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Normalized plastic work of sand under undrained monotonic loading condition

砂の非排水単調載荷条件下における正規化塑性仕事量

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Extensive study was conducted to examine carefully the characteristic of the normalized plastic work under undrained condition. The effect of both the principal stress direction, α, and intermediate principal stress coefficient, b, were explored as well. The results clearly show that undrained normalized work of sand is almost independent of relative density, confining pressure, and the confining stress ratio, but it is not unique with respect to α and b. The slope of normalized work which is known as dilatancy parameter μ reaches minimum value when α is set to value between 60° to 75° and b is near to 0.75.

Key Words : *anisotropy, dilatancy, normalized plastic work, undrained*

1. Introduction

The volumetric strain response or dilatancy of sand due to shearing is a distinctive feature of sand behaviour. To describe the dilatancy behaviour, stress-dilatancy relation is commonly used. Several stress-dilatancy relation have been developed either based on energy principles (Rowe[1]), Schofield and Wroth[2]), Moroto[3])) or regarded as a constraint imposed by internal grain geometry (Matsuoka[4]), Nemat-Nasser[5])).

A stress-dilatancy relation was proposed by Moroto[3]). The stress-dilatancy is modeled based on calculation of volumetric strain due to shearing and is related to shear work which is normalized by mean principal stress p. By compiling the data from drained triaxial test under constant p, the normalized plastic work was found to be independent of stress path and was believed to have unique relationship associated to the disturbance to granular materials during shearing process.

Ghaboussi and Momen[6]) applied a similar approach of Moroto and arranged a number of drained triaxial data for different type of sands. The results led to the conclusion that the normalized work – equivalent deviatoric plastic strain ($\Omega - \bar{\varepsilon}^p$) relationship from which they proposed a step-wise linear equation to model the relationship is independent of relative density, mean principal stress, and over consolidation ratio.

Subsequently, Kabilamany and Ishihara[7]) and Cubrinovski and Ishihara[8]) obtained a nearly unique of the normalized work – equivalent deviatoric plastic strain relationship independent of mean principal stress for drained condition. In addition, Kabilamany and Ishihara[7]) proposed another equation to describe $\Omega - \bar{\varepsilon}^p$ relationships and defined the slope of the curve as dilatancy parameter μ.

Tohwata and Ishihara[9]) performed undrained hollow cylindrical test with cyclical loading and several loading schemes to study shear work and pore water pressure in undrained shear. Based on this observation, they indicated that there is a unique relationship between shear work and pore water pressure at every state of shear stress change. It is independent of shear stress schemes, cyclic shear stress amplitude and number of cycles to liquefaction.

A review of past studies described above indicates the importance of the normalized work and its unique characteristic. However, only a few studies consider undrained condition. This paper presents the study of the characteristic of the normalized work under undrained condition. This study is not limited to the effect of relative density, initial confining pressure, and the confining stress ratio K_c, the investigation on the influence of the principal stress direction and of the magnitude of intermediate principal stress are considered as well.

The direction of principal stress is represented by the angle α. This angle is equal to that of the major principal stress from vertical to the bedding plane, α (**Fig. 1**) and the magnitude of the intermediate principal stress associated to the intermediate principal stress coefficient b, which is equal to $(\sigma_2 - \sigma_3) / (\sigma_1 - \sigma_3)$.

2. Experimental data

All experimental data used in this paper are revealed from undrained hollow cylindrical test conducted by Yoshimine[10] and are shown in **Table 1**. The apparatus used to perform the test was the improved version of hollow cylindrical apparatus described by Ishihara and Tohwata[11] and Pradel *et al.*[12] which is modified by adding automatic stress and strain control by means of personal computer (see for details, Yoshimine[10]).

The specimens were prepared by dry deposition method in 8 layers. The nominal dimension of the hollow cylindrical specimens were 195 mm in height. The outer diameter, D_o, and inner diameter, D_i are 100 mm and 60 mm respectively.

Toyoura sand, Japanese standard sand, was used throughout the testing program. This sand has a mean particle diameter $D_{50} = 0.17$ mm. The minimum and maximum relative density are $e_{min} = 0.597$ and $e_{max} = 0.977$ respectively.

