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T-Stress Effects in Microcrack Shielding Problems of Anisotropic Materials

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This paper is concerned with the effect of the T-stress in microcrack shielding problems in anisotropic materials. The interaction between a macrocrack and associated microcracks in anisotropic elastic material under the T-stress effect is analyzed and compared with the results in the isotropic material. Three cases where a microcrack is in front of the main crack tip, a microcrack is on the side of the main crack tip, and two microcracks are on the both side of the main crack tip are investigated. The analysis is performed by an extended Pseudo-Traction Method. The stress intencity factor of the main crack tip and its incremental value due to the existence of microcracks and the T-stress are computed. The T-stress determines the magnitude of shielding and/or amplification due to the inclined near-tip microcracks. The effect depends on the positon of the microcrack, the inclination angle of the microcrack, the sign of the T-stress, the magnitude of the T-stress and the material anisotropy.

Key Words : *T-stress, Near-tip microcrack, Shielding and amplification effect, Stress intensity factor, J-integral*

1. Introduction

This paper is concerned with the effect of the T-stress in microcrack shielding problems in anisotropic materials. The interaction between a macrocrack and associated microcracks in anisotropic elastic material under the T-stress effect is analyzed and compared with the results in the isotropic material.

T-stress is a background stress field whose normal component is acting parallel to the crack. This is introduced by Rice[1]. T-stress has been studied from the point of view of its effect on the crack growth mechanism. Rice[1] and Larsson & Carlsson[2] found that the sign and the magnitude of the T-stress effects the size and shape of the plane-strain crack tip plastic zone. Bilby et al.[3] showed that the T-stress extends the range of validity of the small-scale yielding conditions at finite strains. In fracture analysis it is known that the T-stress governs the stability of a straight crack path under the mode I loading condition[4),5)]. Tvergaard & Hutchinson[6] studied the effect of the T-stress on mode I crack growth resistance in a ductile solid.

In studies of the microcrack toughness, investigations were performed by continuum model approach[2),3),4)] or discrete model approach[5),6)]. These investigations are associated with the release of a general residual stress field as a result of the existence of microcracks in the neighborhood of a macrocrack tip, but the T-stress effect is neglected. The T-stress may not be negligible when the microcracks are located at some distance, and the T-stress is the same order with the magnitude $K/\sqrt{2\pi r}$.

In the followings the effect of the T-stress in microcrack shielding problems in anisotropic materials will be studied. The interaction between a macrocrack and neighbouring microcracks in anisotropic elastic material will be analyzed in three different arrangements of cracks and the results will be compared with those in the isotropic materials.

The numerical method to analyze the crack interaction problems is based on the Pseudo-Traction Method that is proposed by Ma & Chen[8]. That is extended by taking the T-stress into account in the remote stress field.

2. T-Stress and Shielding Effects

Let us assume a crack parallel to the x-axis in Cartesian coordinate system (x,y). According to Williams[9], the stress distribution near the crack tip is expressed as

$$\begin{bmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{bmatrix} = \frac{K}{\sqrt{2\pi r}} \begin{bmatrix} f_{xx}(\theta) & f_{xy}(\theta) \\ f_{yx}(\theta) & f_{yy}(\theta) \end{bmatrix} + \begin{bmatrix} T & 0 \\ 0 & 0 \end{bmatrix} + \text{other terms} \quad (1)$$

where K is the stress intensity factor. The first term in the right hand side expresses the singularity at the crack tip; the second term is a stress field acting parallel to the crack plane. This stress field is named as "T-stress" by Rice[1]. It is known that T-stress has effects on the crack growth mechanism.

In the investigation of the microcrack shielding effect Chen[7] introduced a relation of the J-integrals:

$$J_t + \Delta J = J_\infty \quad (2)$$

where J_∞ is the contribution from the remote stress field parameterized by K_I^∞ and K_{II}^∞, J_t is the contribution from the macrocrack tip, and ΔJ is the contribution from the existing microcracks (see **Fig. 1**).

