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Reduction of structural vibrations induced by a moving load

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Structural response to the ground excitations and the ground response to a moving load on the soil surface are considered. The reduction of the structural response is achieved by reducing the ground vibrations using a trench and a wave-impeding barrier (wib). The numerical investigation is performed by using a boundary element and finite element method. The response of the structure increases with the moving speed of the load, independent of the reduction approach. The investigation reveals that both trench and wave barrier can be used as a reduction measure.

Key Words: Moving load, reduction of vibrations, wave impeding barrier, BE-FE coupling, trench

1. Introduction

Ground vibrations due to moving load received more attention recently, since people are more aware of their life qualities. The other reason is the increasing speed of the wave source, such as high-speed train. Since the induced ground vibrations increase in general with the speed and the magnitude of the moving load, the effect of the moving heavy load will be recognized easily, especially, in the densely populated regions. The public awareness of their surrounding decreases the acceptance of disturbances due to man-made vibrations. A good overview on the effect of man-made vibrations is given in the proceedings of the Wave2000-meeting[1]. The main objectives of the investigations on the effect of moving loads are the modeling of the load, the proper modeling of the wave propagation in the soil and the reduction measures.

In order to reduce the effect of a moving source we can install reduction devices at the source, in the transmitting medium soil, or we can increase the damping ability of the structures itself, for example, by adding dampers at the structural elements, by using tuned mass dampers or by installing base-isolation systems between the structure foundation and the base. Warburton[2] gives a good description of possible reduction measures for the vibration of structure. Reduction measures in the soil can be achieved by installing a trench, a wall or a row of hollow piles as a wave barrier. A solid wall is stable, however, it is not as effective as a trench. The trench is very effective, however, it is difficult to keep it open. To overcome this difficulty Massarsch[3] invented gas cushion screens to replace a solid wall. Kim et al.[4] suggested -for example- rubber chip barrier instead of a solid wall.

Another possible way to reduce the ground vibrations is to use a wave impeding barrier[5] (wib). It is based on the cut-off frequency of a soil layer over bedrock[6]. The effect of cut-off frequency was described by Lord Kelvin[7] more than one hundred years ago. However, his work maybe overlooked by many scientists, since the title of his works does not say any thing about the cut-off frequency. We know from the case of stationary source that waves with predominant frequencies below the cut-off frequency of a soil layer over bedrock are impeded in their propagation. We also know that a stiff barrier cannot replace the real bedrock, since the barrier will move if it is excited by the incoming waves. However, we still can activate the wave impeding behaviour in the layer between the wave barrier and the soil surface.

Investigations on reduction of traffic induced vibrations are often limited to two-dimensional problems, e.g. Adam et al.[8]. Three-dimensional analysis is limited, e.g. Hung and Yang[9] investigated the influence of the stiffness of a wall, along the path of the moving load, on the effectiveness of the wall. Study on reduction of the effect of moving load on structures in the surrounding is very limited.

This work addresses the ground vibrations induced by a moving load on the soil surface, and their reduction due to a trench and a wib. To the best knowledge of the authors it is the first time that wib for reducing the effect of a moving load is considered. The response of a structure to the ground vibrations is studied as well.

2. Soil-structure system and ground vibrations

2.1 Soil-structure system in Laplace domain

In the analysis the structure with foundation is described by finite elements with continuous mass in the Laplace domain. The dynamic stiffness of the structural members is obtained by solving the equation of motions analytically.

For axial vibrations u_x the solution of the equation of motion

$$m\ \ddot{u}_X = EA\ u_{X,XX} \tag{1}$$

is

$$\frac{m\,s^2}{EA}\,\tilde{u}_X = \tilde{u}_{X,XX} \tag{2}$$

m is the mass per unit length. Dots indicate differentiation with respect to time t, and commas indicate partial derivatives with respect to x. E and A are the modulus of elasticity and cross-section area, respectively.

$s = \delta + i\omega$ is the Laplace parameter, and $i = \sqrt{-1}$.

By introducing the boundary conditions, the dynamic stiffness $\tilde{\mathbf{k}}_X$ for the axial vibrations is determined by the relationship between the axial forces $\tilde{\mathbf{P}}_X$ and the displacements $\tilde{\mathbf{u}}_X$ at the both ends of the structural member

$$\tilde{\mathbf{k}}_X\ \tilde{\mathbf{u}}_X = \tilde{\mathbf{P}}_X \tag{3}$$

The solution of the equation of motion

$$m\ \ddot{u}_y = -EI_z\, u_{y,xxxx} \tag{4}$$

for the transverse vibrations u_y in the Laplace domain is

$$\frac{m\,s^2}{EI_z}\,\tilde{u}_y = -\tilde{u}_{y,xxxx} \tag{5}$$

EI_z is the flexural stiffness. An introduction of the boundary conditions leads to the relationship between the shear forces and bending moments $\tilde{\mathbf{P}}_X$ and the deformations $\tilde{\mathbf{u}}_X$ at the both ends of the structural members

$$\tilde{\mathbf{k}}_y\ \tilde{\mathbf{u}}_y = \tilde{\mathbf{P}}_y \tag{6}$$

For torsional vibrations φ_X of an element of constant torsional stiffness $G\,I_p$ the solution of the equation of motion

$$\rho\, I_p\ \ddot{\varphi}_X = G\, I_p\ \varphi_{X,XX} \tag{7}$$

is

$$\frac{ms^2}{GA}\,\tilde{\varphi}_X = \tilde{\varphi}_{X,XX} \tag{8}$$

By introducing the boundary conditions, the dynamic stiffness $\tilde{\mathbf{k}}_T$ for the torsional vibrations is defined by the relationship between the torsion moment $\tilde{\mathbf{M}}_T$ and the rotations $\tilde{\varphi}_T$ at the both ends of the structural member

$$\tilde{\mathbf{k}}_T\ \tilde{\mathbf{u}}_T = \tilde{\mathbf{P}}_T \tag{9}$$

We obtain then the dynamic stiffness $\tilde{\mathbf{k}}^b$ of the structure by adding the stiffness of each structural member using the direct stiffness method.

