

Numerical Model for Reinforced Concrete Beams with Shear Cracks

せん断ひび割れを有する RC はりの数値モデル

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A numerical simulation model is presented for the analysis of a reinforced concrete beam under monotonic loading based on the finite element method. The present method has an advantage of taking into account the influence of diagonal cracks in addition to flexural cracks in concrete. After having a diagonal crack, the interval corresponding to the diagonal crack with the shear deformation mode is modeled by shear type elements. The stress transfer to the shear reinforcement from the concrete is expressed by this method. The applicability of the present model to a beam with shear reinforcement is discussed. The analytical result with the present model is compared with the test data as well as the design code.

Key Words: finite element, shear deformation, diagonal crack, shear reinforcement

1. Introduction

It can be noticed that there have been several studies made for the shear failure mechanism^(1,2,3,4) of reinforced concrete (RC) beams considering the various modes of shear deformation. Extensive studies of the behavior of reinforced concrete flexural members have clarified the flexural failure mechanism and those well-examined results are now incorporated in the design codes such as American Concrete Institute (ACI) code (ACI 1999) and the Eurocode 2 (European Committee for Standardization (CEN) 1992) and so on.

A reinforced concrete member must resist shearing forces during its lifetime. This shear forces seldom act on their own but rather in combination with flexure, axial load, and torsion. Therefore, it is necessary to examine the possible interaction of shear with other structural mechanisms. In a reinforced concrete beam, it has been shown that the several crack patterns occur depending on the span to depth ratio⁽⁵⁾. Once the crack pattern is stabilized with the increasing load, it remains up to a load level close enough to collapse. The first flexural cracks are consistent with the initial, mostly linear elastic behavior of reinforced concrete in bending and in shear. Then, subsequently more cracks form on the beam. Therefore, flexural type beam elements are

used in the finite element method in the initial state of the loading on the reinforced concrete beam. Using these flexural type beam elements, the prediction of crack pattern is possible up to a certain level of loading. After the formation of diagonal cracks in the beam, flexural type elements are no longer able to give the correct deformation. Therefore, it is necessary to consider the region with the most severe shear cracks so as to deal with them properly. In the structural analysis of a reinforced concrete beam, this kind of prediction of crack pattern and the failure mode is important and useful in the design consideration which usually rely on semi-empirical judgement.

When actual structures fail at the ultimate state, the ductile failure mode is desirable because it gives a warning before the final catastrophic failure. From the present model, the crack pattern of the structure, the region of the predominant diagonal crack can be detected. Then, the region where the significant stress transfer occurs from concrete to stirrups can also be identified. When insufficient stirrups present in the region of the predominant diagonal crack, shear failure may occur without a warning.

The present model has an advantage of simulating the whole range of deformation up to the failure state over a model which is suitable for the ultimate limit state such

as the strut-tie model. Furthermore, it has merits such as simplicity in modeling and computational efficiency over FEM with plane elements in particular in the case of cyclic loading.

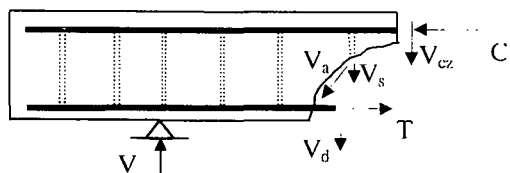


Fig.1 Shear resisting mechanism in RC beam

Fig.1. shows the shear resistance mechanism of a reinforced concrete beam with shear reinforcement after the formation of diagonal cracks in the beam.

In Fig.1, C is the compressive force of concrete, V_{cz} is the shear force of concrete by uncracked concrete, V_a is shear force by aggregate interlock, T and V_d are the forces due to the longitudinal reinforcement and V_s is the shear force carried by the shear reinforcement.

In a concrete beam reinforced with stirrups, the resistance to the total shear force V is divided into the shear force by concrete V_c and the shear force by stirrup V_s . Initially upon loading, the shear reinforcements carry only a small portion of the shear force. The concrete carries the rest of the shear stress. Upon the formation of the first predominant diagonal crack, redistribution of shear stress occurs with some part of the shear being carried by concrete, and the rest being carried by the stirrups. This kind of phenomena has been observed from experimental results¹⁾. The shear load that caused diagonal or inclined cracking V_{cr} is considered to be the ultimate capacity of concrete to resist shear⁶⁾. The total shear resisting capacity by the concrete and the stirrups of the beam remains constant once the stirrups have yielded.