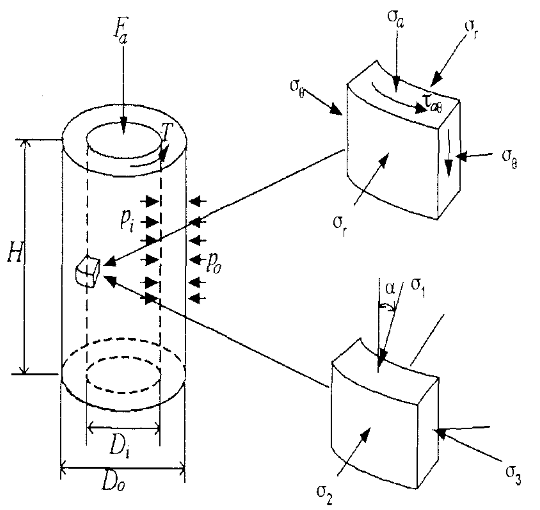

Fig. 1 Stress state within hollow cylindrical specimen

3. Stress and strain

Fig. 1 shows a hollow cylindrical specimen with height H, inner diameter d_i and outer diameter d_o. It is subjected to four loading components as follows: Axial load F_a, torque T, and inner and outer pressures, p_i and p_o. Application of these loads in-

Table 1 Test results by Yoshimine[10]

Shear mode		Initial state			M	$\left(\frac{q}{p}\right)_{max}$
		e	Dr (%)	p_c (kPa)		
$K_c = 1.0$						
Simple shear		0.835	37.4	100	1.113	1.221
Simple shear		0.88	25.5	100	1.131	1.28
$K_c = 0.5$						
Simple shear		0.835	37.4	100	1.138	1.269
Simple shear		0.878	26.1	100	1.109	1.193
b	$\alpha(°)$					
0.0	0	0.889	23.2	50	1.29	1.391
0.0	0	0.825	40	100	1.257	1.344
0.0	0	0.879	25.8	100	1.268	1.335
0.0	0	0.916	16.1	100	1.265	1.377
0.0	0	0.889	23.2	100	1.263	1.339
0.0	0	0.87	28.2	300	1.255	1.317
0.0	15	0.823	40.5	100	1.26	1.358
0.0	30	0.82	41.3	100	1.207	1.32
0.0	45	0.829	38.9	100	1.152	1.27
0.25	0	0.821	41.1	100	1.193	1.309
0.25	15	0.817	42.1	100	1.19	1.315
0.25	30	0.818	41.8	100	1.086	1.265
0.25	45	0.859	31.1	100	1.1	1.232
0.25	45	0.826	39.7	100	1.088	1.215
0.25	45	0.813	43.2	100	1.084	1.215
0.25	45	0.828	39.2	100	1.1	1.228
0.25	45	0.818	41.8	100	1.086	1.216
0.5	15	0.825	40.0	100	1.073	1.184
0.5	30	0.824	40.3	100	1.038	1.161
0.5	45	0.821	41.1	100	1.012	1.106
0.5	60	0.828	39.2	100	0.958	1.067
0.5	75	0.823	40.5	100	1.001	1.188
0.75	30	0.818	41.8	100	0.986	1.083
0.75	45	0.83	38.7	100	0.965	1.028
0.75	60	0.819	41.6	100	0.892	0.97
0.75	75	0.829	38.9	100	0.891	0.966
0.75	90	0.817	42.1	100	0.931	1.035
1	45	0.821	41.1	100	0.97	0.99
1	60	0.826	39.7	100	0.9	0.995
1	75	0.825	40.0	100	0.902	0.975
1	90	0.826	39.7	100	1.01	0.985

duces four stress components: The axial stress σ_a, the radial stress σ_r, the circumferential stress σ_θ, and the torsional shear stress $\tau_{a\theta}$. Calculation of the stress components from loading components is following the procedure explained by Pradhan *et al.*[13] and Pradel *et al.*[12].

The inner and outer cell pressure were chosen so that the average radial stress was equal to the intermediate principal stress σ_2 and that the major and minor principal stress axis were contained in the plane of the axial and circumferential stresses.