The wave transmitting behavior of the soil is described by boundary element method. We then obtain the dynamic stiffness of the soil $\tilde{\mathbf{k}}^s$. In order to couple the two subsystems structure with foundations and soil, the degree-of-freedom (DOF) of the structure is divided into contact DOF (CDOF) of the foundations and the DOF of the rest of the structure. The DOF of the soil elements at the interface between foundation and soil are transformed into the CDOF $\tilde{\mathbf{U}}^s_c$. We thereby obtain the transformed stiffness matrix $\tilde{\mathbf{k}}^s_{cc}$. The coupling of the structure and soil is achieved by equating the displacements and by equilibrating the forces at the soil-foundation interface. We obtain the governing equation of the soil-structure system.

$$\begin{bmatrix} \tilde{\mathbf{k}}^b_{bb} & \tilde{\mathbf{k}}^b_{bc} \\ \tilde{\mathbf{k}}^b_{cb} & \tilde{\mathbf{k}}^b_{cc} + \tilde{\mathbf{k}}^s_{cc} \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{U}}^b_b \\ \tilde{\mathbf{U}}^b_c \end{bmatrix} = \begin{bmatrix} \tilde{\mathbf{P}}^b_b \\ \tilde{\mathbf{P}}^b_c \end{bmatrix} \tag{10}$$

The ground excitation $\tilde{\mathbf{P}}^b_c$ of the footings is determined in the time domain by using another boundary element program.

2.2 Determination of soil vibration in time domain

In order to determine the soil vibration and the foundation excitation a boundary element method in time domain is used. The differential equation of *Lame-Navier* describes the displacement field u_i of a linear elastic homogeneous continuum subjected to the body forces per unit mass f_i:

$$(c_p^2 - c_s^2)\,u_{k,ki} + c_s^2\,u_{i,kk} - \ddot{u}_i + f_i = 0, \quad i,k = 1,2,3 \tag{11}$$

The commas and dots indicate space and time derivatives, respectively. The constant parameters

$$c_p = \sqrt{\frac{\lambda + 2\mu}{\rho}} \quad \text{and} \quad c_s = \sqrt{\frac{\mu}{\rho}} \tag{12}$$

represent the dilatational and the shear wave velocity where ρ is the mass density and λ and μ are *Lame*'s constants.

For the existence of a unique solution, the following initial and boundary conditions have to be considered:

$$u_i(\mathbf{x},t=0)=u_i^0(\mathbf{x}),\dot{u}_i(\mathbf{x},t=0)=v_i^0(\mathbf{x})\quad \forall\mathbf{x}\in\Psi,$$
$$u_i(\mathbf{x},t)=\bar{u}_i(\mathbf{x},t)\quad \forall\mathbf{x}\in\Gamma_1, \quad (13)$$
$$t_i(\mathbf{x},t)=\sigma_{ik}n_k=\bar{t}_i(\mathbf{x},t)\quad \forall\mathbf{x}\in\Gamma_2,$$

where $\Gamma=\Gamma_1\cup\Gamma_2$ is the boundary of the domain Ψ, t_i represent the traction and n_i are the direction cosines of the outward normal to the boundary.

Using *Stokes*'[10] full-space fundamental solution together with the *Reciprocal Theorem* of elastodynamics, the *Somigliana* identity can be derived[11] and formulated for the boundary of the domain. When body forces are neglected and homogeneous initial conditions apply, it has the following form:

$$c_{ik}u_k(\xi,t)=\int_{\Gamma_x}\int_0^t u_{ik}^*(\mathbf{x},t,\xi,\tau)\,t_i(\mathbf{x},\tau)\,d\tau\, d\Gamma_{\mathbf{x}} - \int_{\Gamma_x}\int_0^t t_{ik}^*(\mathbf{x},t,\xi,\tau)\,u_i(\mathbf{x},\tau)\,d\tau\, d\Gamma_{\mathbf{x}}, \quad (14)$$

where t_{ik}^* is the fundamental solution for the traction, which corresponds to the fundamental solution for the displacements u_{ik}^* and Γ_x indicates the integration over the boundary with respect to $\mathbf{x}$. The boundary matrix c_{ik} includes the integral-free terms, which depend on the geometry in the vicinity of the source point ξ. For smooth boundaries c_{ik} is equal to δ_{ik} if $\xi\in\Psi$ and $0.5\,\delta_{ik}$ if $\xi\in\Gamma$. For each point on the boundary Γ either the displacement or the traction is known, and equation (14) has to be solved to find the unknown boundary values.