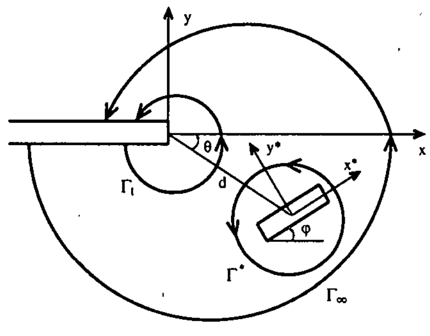

Fig. 1 Different J-integral contours in a macro-microcrack shielding problem

Eq (2) reveals that there is a wastage ΔJ when the remote J-integral J_∞ transmits across the microcracking zone from the infinity to the macrocrack tip, that is different from the relation $J_t = J_\infty$ used in the continuum modeling[4),12),13]. The wastage ΔJ due to the microcracks can be used as a criterion to determine the microcrack shielding or amplification effect[7].

For a finite crack of length $2a$ in an infinite plate, the T-stress depends on the applied remote stress. When the plate is loaded by a stress σ normal to the crack plane and $R\sigma$ parallel to it, we have

$$T = (R-1)\sigma, \qquad K_I = \sigma\sqrt{\pi a} \quad (3)$$

Thus, a biaxial parameter B can be defined as

$$B = \frac{T\sqrt{\pi a}}{K_I} = R - 1 \quad (4)$$

and in the following examples B is used to define the magnitude of the remote T-stress T^∞ as:

$$B = \frac{T^\infty\sqrt{\pi a}}{K_I^\infty} \quad (5)$$

From Eq (4), $-1 \le B \le 1$ for $0 \le R \le 2$; thus, it is appropriate to take values of B in the interval $(-1, 1)$ in the following calculation in anisotropic and isotropic materials. The remote stress field is also parameterized by K_I^∞ and B ($K_{II}^\infty = 0$).

3. Analysis Method

The numerical method to analyze the microcrack shielding problems is based on the method proposed by Ma & Chen[8]. The outline of the method is presented here.

Consider an interacting problem between a semi-infinite crack and N randomly distributed microcracks as shown in **Fig. 2**. The remote loads are K_I^∞

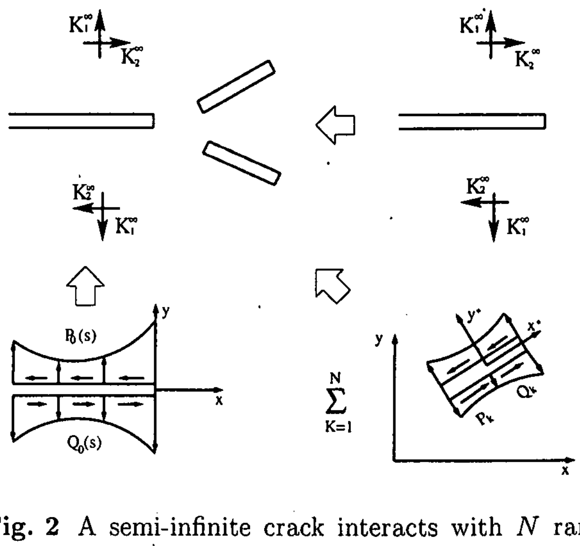

Fig. 2 A semi-infinite crack interacts with N randomly distributed microcracks

and K_2^∞. By using the principle of superposition and the Pseudo-Traction Method[17], the total stress state is decomposed into N+2 sub-states

$$\sigma_{ij} = \sigma_{ij}^\infty + \sigma_{ij}^0 + \sum_{k=1}^{N} \sigma_{ij}^k \quad (6)$$

where each stress state implies the followings as:

1. σ_{ij}^∞ is the stress state around the semi-infinite crack that is due to the remote loads K_1^∞ and K_2^∞. The lateral stress component that causes T-stress is added to this stress state.
2. σ_{ij}^0 is the stress caused by pseudo-tractions $P_0(s)$ and $Q_0(s)$ on the faces of the semi-infinite crack.
3. σ_{ij}^k is the stress caused by pseudo-tractions $P_k(t_k)$ and $Q_k(t_k)$ on the faces of the k-th microcrack.