The principal stresses have been calculated from the average axial stress, the average circumferential stress, the average radial stress, and the average shear stress as follows:

$$\sigma_1 = \frac{(\sigma_a - \sigma_\theta)}{2} + \sqrt{\left(\frac{\sigma_a - \sigma_\theta}{2}\right)^2 + \tau_{a\theta}^2} \quad (1)$$

$$\sigma_2 = \sigma_r \quad (2)$$

$$\sigma_3 = \frac{(\sigma_a - \sigma_\theta)}{2} - \sqrt{\left(\frac{\sigma_a - \sigma_\theta}{2}\right)^2 + \tau_{a\theta}^2} \quad (3)$$

where σ_1 is major principal stress, σ_2 is intermediate principal stress, and σ_3 is minor principal stress.

The effective mean principal stress and equivalent deviatoric stress are expressed as follows:

$$p = \frac{1}{3}\sigma_{ii} = \frac{(\sigma_1 + \sigma_2 + \sigma_3)}{3} \quad (4)$$

$$q = \left(\frac{3}{2}S_{ij}S_{ij}\right)^{\frac{1}{2}} = \sqrt{\frac{1}{2}\left((\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right)} \quad (5)$$

in which S_{ij} is deviatoric stress.

The equivalent deviatoric plastic strain increment is calculated incorporating principal strain components as follows:

$$d\bar{\varepsilon} = \left(\frac{2}{3}de_{ij}de_{ij}\right)^{\frac{1}{2}} = \sqrt{\frac{2}{9}\left((d\varepsilon_1 - d\varepsilon_2)^2 + (d\varepsilon_2 - d\varepsilon_3)^2 + (d\varepsilon_3 - d\varepsilon_1)^2\right)} \quad (6)$$

where de_{ij} denotes deviatoric strain increment components.

The angle α is related to stress components by

$$\alpha = \frac{1}{2}tan^{-1}\left(\frac{2\tau_{a\theta}}{\sigma_a - \sigma_\theta}\right) \quad (7)$$

4. Plastic work

The total work done per unit volume of deformable body during strain increment can be expressed as

$$dW = \sigma_{ij}d\varepsilon_{ij} \quad (8)$$

where $d\varepsilon_{ij}$ is the total strain increment which consists of both elastic and plastic component. It is usual to assume that the total strain increment can be decomposed as two parts, elastic part, $d\varepsilon_{ij}^e$, and plastic part, $d\varepsilon_{ij}^p$ as:

$$d\varepsilon_{ij} = d\varepsilon_{ij}^e + d\varepsilon_{ij}^p \quad (9)$$

in which superscripts e and p denote the elastic and plastic part respectively.

Substituting the total strain in Eq. (9) to Eq. (8), it can be written as:

$$\begin{aligned} dW &= \sigma_{ij}\left(d\varepsilon_{ij}^e + d\varepsilon_{ij}^p\right) \\ &= \sigma_{ij}d\varepsilon_{ij}^e + \sigma_{ij}d\varepsilon_{ij}^p \\ &= dW^e + dW^p \end{aligned} \quad (10)$$

The quantity dW^e is the increment of the elastic energy stored which is recoverable. The quantity dW^p is the increment of the plastic work and is not recoverable, as the plastic deformations are irreversible.

The plastic work is obtained by first separating elastic strain part from the total strain. The incremental elastic relations are given by Hooke's law

$$d\sigma_{ij} = D_{ijkl}d\varepsilon_{kl}^e \quad (11)$$

here, D_{ijkl} is elasticity matrix, which is a function of Poisson's ratio ν and elastic shear modulus G. In this paper, Poisson ratio is assumed constant during loading and has a value of 0.2 for the present sand (Gutierrez[14]).

The elastic shear modulus is defined as (Iwasaki and Tatsuoka[15])

$$G = 88290\,\frac{(2.17 - e)^2}{1 + e}\left(\frac{p}{98.1}\right)^{0.4} \quad (12)$$

where the unit of G is kPa and e is void ratio.

With reference of Eq. 10, the increment of plastic work for hollow cylindrical specimen is modified by considering the equivalent deviatoric plastic strain part only as

$$dW^p = \sigma_a d\varepsilon_a^p + \sigma_r d\varepsilon_r^p + \sigma_\theta d\varepsilon_\theta^p + \tau_{a\theta}d\gamma_{a\theta}^p \quad (13)$$

To examine the behaviour of the normalized work, first we make definition and calculation for these parameter.