The boundary integral equation (14) is discretized and solved numerically. Space constant elements are used for the discretization of space. Linear shape functions are used for the description of the time dependence of the displacements and constant shape functions for the description of the time dependence of the traction. The following discrete form is obtained from the time-domain formulation:

$$c_{ik}u_i(\xi,t_N)+\sum_{l=1}^{L}\sum_{m=1}^{N}\int_{\Gamma_l}T_{ik}^{(N-m+1)}(x,\xi)\,d\Gamma_{lx}{}^{(l)}u_i^{(m)} = \sum_{l=1}^{L}\sum_{m=1}^{N}\int_{\Gamma_l}U_{ik}^{(N-m+1)}(x,\xi)\,d\Gamma_{lx}{}^{(l)}t_i^{(m)} \quad (15)$$

where T_{ik} and U_{ik} are the traction and displacement kernels, resulting from the fundamental solutions' temporal integration. The outer summation in equation (15) is carried out over the total number of elements L and the inner summation is carried out over the number of time steps N. *Gaussian* quadrature can be used to evaluate the integrals over the boundary Γ_l. The quadrature scheme has to be modified for $m=N$ to account for the singularities of T_{ik}^1 and U_{ik}^1, which occur when the distance between the source and the field point approaches zero[12]. After integration, equation (15) can be written in matrix notation as:

$$\mathbf{U}^1\mathbf{t}^N=\mathbf{T}^1\mathbf{u}^N+\mathbf{E}^N,\ \mathbf{E}^N=\sum_{m=2}^{N}\mathbf{T}^m\mathbf{u}^{N-m+1}-\mathbf{U}^m\mathbf{t}^{N-m+1} \quad (16)$$

where $\mathbf{U}^m$ and $\mathbf{T}^m$ are the coefficient matrices of the system at time $m\Delta t$. For the current time step N, all traction vectors $\mathbf{t}^m$, $m=1$ to N, and previous displacement vectors $\mathbf{u}^m$, $m=1$ to N-1, are known. Details about the derivation are given by the first author [13].

3. Reduction of soil vibrations and structural responses

3.1 System considered

The considered structure is a three-storey steel frame structure with four rigid surface foundations (Fig. 1(a)). Each structural member has a length of 3.5m. The mass per unit length is 74.4kg/m. The flexural stiffness EI_y = 4921.214kNm², and EI_z = 34438.95kNm². The torsional constant I_p = 7.4e-7m⁴. EA = 1991619kN. The double T-profile of the steel members is indicated in the figure. The y- and z-axis is the axis perpendicular and parallel to the flange, respectively.

The soil is assumed to be a half-space. The propagation velocity of shear waves in the soil is 130m/s. The soil density is 1800kg/m³, and the Poisson ratio is 0.33.

It is assumed that the structure has no material damping. The structure will therefore experience only radiation damping due to the propagation of waves in the soil. The ground motions are produced by a moving load with a constant speed of 300, 400 and 500km/h. The load path, indicated by a black line, is located 7m from the structure. Since each of the structural foundations has different relative distance to the vibration source, they will therefore experience different ground excitations. The wib has a density of 2000kg/m³, and the shear wave velocity of 250m/s.

The considered reduction approaches are a wave impeding barrier (wib) and an open trench. The trench has a depth of 4.5m, and a width of 0.5m (Fig. 1(b)). The distance between the load path and the right boundary of the trench is 1.75m. The wib is installed below the load path at a depth of 1.5m. The wib block has a width of 3.0m and a thickness of 0.8m (Fig. 1(c)).

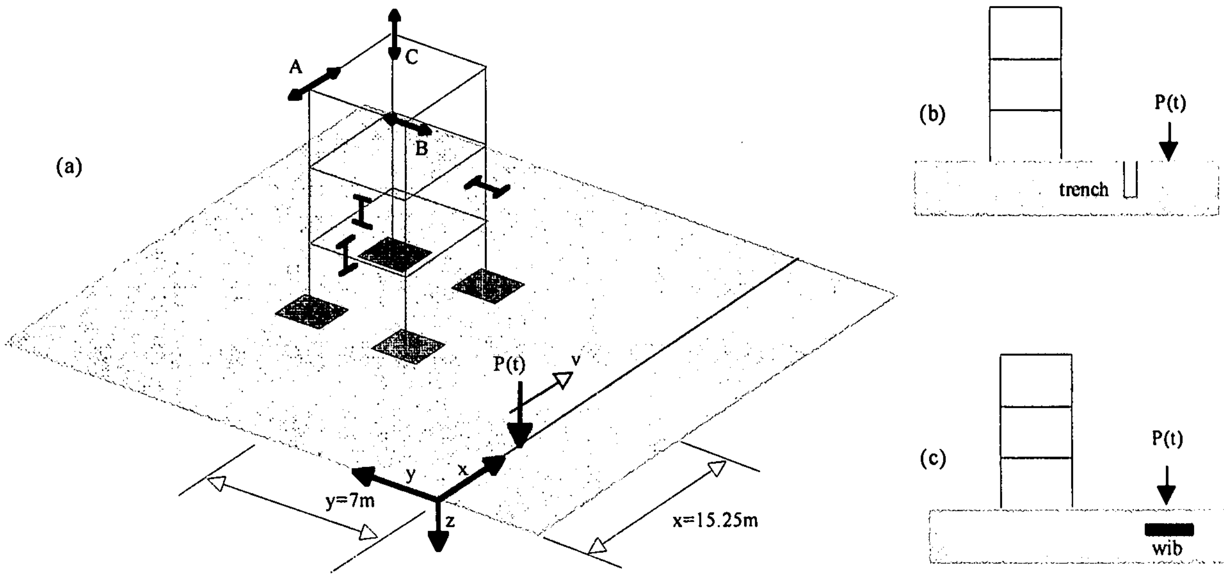

Fig. 1 Structure, foundations and subsoil. (a) Half-space without reduction measures, (b) with trench and (c) with wib