The boundary condition on the semi-infinite crack surface becomes

$$n_j(s)\sigma_{ij}^0(s) + \sum_{k=1}^{N} n_j(s)\sigma_{ij}^k(s) = 0 \quad (7)$$

and on the k-th microcrack surface becomes

$$n_j(t_k)\sigma_{ij}^\infty(t_k) + n_j(t_k)\sigma_{ij}^0(t_k) + \sum_{l=1}^{N} n_j(t_k)\sigma_{ij}^l(t_k) = 0 \quad (8)$$

where n_j is the unit normal vector on the crack surface, and $-\infty < s < 0$, $-a_k < t_k < a_k$. These boundary conditions lead to $N+1$ integral equations with respect to the pseudo-tractions as

$$\begin{aligned}&(P_0(s), Q_0(s)) \\ &\quad + \sum_{k=1}^{N} \int_{-a_k}^{a_k} (P_k(t_k), Q_k(t_k)) f_{,k0}^*(t_k, s)\, dt_k \\ &= (0, 0)\end{aligned} \quad (9)$$

$$\begin{aligned}&\int_{-\infty}^{0} (P_0(s), Q_0(s)) f_{,0k}^*(s, t_k)\, ds \\ &\quad + (P_k(t_k), Q_k(t_k)) \\ &\quad + \sum_{\substack{i\neq k\\ i=1}}^{N} \int_{-a_k}^{a_k} (P_i(t_i), Q_i(t_i)) f_{,ik}^*(t_i, t_k)\, dt_i \\ &= (p_k(t_k), q_k(t_k))\end{aligned} \quad (10)$$

where $f_{,k0}^*, \cdots, f_{,ik}^*$ are the fundamental solutions (influence coefficients) that are obtained by the Pseudo-Traction Method[17]. $p_k(t_k)$ and $q_k(t_k)$ are the released stress components on the microcrack surface

$$p_k(t_k) = -n_i(t_k)n_j(t_k)\sigma_{ij}^\infty(t_k) \quad (11)$$

$$q_k(t_k) = -n_i(t_k)\tau_j(t_k)\sigma_{ij}^\infty(t_k) \quad (12)$$

where τ_i is the unit tangential vector.

To discretize Eq (9) and Eq (10), Chebyshev expansions of the pseudo-tractions are introduced. On the surface of the semi-infinite crack, parameter u is introduced as

$$s = (u-1)/(u+1), \quad -1 < u < 1 \quad (13)$$

and the Chebyshev numerical summation method is used. Eq (9) and Eq (10) are discretized into linear equations of $2M(N+1)$ unknown pseudo-tractions

$$\begin{aligned}&\frac{(u_m+1)^2}{2}(P_0(u_m), Q_0(u_m)) \\ &\quad + \sum_{k=1}^{N}\sum_{l=1}^{M}(P_k(t_k^l), Q_k(t_k^l)) f_{,k0}^*(t_k^l, u_m)\delta_k^l \\ &= (0, 0) \qquad (m = 1, 2, ...M)\end{aligned} \quad (14)$$

$$\begin{aligned}&\sum_{m=1}^{M}(P_0(u_m), Q_0(u_m)) f_{,0k}^*(u_m, t_k^n)\delta_0^m \\ &\quad + (P_k(t_k^n), Q_k(t_k^n)) \\ &\quad + \sum_{\substack{i\neq k\\ i=1}}^{N}\sum_{r=1}^{M}(P_i(t_i^r), Q_i(t_i^r)) f_{,ik}^*(t_i^r, t_k^n)\delta_k^r \\ &= (p_k(t_k^n), q_k(t_k^n)) \qquad (n = 1, 2, ...M)\end{aligned} \quad (15)$$

where M is the number of the Chebyshev collocation points, and $M = 21$ in the following computations. The parameters are defined by

$$u_m = \cos\frac{(2m-1)\pi}{2M}, \quad \delta_0^m = \frac{\pi}{M}\sin\frac{(2m-1)\pi}{2M} \quad (16)$$

$$t_k^n = a_k \cos\frac{(2n-1)\pi}{2M}, \quad \delta_k^n = \frac{\pi a_k}{M}\sin\frac{(2n-1)\pi}{2M} \quad (17)$$

Once the unknown pesudo-tractions are obtained, the changes of the SIFs at the main crack tip can be obtained by the following formulas[8]

$$\Delta K_I = \frac{2\sqrt{2\pi}}{M}\sum_{m=1}^{M} P_0(u_m)(1+u_m) \quad (18)$$

$$\Delta K_{II} = \frac{2\sqrt{2\pi}}{M}\sum_{m=1}^{M} Q_0(u_m)(1+u_m) \quad (19)$$