Recalling the second term on the right hand side of Eq. 10 and expressing σ_{ij} and $d\varepsilon_{ij}^p$ in terms of their respective hydrostatic and deviatoric components, we obtain

$$\begin{aligned} dW^p &= (S_{ij} + p\,\delta_{ij})\left(de_{ij}^p + \frac{1}{3}d\varepsilon_v^p\delta_{ij}\right) \\ &= p\,d\varepsilon_v^p + S_{ij}\,de_{ij}^p \end{aligned} \quad (14)$$

where δ_{ij} is Kronecker delta.

The dissipated energy in Eq. 14 is divided with the mean stress p and all volumetric strain are due to

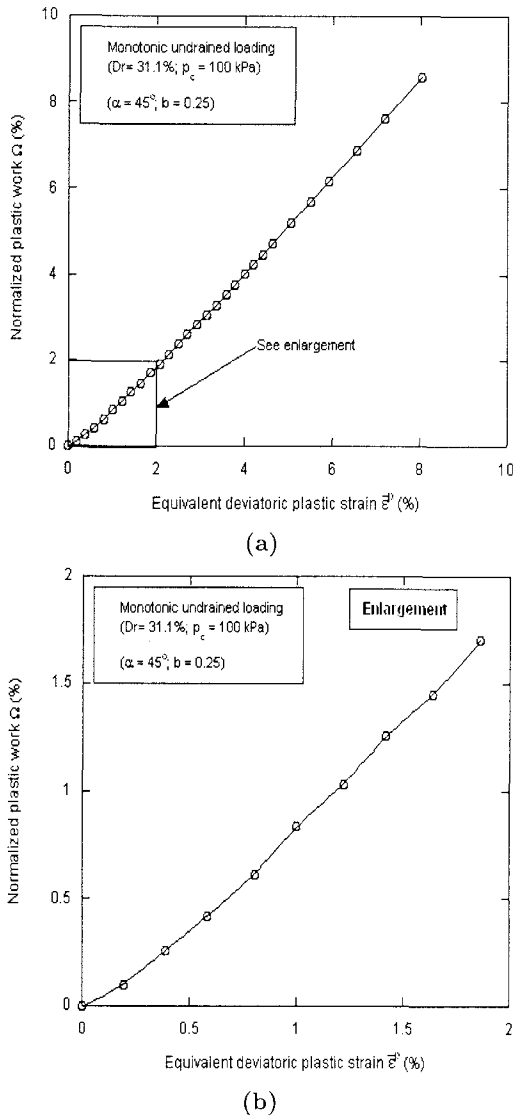

Fig. 2 Normalized plastic work versus equivalent deviatoric plastic strain for undrained torsional test sample of Toyoura sand

dilatancy alone (Moroto[3]), we obtain the normalized increment of shear work as

$$d\Omega = \frac{dW^p}{p} = d\varepsilon_v^p + \frac{S_{ij}\ de_{ij}^p}{p} \tag{15}$$

Then, based on the assumption of Moroto[3] that there is a unique relation between the normalized plastic work increment $d\Omega$ and the equivalent deviatoric plastic strain increment $d\bar{\varepsilon}^p$, we obtain the following equation

$$d\Omega = \mu\ d\bar{\varepsilon}^p \tag{16}$$

Finally, by substituting Eq. 16 into Eq. 15 we get an equation which describes the stress-dilatancy

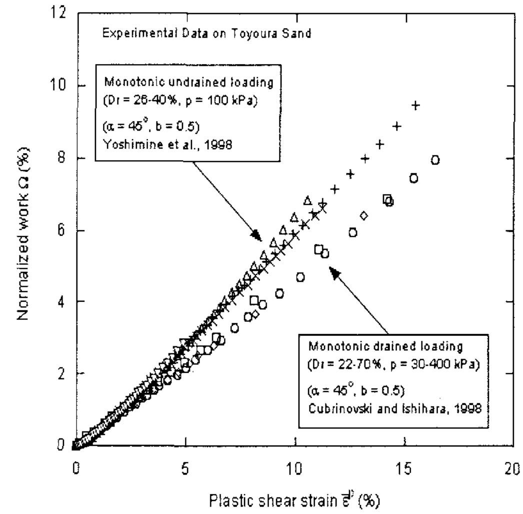

Fig. 3 Normalized work under drained and undrained condition

relation as

$$\frac{d\varepsilon_v^p}{d\bar{\varepsilon}^p} = \mu - \frac{S_{ij}\ de_{ij}^p}{p\ d\bar{\varepsilon}^p} \tag{17}$$