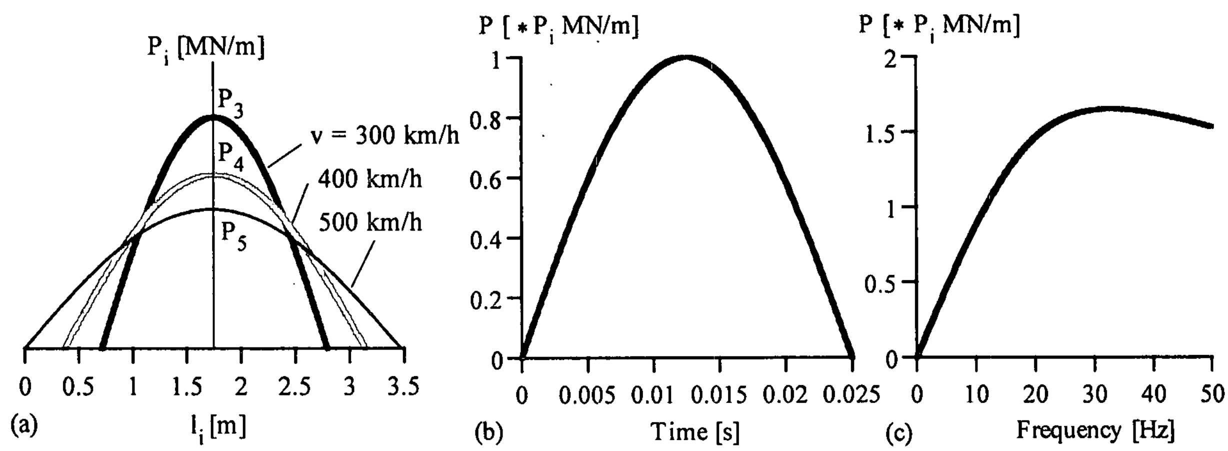

Fig. 2 Moving load. (a) Distribution along the load path, (b) the time history and (c) the corresponding response spectrum

Since most of the traffic loads have a wide range of frequency contents with predominant frequencies above 20Hz, a load with a half-sine time history in Fig. 2(b) is chosen. The response spectrum in Fig. 2(c) clearly shows the predominant frequencies in the high frequency range. In order to have the same load shape and duration (see Fig. 2(b)) at each locations of the load path, we have to increase the distribution length of the load in the load path, and the maximum load P_i therefore decreases with a growing speed of the load (Fig. 2(a)). The maximum of the load for the considered load speed is given in Table 1.

3.2 Ground response

Fig. 3 displays the response of the ground to the passing load. The considered location is 7m left from the load path and 18.75m from the starting point of the load. The direction perpendicular to the load path is indicated as the y direction, and the moving direction of the load as the x direction. The response clearly shows the dependency of the ground vibrations on the speed of the moving source. Since the surface wave in the soil has a velocity of about 402km/h, we can expect an amplification

Table 1 Change of the load maximum with the load speed

Source speed [km/h]	300	400	500
P_i [MN/m]	0.7540	0.5655	0.4524

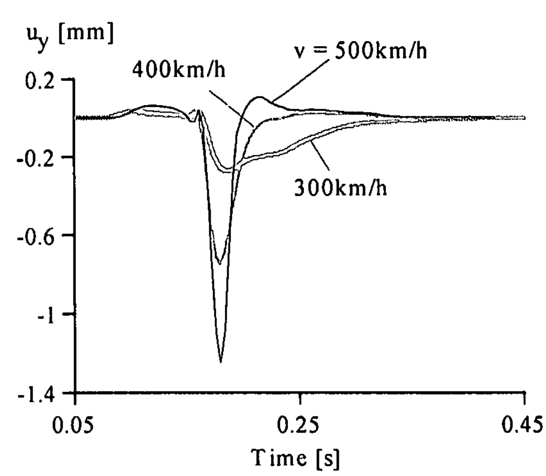

Fig. 3 Influence of the moving speed on the surface vibration of the half-space

of the ground vibrations at a moving speed of 400km/h. This is also the case, when we consider the ground motions at the load path (not presented here). However, at a certain distance from the load path the ground response increases with the load speed. The higher the speed is, the stronger the ground response in the surrounding area will be.

Fig. 4 shows the frequency content of the ground acceleration, which represents the ground excitation of structure in the neighbourhood of the wave source. The source moves at a speed of 500km/h. The ground motions in all x-, y- and z-direction have the same frequency content as the source. Since a response spectrum displays the maximum response of a single-degree-of-freedom (SDOF) system to the ground motions as a function of its natural frequency, the figure therefore indicates that a structure will be strongest excited in the vertical direction. Compared to the excitation in the y-direction the excitation in the x-direction is stronger.