To calculate the J-integrals, consider the integration contours shown in **Fig. 1**. Γ_t, Γ^* and Γ_∞ are enclosing the macrocrack tip, microcrack and remote field, respectively. Asssuming that the stress intencity factors by the remote load are K_I^∞ and K_{II}^∞, the J-integrals on the contours Γ_∞, Γ_t and Γ^* are expressed by

$$\begin{aligned}J_\infty = -\frac{b_{11}}{2}\Im\,&\big[(K_I^\infty)^2(s_1+s_2)\bar{s}_1\bar{s}_2 \\ &+ 2K_I^\infty K_{II}^\infty \bar{s}_1\bar{s}_2 - (K_{II}^\infty)^2(s_1+s_2)\big]\end{aligned} \quad (20)$$

$$J_t = -\frac{b_{11}}{2}\Im\left[(K_I^\infty + \Delta K_I)^2(s_1+s_2)\bar{s}_1\bar{s}_2 + 2(K_I^\infty + \Delta K_I)(K_{II}^\infty + \Delta K_{II})\bar{s}_1\bar{s}_2 + (K_{II}^\infty + \Delta K_{II})^2(s_1+s_2)\right] \quad (21)$$

$$\Delta J = J_1^* \cos\varphi - J_2^* \sin\varphi \quad (22)$$

where s_1, s_2, $\bar{s}_1$ and $\bar{s}_2$ are the roots of the characteristic equation

$$b_{11}s^4 + (2b_{12} + b_{66})s^2 + b_{22} = 0 \quad (23)$$

where $b_{11} = 1/E_{11}$, $b_{12} = -\nu_{12}/E_{11}$, $b_{22} = 1/E_{22}$ and $b_{66} = 1/\mu_{12}$. E_{11}, E_{22}, ν_{12} and μ_{12} are the material constants. J_1^* and J_2^* are x^* and y^* components, respectively, of the J_k^* vector in the local coordinate system (x^*, y^*), that can be easily calculated once the stress (strain) field is obtained.

4. Numerical Results and Discussion

Three cases of microcrack shielding problems are examined both for isotropic and orthotropic materials: a microcrack is at the co-axial front of the macrocrack tip, a microcrack is on the side of the macrocrack tip, and two microcracks are on the both sides of the macrocrack tip. In each case the stress intensity factors and J-integrals are computed for various inclining angles. The material parameters, elastic modulus E and Poisson's ratio ν, are shown in **Table 1**.

Table 1 Material parameters of orthotropic and isotropic materials

Orthotropic Material	
E_1	$1.72 \times 10^{10}\mathrm{N/m^2}$
$E_2 = E_3$	$0.496 \times 10^{10}\mathrm{N/m^2}$
ν_{12}	0.32
ν_{23}	0.44
Isotropic Material	
E	$0.372 \times 10^{10}\mathrm{N/m^2}$
ν	0.345

4.1 Macrocrack with a microcrack in front of the tip

A macrocrack with a microcrack of length $2a$ that is at the co-axial front of the macrocrack tip is considered as shown in **Fig. 3**. The distance between the crack tip and the center of the microcrack is $d/a = 1.5$.

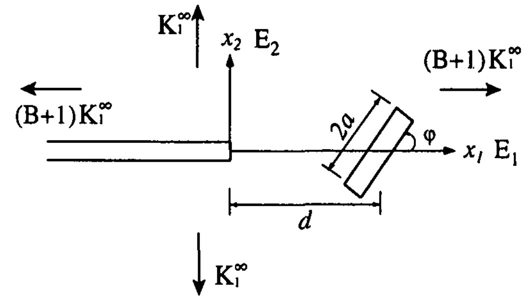

Fig. 3 Macrocrack with a microcrack of length $2a$ in the co-axial front of the tip ($d/a = 1.5$)

The stress intensity factors of the macrocrack and the J-integrals for the inclination angle of the microcrack are shown in **Fig. 4** and **Fig. 5**, respectively, where the solid line indicates the result of the orthotropic material and the dotted line indicates the result of the isotropic material.

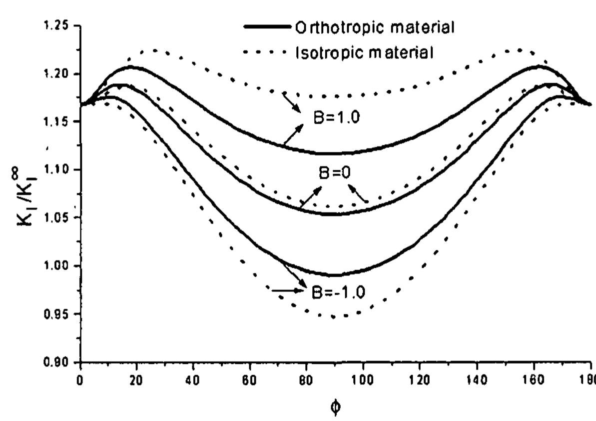

Fig. 4 The stress intensity factor K_I/K_I^∞ of the macrocrack-tip versus the microcrack inclination angle for $d/a = 1.5$