5. Experimental results

5.1 Effect of initial relative density, confining pressure, and the confining stress ratio K_c on $\Omega - \bar{\varepsilon}^p$ relation

Fig. 2(a) shows a plot of the normalized plastic work versus equivalent deviatoric plastic strain which is calculated from undrained torsional test. It is clearly seen that the slope μ of the curve is not constant, but rather it is a function of the equivalent deviatoric plastic strain $\bar{\varepsilon}^p$. The change of μ takes place mostly at small strain level and it eventually takes a constant value when the shear strain becomes large and it can be seen more clearly in **Fig. 2(b)** which shows the enlargement of the normalized plastic work curve for relatively small strain. The increment of μ is relatively small, but it have to be considered carefully due to its effects on stress-strain behavior of sand (Gutierrez[14]),Kabilamany and Ishihara[7]) and Cubrinovski and Ishihara[8]))

The comparison between normalized work under drained and undrained condition is plotted in **Fig. 3**. Since the reported data for drained condition is for two dimensional case, equivalent deviatoric plastic strain is also calculated in two dimensional condition using the following equation:

$$\bar{\varepsilon}^p = \sqrt{\left(\frac{\varepsilon_a - \varepsilon_\theta}{2}\right)^2 + (\gamma_{a\theta})^2} \tag{18}$$

It clearly shows that for strain up to 3%, normalized

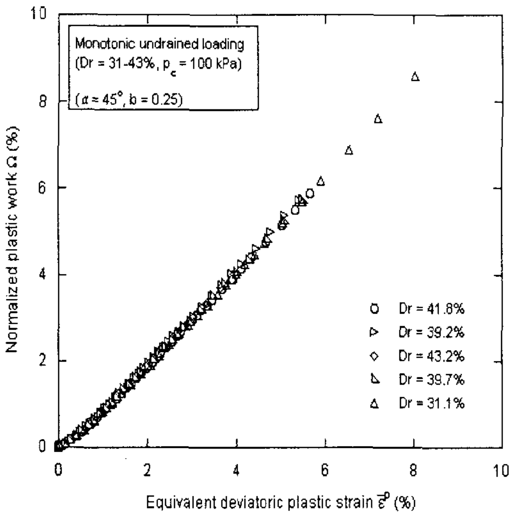

Fig. 4 Normalized shear work curve for different initial relative density Dr

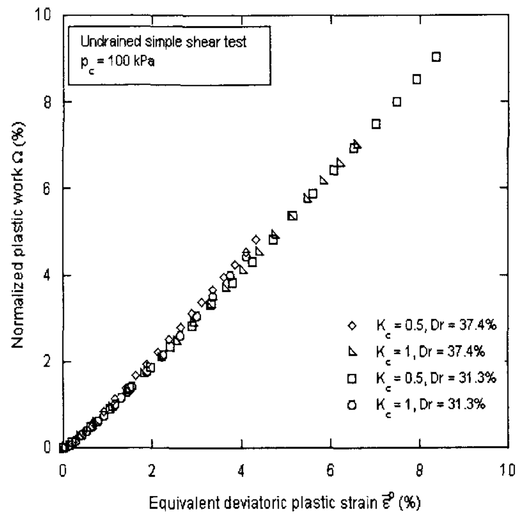

Fig. 5 Effect of confining stress ratio on the normalized plastic work

work is unique irrespective to drainage condition of test. However, at medium to large strain, normalized work in undrained condition is become larger than in drained condition. It is probably related to ununiform deformation at medium to large strain and to particle crushing at bigger stresses applied. Lade *et al.*[16] showed that on the basis of effective mean pressure, particle crushing in undrained condition is greater than one in drained condition. They also obtained that the number of particle crushing is unique related proportionally to energy input per unit volume of specimen during the test. Coop[17] confirmed that particle crushing exists even at relatively low stress. Hence, it can be deducted that in experimental work from which the data for this paper are obtained, there is particle crushing. And because of the amount of particle crushing in undrained condition is more than one in drained condition, the energy is also bigger. Consequently, the normalized work for undrained condition is greater than one in drained condition.