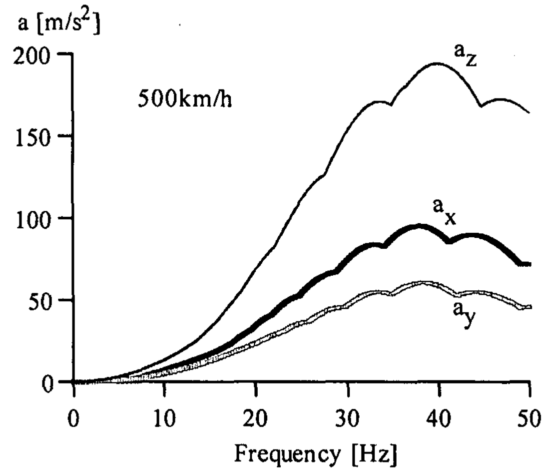

Fig. 4 Response spectra of the ground accelerations in the x-, y- and z-direction at x=18.75m and y=7m of the half-space

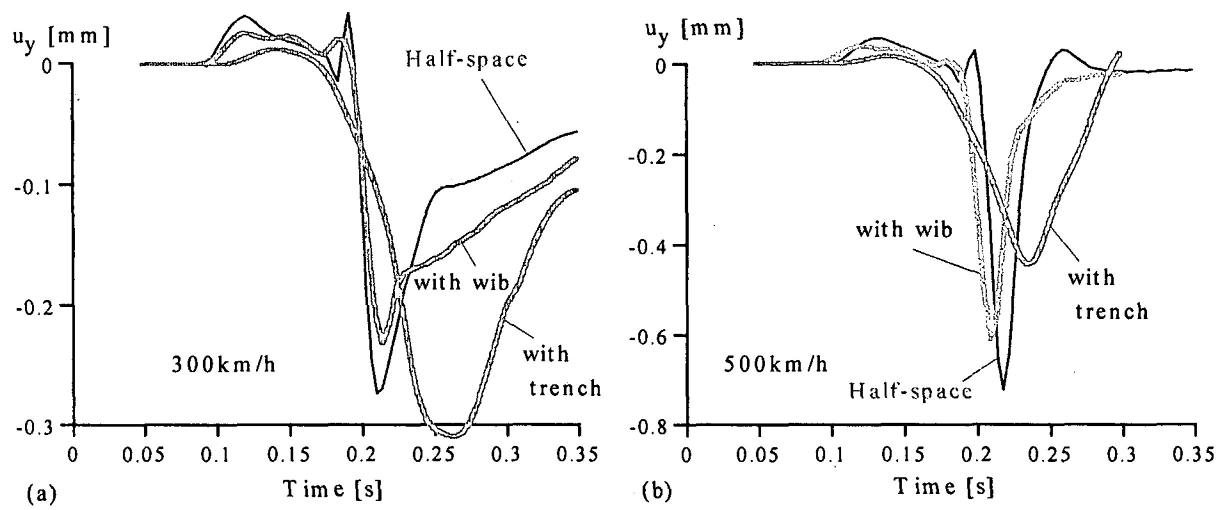

Fig. 5 Influence of the reduction measure on the ground response at x=18.75m and y=14m

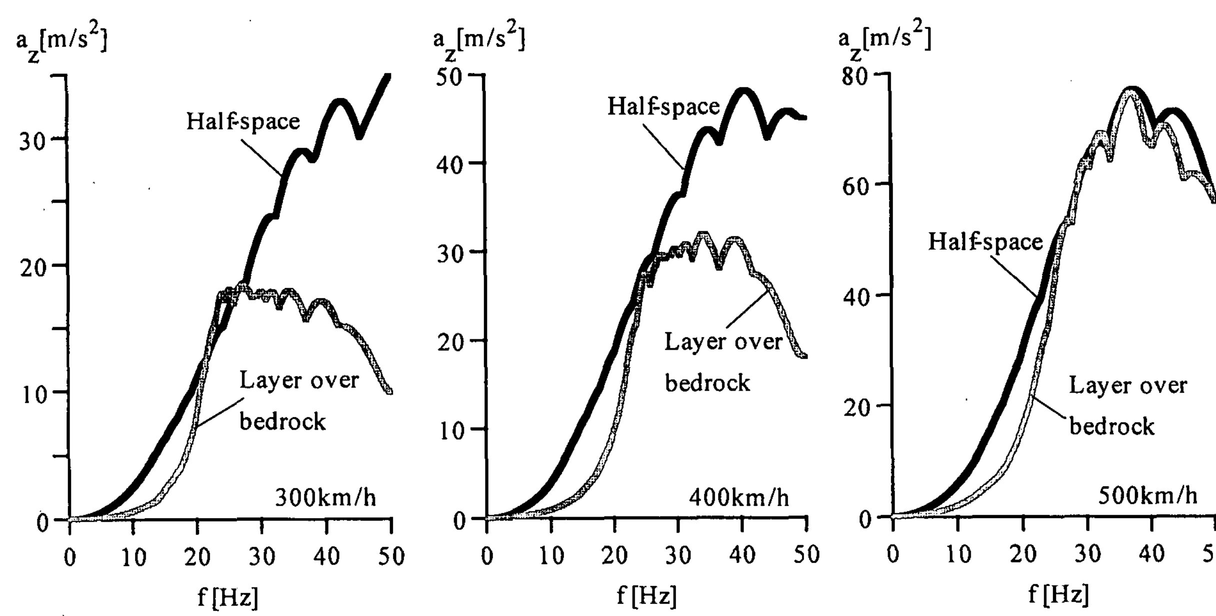

Fig. 6 Influence of the moving speed and bedrock on the vertical ground response accelerations a_z at x=18.75m and y=14m