In **Fig. 4** the shapes of the stress intencity factor graphs are similar both in isotropic and anisotropic cases, that is, the tendencies of the T-stress effect on the stress intensity factor of the macrocrack is similar for both materials. It can be seen that the results are independent of the T-stress values when $\phi = 0°(= 180°)$, i.e. the T-stress has no effect for the parallel microcracks. However, there exists obvious difference between the results with the T-stress and those without it for other inclination angle ϕ, and the effect of the T-stress is the most significant when $\phi = 90°$. As shown in **Fig. 4**, the T-stress has substantial influences on shielding or amplification effect by the inclined microcrack, and the larger the value of $|B|$ the more influencial. The positive T-stress ($B > 0$) increases the amplification effect, whereas the negative T-stress ($B < 0$) decreases the amplification effect.

In **Fig. 4** there exists obvious difference between the T-stress effects in isotropic and orthotropic materials. The magnitude of the difference depends on the value of B. The difference is minimum when $B = 0$, i.e., $T = 0$, and the difference increase as the value of $|B|$. The difference of $B = 1$ case is larger than that of $B = -1$ case. In the orthotropic material case, the amplification effect decreases when $B = 1$, whereas the shielding effect decreases when $B = -1$. These facts suggest that the material anisotropy affects the T-stress effect strongly. In this case, where the anisotropy is given by **Table 1**, i.e., $E_1 > E_2$, the anisotropy decreases the T-stress effect.

Fig. 5 shows the J-integrals J_t and ΔJ for various inclination angle ϕ in the orthotropic material with $d/a = 1.5$. The relation Eq (2) is still valid if the T-stress is considered, although the T-stress influences the values of J_t and ΔJ.

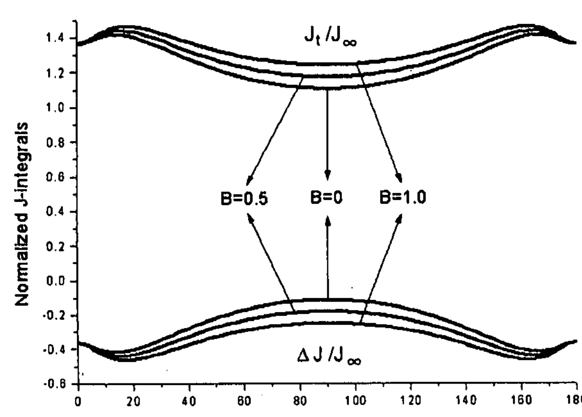

Fig. 5 The J-integrals J_t and ΔJ (nomalized by J_∞) v.s. the microcrack inclination angle ϕ in orthotropic materials

4.2 Macrocrack with a microcrack on the side of the tip

In this case, a macrocrack with a microcrack of length $2a$ on the side of the crack tip is considered as shown in **Fig. 6**. The distance from the crack tip

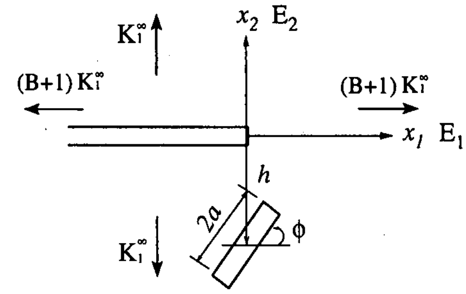

Fig. 6 Macrocrack with a microcrack of length $2a$ on the side of the macrocrack tip ($h/a = 1.5$)

to the center of the microcrack is $h/a = 1.5$ in this example.

The numerical results of the stress intensity factors of the macrocrack and the J-integrals are shown in **Fig. 7** and **Fig. 8**, respectively, for the microcrack inclination angle ϕ.