Fig. 4 demonstrates the normalized plastic work curve versus equivalent deviatoric plastic strain for various initial relative density. The range of relative densities is narrow. However, in that range of the experimental data used, it clearly shows that initial relative density has no effect on the normalized plastic work and equivalent deviatoric plastic strain relationship. Furthermore, this figure indicates that the uniqueness of the normalized plastic work under drained condition irrespective of relative density is also valid under undrained condition.

The effect of initial anisotropy is investigated using data from simple shear test in which the initial anisotropy consolidation state was reached by consolidated anisotropically. The result is presented in **Fig. 5** where two value of K_c are used, i.e., 1 and 0.5. The graphs are rather scattered, especially for equivalent deviatoric plastic strain greater than 5%, however it could be considered as a unique relationship and be concluded that the normalized plastic work is independent of K_c.

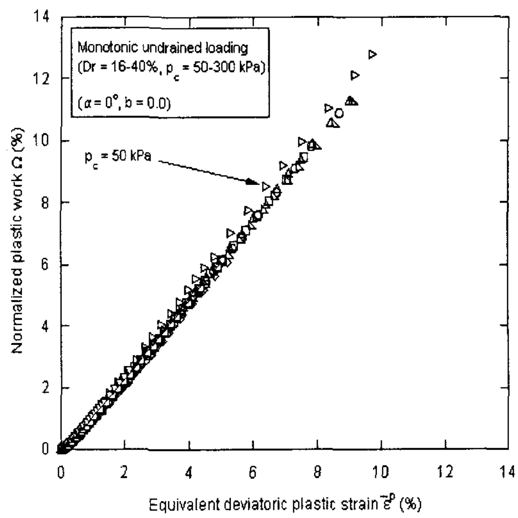

Fig. 6 Normalized shear work curve for different initial confining pressure p_c and relative density Dr

Because of the difficulties to keep p constant during the process of undrained test, instead of examining the effect of mean pressure, the influence of initial confining pressure is investigated. **Fig. 6** prompts

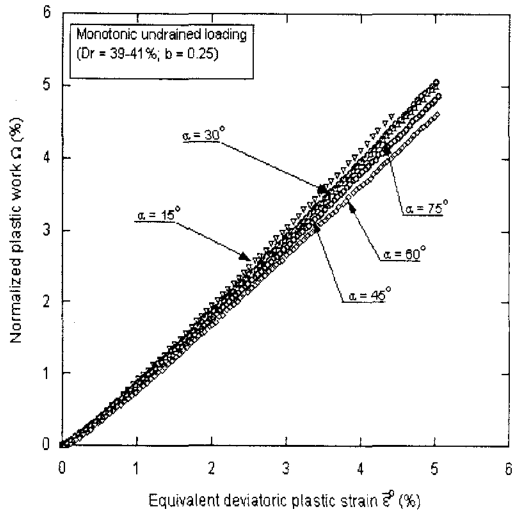

Fig. 7 Effect of α on the normalized plastic work

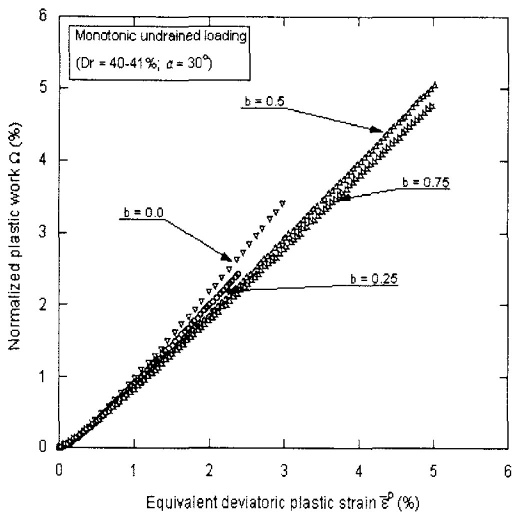

Fig. 8 Effect of b on the normalized work

the normalized plastic work curve with various both initial confining pressure p_c and the initial relative density Dr. It can be seen that the normalized plastic work is almost unique, however some deviation data takes place for which initial confining pressure is 50 kPa. Since **Fig. 4** shows that relative density does not affect the normalized plastic work, this discrepancy is probably related to the influence of a small initial confining pressure.