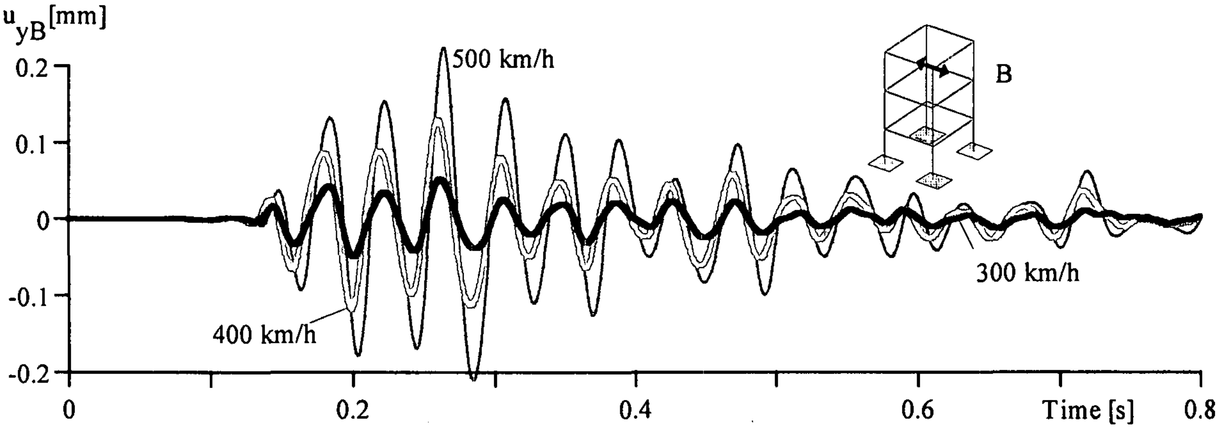

Fig. 7 Influence of the moving speed on the structural response

The reduction effect of the trench and the wave barrier on the ground responses is presented in Fig. 5. While the trench can clearly reduce the ground response in the x- and z-direction in all considered load speeds, in case of the load velocity of 300km/h the ground response in the y-direction is amplified. The wave barrier can reduce the ground response, but not as strong as the trench.

In Fig. 6 the effect of bedrock at a depth location of 3m on the ground response is presented. Since bedrock represents an ideal wave-impeding barrier, the effect of the bedrock can reflect the expected influence of a wave-impeding barrier. The results show that the effect of the bedrock depends on the load speed. While in case of 300km/h and 400km/h load speed the ground vibrations below and above the layer natural frequency of 20Hz are weaker than those without bedrock. However, in case of 500km/h weaker ground response can only be observed below the natural frequency of the layer. Above the layer frequency almost the same ground vibrations are observed.

3.3 Structural response

Fig. 7 shows the displacement response of the structure at the location B (see also Fig. 1(a)) in the y-direction. Ground excitations in all three directions are taken into account.
Corresponding to the ground response the largest response of the structure occurs at the load speed of 500km/h. The maximum response u_y of 0.224 mm occurs at 0.264 s. It is 1.29 times the maximum response caused by the moving load with the speed of 400 km/h., and 4.24 times the one due to the load with a speed of 300km/h.

In the earthquake engineering it is common to consider the horizontal ground motions as the main damaging forces of structures. The reason is that the horizontal ground motions of an earthquake have generally low dominant frequencies, and the fundamental structural frequency normally belongs to the horizontal global vibrations of the structure.

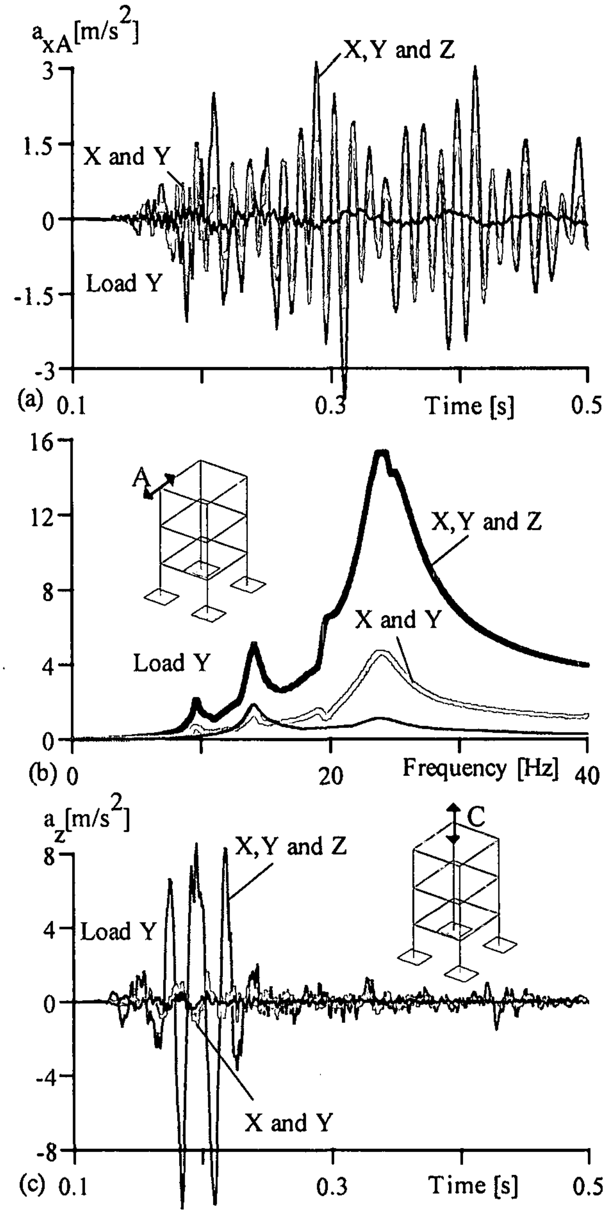

Fig. 8(a)-(c) Influence of the three-dimensional ground excitations on the induced vibrations

