 E+2	200.0	420.0	0.8
<b>175x10/250x8</b>	17.5	1.0	25.0	0.8	1.6 E+1	1.5 E+5	0.9 E+3	300.0	630.0	0.9
<b>300x12/400x12</b>	30.0	1.2	40.0	1.2	5.8 E+1	2.3 E+6	5.4 E+3	400.0	840.0	1.2
<b>250X8/300X10</b>	25.0	0.8	30.0	1.0	1.9 E+1	5.0 E+5	2.1 E+3	350.0	735.0	1.1
<b>275X12/350X10</b>	27.5	1.2	35.0	1.0	4.3 E+1	13.6 E+5	4.2 E+3	300.0	630.0	1.4

**Table A.1 (Cont.)** Cross-Sections data for cantilever beams under uniform bending.

Section	Q1	Q2	$C_b$	$M_{cr}(t.m)$
<b>150x10/200x8</b>	7.20 E+5	4.80 E+5	1.00	10.95
<b>175x10/250x8</b>	6.00 E+5	3.60 E+5	1.00	9.84
<b>300x12/400x12</b>	74.00 E+5	110.00 E+5	1.00	42.50
<b>250X8/300X10</b>	12.00 E+5	15.00 E+5	1.00	16.50
<b>275X12/350X10</b>	76.00 E+5	150.00 E+5	1.00	48.00

**Table A.2** Cross-Sections data for cantilever beams under uniform load.

Section	$b_f$	$t_f$	$h$	$t_w$	$J$	$C_w$	$I_y$	$Lu$
<b>150x10/200x8</b>	15.0	1.0	20.0	0.8	1.3 E+1	0.6 E+5	5.6 E+2	200.0
<b>175x10/250x8</b>	17.5	1.0	25.0	0.8	1.6 E+1	1.5 E+5	0.9 E+3	300.0
<b>300x12/400x12</b>	30.0	1.2	40.0	1.2	5.8 E+1	2.3 E+6	5.4 E+3	400.0
<b>275X14/350X10</b>	27.5	1.4	35.0	1.0	6.2 E+1	1.6 E+6	4.8 E+3	300.0
<b>200X10/300X10</b>	20.0	1.0	30.0	1.0	2.3 E+1	3.2 E+5	1.3 E+3	300.0

**Table A.2 (Cont.)** Cross-Sections data for cantilever beams under uniform load.

Section	$K$	$\epsilon$	$W_{cr} (t/m)$	$M_{cr}(t.m)$
<b>150x10/200x8</b>	1.70	-0.55	22.30	44.50
<b>175x10/250x8</b>	1.60	-0.52	9.10	41.10
<b>300x12/400x12</b>	2.50	-0.80	16.10	128.40
<b>275X14/350X10</b>	2.70	-0.87	36.30	163.40
<b>200X10/300X10</b>	1.90	-0.63	13.00	58.50

Notes:

All units are ton-cm unless other wise noted.

**Table A.3** Cross-Sections data for cantilever beams under end concentrated load.

Section	$b_f$	$t_f$	$h$	$t_w$	$J$	$C_w$	$I_y$	$Lu$
150x10/200x8	15.0	1.0	20.0	0.8	1.3 E+1	0.6 E+5	5.6 E+2	200.0
175x10/250x8	17.5	1.0	25.0	0.8	1.6 E+1	1.5 E+5	0.9 E+3	300.0
300x12/400x12	30.0	1.2	40.0	1.2	5.8 E+1	2.3 E+6	5.4 E+3	400.0
275X14/350X10	27.5	1.4	35.0	1.0	6.2 E+1	1.6 E+6	4.8 E+3	300.0
200X10/300X10	20.0	1.0	30.0	1.0	2.3 E+1	3.2 E+5	1.3 E+3	300.0

**Table A.3 (Cont.)** Cross-Sections data for cantilever beams under end concentrated load.

Section	$K$	$\varepsilon$	$P_{cr} (t)$	$M_{cr}(t.m)$
150x10/200x8	1.70	-0.55	12.80	25.60
175x10/250x8	1.60	-0.52	7.90	23.70
300x12/400x12	2.50	-0.80	17.80	71.20
275X14/350X10	2.70	-0.87	30.00	89.90
200X10/300X10	1.90	-0.63	11.10	33.30

Notes:

All units are ton-cm unless other wise noted.