5.2 Effects of α and b on $\Omega - \bar{\varepsilon}^p$ relation

The data obtained from undrained hollow cylindrical - torsional test at which both the angel of maximum principal stress from vertical to bedding plane α and the intermediate principal stress coefficient b were fixed throughout the testing program was used. The data allows to observe the effect of each factor α and b separately or of mutually combination of both factors.

The effects induced by α on undrained normalized plastic work for a constant b are shown in **Fig. 7**. In this case, b is set to be 0.25 on a tentative basis. At a very small strain, the curves are almost identical, however, they become different proportional to increasing equivalent deviatoric plastic strain. Paying attention at large strain, apparently the normalized plastic work decrease with increasing α and reaches a minimum value when α equal to 60° and it turn to increase after α become larger than 60°. In this figure, the normalized plastic work for α equal to 30° and 75° are almost coincide.

Test series at which α was fixed to value of 15°, on a tentative basis, and various b are demonstrated in **Fig. 8**. Focusing on normalized plastic work at large strain, it is clearly seen that the normalize shear work decrease along the increasing of b. Though the limitation of data for $\alpha = 15°$ in which b only in the range of $0 - 0.75$, the effect of b bigger than 0.75 will be discussed further in the rest of this section regarding other value of α.

For better examination of the effect of α and b, it is better to explore the characteristic of the normalized plastic work at large strain. With reference of stress dilatancy relation of Eq. 17, the parameter μ at large strain, M, specifies stress ratio q/p at which plastic volumetric strain increment equal to zero. Since the vanishing of volumetric strain indicates the existence of the transition of deformation mode, the value of M exhibits the same implication with the angle of phase transformation (Ishihara et al.[18]).

From now on, the discussion is emphasized to observe the slope of normalized plastic work at large strain M.

The behaviour of M for different value of α is prompted in **Fig. 9**. The data available were not fully covered the full range of variation for each value of α and b, but they nevertheless exhibit a consistent tendency of M along the increasing of α for constant b. The value of M varies with α and has the minimum value when α set to value in the range of $60° - 75°$.

Miura *et al.*[20] clarified that the stress ratio at failure $\sin\phi = (\sigma_1 - \sigma_3)/(\sigma_1 + \sigma_3)$ has the minimum value when principal stress axis are orientated to the direction of $2\alpha = 120°$ to 150° or on the other word the direction of major principal stress axis relative to the vertical is between $\alpha = 60°$ to 75°. The reason is that when the direction is fixed to the value of 60° to 75°, a shear plane nearly parallel to the bedding plane has appeared, whereas the resistance against shear stress is minimum on the bedding plane.