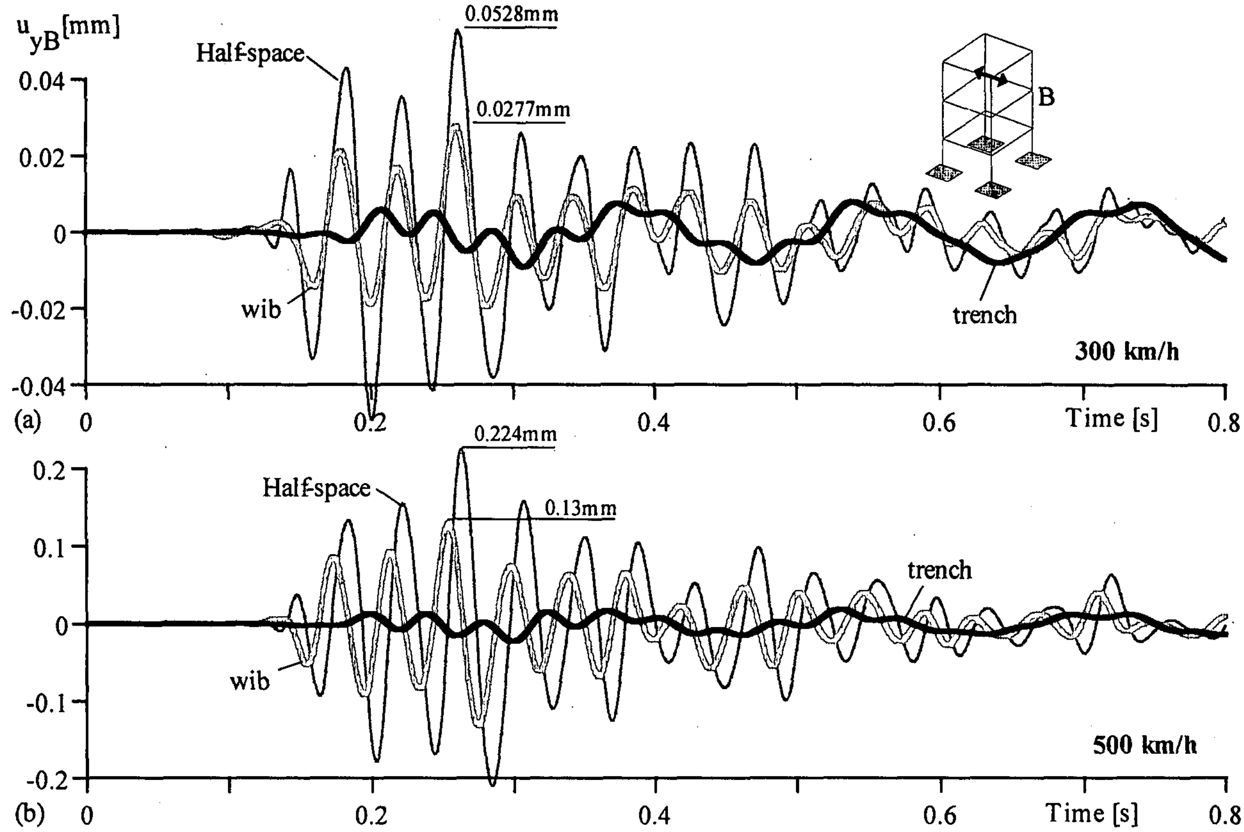

Fig. 9(a) and (b) Reduction of the structural vibrations due to the trench and the wave barrier

Therefore in the design of structures against earthquakes mainly horizontal ground motions are considered. In most of the design regulations the effect of vertical ground motions may be neglected. In contrast, it is commonly believed that in case of man-made induced vibrations the vertical ground motions are most significant for the structures, as we can see in Fig. 4.

In order to indicate the influence of a simultaneous ground excitation on the response of the structure, the effect of the horizontal ground motions in the y-direction, in the x- and y-direction as well as all together in x-, y- and z-direction is presented in Fig. 8. The fundamental frequency of the structure in the x- and y-direction is 3.2Hz and 6.3Hz, respectively. Because of the high frequency content of the ground excitations, as we can see in Fig. 4, we can expect that both fundamental modes in the x- and y-direction will not significantly excited by the ground motions. The response in Fig. 8(a) is therefore mainly determined by the higher vibration modes. Since the response in the x-direction is considered, the ground motions in the y-direction consequently have no significant influence, and also in case of a simultaneous x- and y-excitation. However, a simultaneous x-, y- and z-excitation causes a maximum response, which is 78% larger than the response due to the x- and y-excitation.