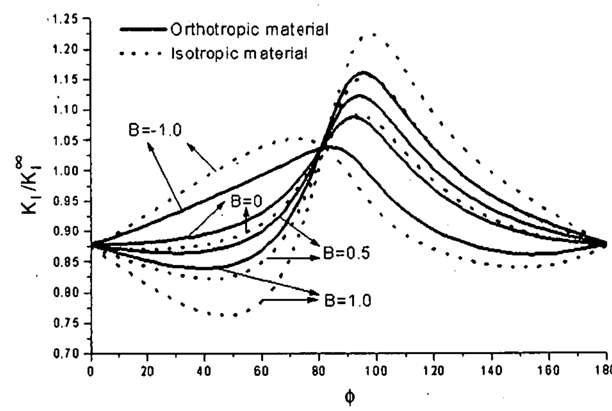

Fig. 7 The stress intensity factors K_I/K_I^∞ of the macrocrack-tip versus the microcrack inclination angle for $h/a = 1.5$

From the results of orthotropic and isotropic materials (see **Fig. 7**), like in the previous case, the T-stress has no effect on the stress intensity factors of the macrocrack if the microcrack is parallel to the macrocrack, i.e., $\phi = 0°$. The effect of the T-stress depends on the sign and the magnitude of the T-stress as well as on the inclination of the microcrack. The curves cross at the same point both in the isotropic and the orthotropic materials, thus the T-stress effect is small around a cirtain angle ($\phi \simeq 80°$) of microcrack inclination.

There exists obvious difference between the results of the orthotropic and isotropic materials (see **Fig. 7**). This difference depends not only on the magnitude

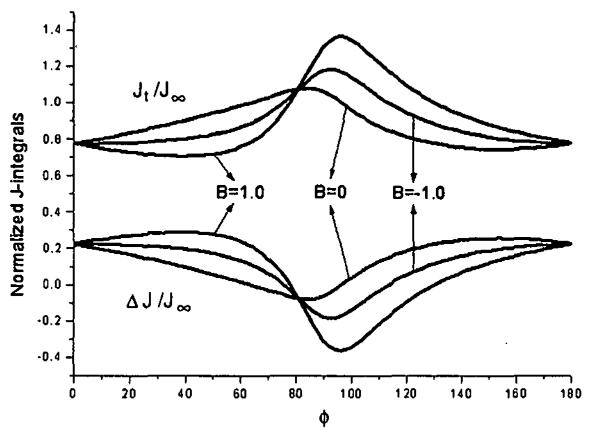

Fig. 8 The J-integrals J_t and ΔJ (normalized by J_∞) versus the microcrack inclination angle in orthotropic materials

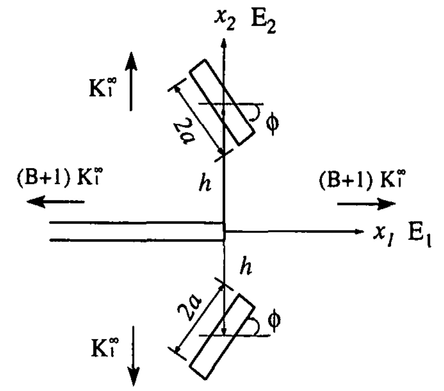

Fig. 9 Two symmetrically arranged microcracks of length $2a$ centered above and below the macrocrack-tip ($h/a = 1.5$)

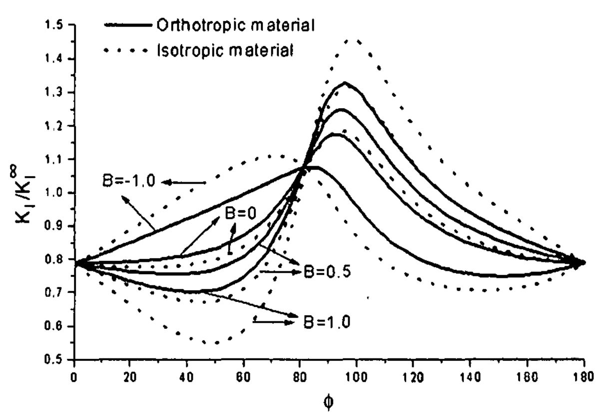

Fig. 10 The stress intensity factors $K_I/K_I^\infty$of the macrocrack-tip versus the microcrack inclination angle for $h/a = 1.5$

of the T-stress but also on its sign. Although the difference is small when $B = 0$, the difference increase with the magnitude of $|B|$. This anisotropy effect is larger when $B = 1$ than when $B = -1$. The effect of T-stress is larger in the isotropic case than in the orthotropic case with the present material constants, i.e., $E_1 > E_2$, as in the previous problem.