Since stress ratio is proportional to normalized plas-

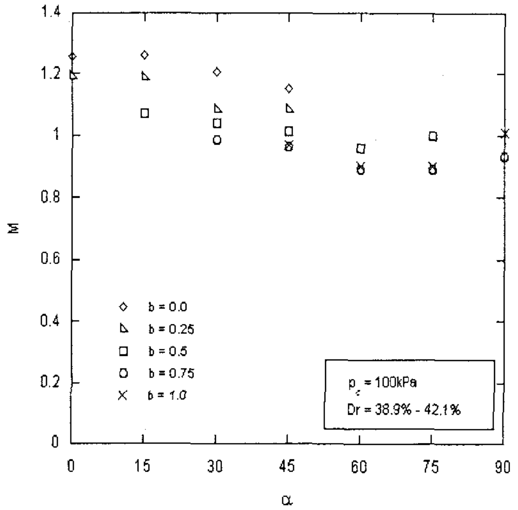

Fig. 9 Value of M for different α

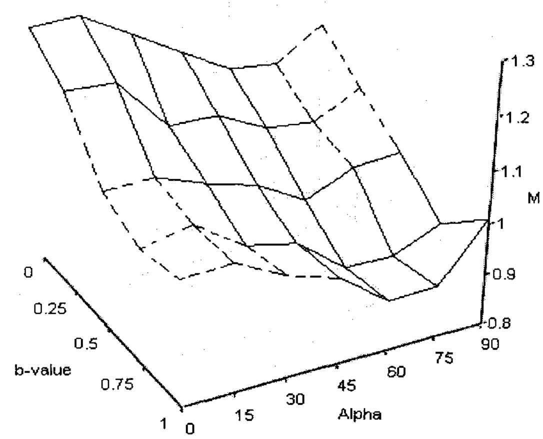

Fig. 11 Value of M for different α and b

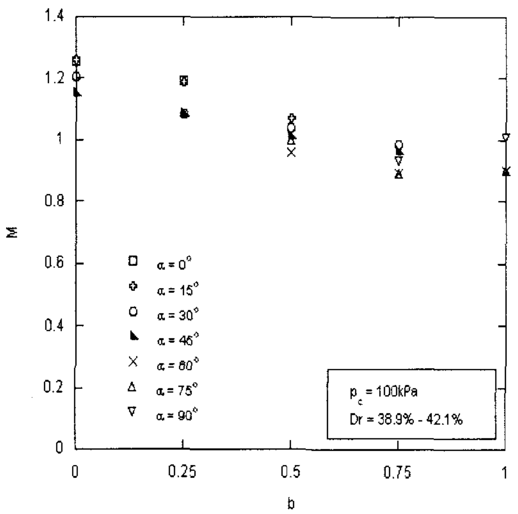

Fig. 10 Value of M for different b

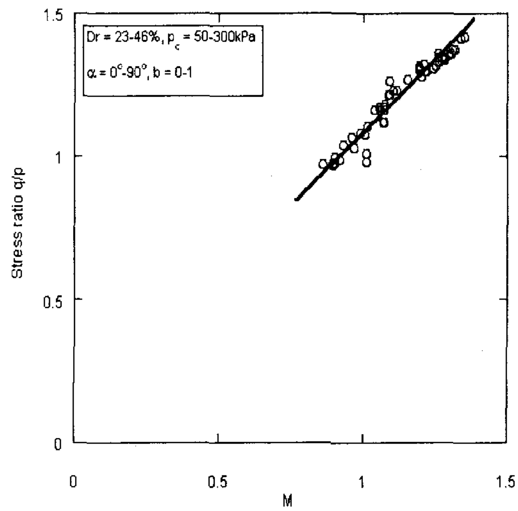

Fig. 12 Relation between M and stress ratio q/p

tic work (Eq. 15), the result in which M has minimum value when α in the range of 60° to 75° is consistent to the work done by Miura *et al.*[20] described above.

Fig. 10 demonstrates the value of M for different b to which α kept to be constant. The result is similar to one which is shown in **Fig. 8** where the value of M decrease gradually proportional to the increasing b and it turn to increase after the value of b is bigger than 0.75. It is well recognized that the intermediate principal stress coefficient has strong influence on soil behaviour under loading, however the mechanism remains not fully understood yet.

The effects of mutual combination of both α and b can be shown in the $M - \alpha - b$ space in **Fig. 11**. The dash lines connect the missing data obtained by extrapolation with assumption that the trend value of M is consistent following the existing data. The minimum value of M is attained when α is in the range of 60° to 75° and b is set to 0.75.

5.3 $M - (q/p)_{max}$ relationship

It is worthwhile evaluating the relation between M and maximum stress ratio. From **Fig. 12**, clearly it can be deducted that M has a linear relation with maximum stress ratio and ratio between M and maximum stress ratio is 0.9.

6. Conclusion

A careful study was conducted to evaluate the characteristic of the normalized plastic work for undrained condition in which the effects of both the direction of principal stress and the intermediate principal stress were considered. Based on the test results described in the present paper, the following conclusion can be drawn.

1. The relationship between normalized plastic work and equivalent deviatoric plastic strain is unique irrespective of drain conditions for strain up to 3%. At medium to large strain, the normalized work under undrained condition has a bigger value comparing with one under drained condition.
2. Undrained normalized plastic work is independent of the initial confining stress ratio, and initial confining pressure. However it is not unique with respect to the principal stress direction and the intermediate principal stress coefficient.
3. The parameter M has the minimum value when the maximum principal stress direction to vertical axis is orientated to the value between 60° and 75° and when the intermediate principal stress coefficient is set to 0.75.

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