Fig. 8(b) shows the response spectrum of the horizontal induced vibrations, which are displayed in the Fig. 8(a). The response due to the y-excitation shows that the rotation mode with the frequency of 13.5Hz is strongest excited. The response due to the x- and y-excitation excites additionally the second mode in the x-direction with the frequency of 9.7Hz, the second mode in the y-direction with the frequency of 20.36Hz, the third mode in the x-direction with the frequency of 20.5Hz as well as the rotation mode with the frequency of 25.7Hz. Although the considered response is a horizontal response, the vertical ground motions cause a remarkable amplification. The possible reason is the difference in the arriving time and the time history of the strong vertical ground motions. Consequently, the structure experiences strong rocking vibrations.

Fig. 8(c) shows the influence of the simultaneous ground excitation on the vertical induced vibrations at the location C. Compared to the effect of the horizontal ground motions the influence of the vertical ground motions is in the considered case, as expected, significant.

Fig. 9 shows the reduction effect of the trench and the wave barrier on the horizontal displacement in the y-direction at the location B. A comparison of the results indicates that a trench is very effective in reduction the effect of a moving load. However, it is difficult in practice to keep a trench open. The results also show that a wave barrier can be used as a reduction measure as well. However, the effectiveness of a wave barrier decreases with a growing load speed. In the considered case of 300km/h the largest displacement u_y without any reduction measure occurs at 0.261s and has the value of 0.0528mm. With wib the largest response is 0.52 times the response without wib. With trench the largest response occurs at 0.374s and is 0.14 times the response without any reduction measure. In case of 500km/h, however, the largest response without wib occurs at 0.264s and has the value of 0.224mm. The response with wib as reduction measure occurs at 0.254s and has the value of 0.13mm. It is 0.58 times the response without reduction measure. The decrease of the effectiveness is expected, as we have seen in the influence of bedrock on the wave propagation in the surrounding in Fig. 6.

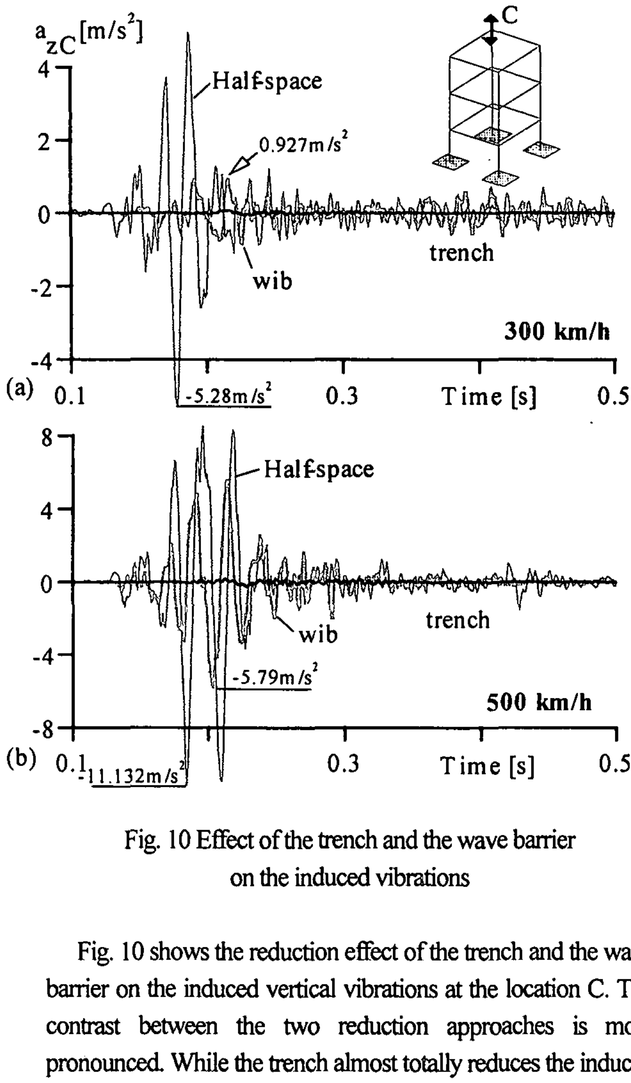

Fig. 10 Effect of the trench and the wave barrier on the induced vibrations

Fig. 10 shows the reduction effect of the trench and the wave barrier on the induced vertical vibrations at the location C. The contrast between the two reduction approaches is more pronounced. While the trench almost totally reduces the induced vibrations, the wave barrier reduces in case of 300km/h the largest induced vibration from 5.28m/s^2 at 0.178s to 0.927m/s^2 at 0.216s. In case of 500km/h wib reduces the induced vibration from –11.132m/s^2 at 0.184s to –5.79m/s^2 at 0.203s.

4. Conclusion

In the neighbourhood of the load path the ground responses increase with the load speed, if the load speed exceeds the velocity of the surface waves. The vibration in the vertical direction is the strongest. The vibration in the direction of the moving load is stronger than the one perpendicular to the load path. All ground responses have almost the same frequency content as the moving load. Trench is more effective than wib in reduction of the effect of the moving load. The effectiveness of wib decreases with the growing load speed.

In the considered case corresponding to the ground responses the reduction of the response of the structure by using wib is stronger in case of 300km/h load speed than in case of 500km/h.

Further investigations of the effect of the reduction measures in a more complicated soil profile are necessary.

Acknowledgments

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