Fig. 8 shows the results of the J-integrals J_t and ΔJ normalized by J_∞ for $h/a = 1.5$. It can also be seen that the relation Eq (2) of the J-integrals is still held even if the T-stress is considered while the T-stress influences the values of J_t and ΔJ.

4.3 Macrocrack with two microcracks on the both sides of the tip

A macrocrack with two symmetrically arranged microcracks both above and below the macrocrack tip with the same distance $h/a = 1.5$ is examined here, as shown in **Fig. 9**.

The stress intensity factors of the isotropic and orthotropic materials with T-stress are plotted in **Fig. 10**, and the J-integrals are shown in **Fig. 11**.

The results are similar with the previous results of the sided single microcrack case, but the peek-to-peek value is larger than that of the previous problem (**Fig. 10**). In the isotropic case, the stress intensity factor $K_I/K_I^\infty \simeq 0.78$ at $\phi = 0$, the maximum value is $K_I/K_I^\infty \simeq 1.46$ when $B = 1$, and the minimum value is $K_I/K_I^\infty \simeq 0.54$. On the other hand, in the single microcrack case (**Fig. 7**), these values are $K_I/K_I^\infty \simeq 0.86, 1.23$ and 0.76. The difference of these values are due to the number of microcracks. Actually, in **Fig. 11**, $\Delta J/J_\infty \simeq 0.38$ at $\phi = 0$ and the minumum value is about -0.76 when $B = 1$. They are almost twice of the coresponding values of $\Delta J/J_\infty$ in **Fig. 8**: $\Delta J/J_\infty \simeq 0.23$ at $\phi = 0$ and the minumum value is about -0.36.

In **Fig. 10** the graph figures are similar both in the isotropic and othtropic cases, while the values differ. All graphs cross at the same point where $\phi \simeq 80°$ like in the single microcrack case. The effect of T-stress is larger in the isotropic case than in the orthotropic case. The difference between the isotropic and the orthotropic values are larger when $B = 1$ than when $B = -1$. The consistent relation Eq (2) of the J-integral is also valid in this problem (**Fig. 11**).

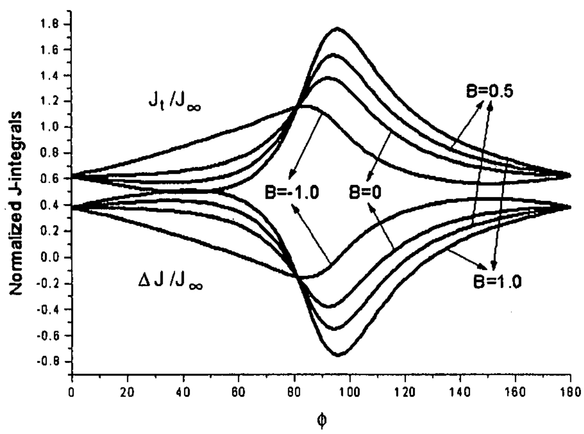

Fig. 11 The J-integrals J_t and ΔJ (normalized by J_∞) versus the microcrack inclination angle in orthotropic materials

5. Conclusions

The T-stress effect in microcrack shielding problems in orthotropic materials is studied by considering the interaction between the macrocrack and the near-tip microcracks. Three cases of the crack interaction problem are analyzed by an extended Pseudo-Traction Method,

The results are summarized as follows:

1. The T-stress determines the magnitude of shielding and/or amplification due to the inclined near-tip microcracks. The effect depends on the positon of the microcrack, the inclination angle of the microcrack, the sign of the T-stress, the magnitude of the T-stress and the material anisotropy.
2. When the microcrack is in front of the main crack tip, the maximum T-stress effect occurs if the microcrack is perpendicular to the main crack. On the other hand, when the microcrack is on the side of the main crack tip, there is a stational angle (near the perpendicular position) where T-stress does not effect. The T-stress effect changes its sign in both sides of the angle.
3. The effect changes its sign depending on the sign of the T-stress, and the magnitude of the effect is increased as the absolute value of the T-stress is larger.
4. The material anisotropy effects on the magnitude of T-stress effect. In the present examples, the material stronger axis is parallel to the main crack, and the T-stress effect is weaker than in the isotropic material case.
5. The conservation relation $J_\infty = J_t + \Delta J$ is satisfied in all cases, although the T-stress influences the contributions to the J-integral induced from the microcracks (ΔJ) and the macrocrack tip (J_t).

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