## APPENDIX B

Regression Statistics	
Multiple R	
<b>R Square</b>	
Adjusted R Square	
Standard Error	
Observations	

ANOVA					
	df	SS	MS	F	Significance F
Regression					
Residual					
Total					

	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%	Lower 95.0%	Upper 95.0%
Intercept								
X Variable 1								
X Variable 2								
X Variable 3								

**Fig.B1** Model of regression analysis output.

Regression Statistics	
Multiple R	0.97
<b>R Square</b>	<b>0.94</b>
Adjusted R Square	0.94
Standard Error	0.12
Observations	<b>115</b>

Regression Statistics	
Multiple R	0.97
<b>R Square</b>	<b>0.94</b>
Adjusted R Square	0.94
Standard Error	0.10
Observations	<b>120.00</b>

Regression Statistics	
Multiple R	0.98
<b>R Square</b>	<b>0.95</b>
Adjusted R Square	0.95
Standard Error	0.07
Observations	<b>120.00</b>

**Fig.B2** Regression statistics for uniform moment, uniform load, and end concentrated load respectively.

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## SYMBOLS

$a$	Stiffeners spacing, (cm).
$A_f$	Area of compression flange (cm <sup>2</sup> ).
$b_f$	Compression flange width, (cm).

$C_b$	Coefficient depending on the type of load and support conditions.
$C_w = I_w$	Warping torsion constant, (cm <sup>6</sup> ).
$d$	Beam total depth, (cm).
$E$	Modulus of elasticity, 2100 t/cm <sup>2</sup>
$f_1$	Inverse of stiffeners number.
$f_2$	Beam aspect ratio.
$f_3$	Ratio between stiffener torsion and bending rigidity.
$G$	Shear modulus, (t/cm <sup>2</sup> )
$h$	Web height, (cm).
$I_y$	Moment of inertia of the cross section about the minor axis of the beam, (cm <sup>4</sup> ).
$J$	Uniform torsion constant, (cm <sup>4</sup> ).
$L$	Beam span, (cm).
$M_{cr \text{ without stiffeners}}$	Critical moment for the case of beam without stiffeners, (t.m).
$M_{cr}$	Critical moment, (t.m).
$P_{cr}$	Critical concentrated load, (ton).
$Q_1$	Term that represents the uniform Torsion resistance, (t <sup>2</sup> .cm <sup>2</sup> ).
$Q_2$	Term that represents the non uniform (warping) torsion resistance, (t <sup>2</sup> .cm <sup>2</sup> ).
$Q$	end concentrated load, (ton).
$q$	uniformly distributed load, (t/m).
$S$	The magnification in the critical Moment.
$S = \frac{M_{cr \text{ with stiffeners}}}{M_{cr \text{ without stiffeners}}}$	
$t_f$	Flange thickness, (cm).
$t_s$	Stiffener thickness, (cm).
$t_w$	Web thickness, (cm).
$W_{cr}$	Critical uniform load, (ton).
$y_Q$	Load height, (cm).
$\varepsilon$	A dimensionless load height parameter.
$\phi_L$	Beam twisting angle, (radian).
$\eta$	Factor which represents the magnification in beam uniform torsion resistance that resulted in using vertical web stiffeners.
$\beta$	Factor which represents the magnification in cantilever beam resistance for lateral torsional buckling that resulted in using vertical web stiffeners.
$\alpha$	Factor, which represents the magnification in beam non uniform (warping) torsion resistance that resulted in using vertical web stiffeners.

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