The shear resistance by concrete is considered to be influenced by various shear mechanisms. The joint ACI-ASCE committee 426 lists three separate shear mechanisms for concrete contribution to shear resistance. They are summarized as follows.

- 1) Uncracked concrete: Uncracked members or parts of a cracked member resist as shown in Fig.1 (V_{cz}).
- 2) Aggregate interlock: The contribution from aggregate interlock between two slip surfaces in the cracked parts of the beam as shown in Fig.1 (V_a) depends on the crack width and the roughness of the slip surfaces.
- 3) Dowel action: The longitudinal reinforcement resists a part of the shear force by dowel action of the rebar. The dowel force (V_d) in the longitudinal reinforcement bar depends on the relative stiffness of the part of rebar crossing the crack¹⁾.

An experimental work⁵⁾ has been conducted for the crack pattern of RC beams under two-point loading with various shear span to effective depth ratios. Typical crack patterns are illustrated in Fig.2.

Flexural stresses and shear stresses are combined to create biaxial state of stresses. When the principal tensile stress exceeds the tensile strength of concrete, a crack forms perpendicularly to the direction of the principal tensile stress. In a region of large bending moment, these stresses are greatest at the extreme tensile fiber of the member and cause flexural cracks perpendicular to the axis of the member. These flexural cracks can be seen from the crack pattern at the bottom of the beam in Fig.2 (a, b). In the region of high shear stresses near the mid height of the rectangular beam, significant principal tensile stresses occur at an angle to the axis of the member. This may result in a diagonal tension crack.

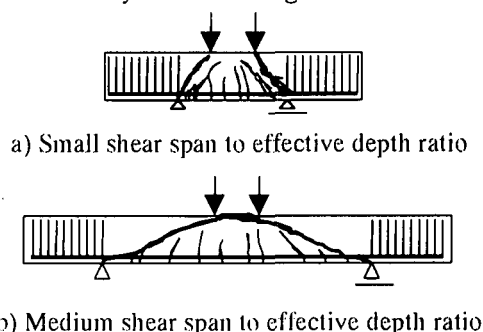


Fig.2 Typical crack pattern of simply supported beam under two-point loading

After formation of these flexural cracks and diagonal tension cracks in a concrete beam, the predominant diagonal crack can be seen as shown in Fig.2. This diagonal crack is responsible for the shear failure of the beam. At this state, the shear resistance by the concrete reaches nearly close to a constant value. Until the diagonal crack forms, the shear stress carried by the stirrup is assumed to be small¹⁾. After the formation of diagonal cracks, the stirrups are assumed to take the shear stress imposed by the external load on the reinforced concrete beam.

2. Numerical model

2.1 Stresses in beam element

The beam is divided into a number of elements for the finite element analysis along the beam having each node with the three degrees of freedom as shown in Fig.3 where u_1, u_2 are the axial nodal displacements along the x -axis, v_1, v_2 are the transverse displacements along the y -axis and θ_1, θ_2 are the rotation at the node 1 and 2 respectively.

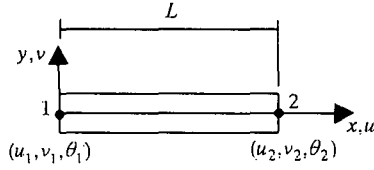


Fig.3 Beam element for finite element analysis

The displacements of the flexural type beam element are given as follows.

$$u(x) = (1 - \xi)u_1 + \xi u_2 \quad (1)$$

$$v(x) = (1 - 3\xi^2 + 2\xi^3)v_1 + L(\xi - 2\xi^2 + \xi^3)\theta_1 + (3\xi^2 - 2\xi^3)v_2 + L(-\xi^2 + \xi^3)\theta_2 \quad (2)$$

where $\xi = x/L$, and L is the length of element.

Following the conventional beam theory, the displacement $U(x, y)$ across the cross section at an arbitrary point is given in Eq. (3). Then, the stress-displacement relationship for the normal stress is given by Eq. (4).

$$U(x, y) = u(x) - y \frac{dv}{dx} \quad (3)$$

$$\sigma_x = E_c \left(\frac{du}{dx} - y \frac{d^2v}{dx^2} \right) \quad (4)$$

where σ_x is the normal stress and E_c is the tangent modulus of concrete for given strain.

From the assumed displacement field, the shear stress is not properly obtained. In order to evaluate the shear stress across the cross section, the multi-segment model⁽⁸⁾ is introduced. Using the equilibrium at a point of the cross section and the shear flow continuity requirement, the shear stress is calculated. This method can be applied to multi-cell box type beam structures in general. Applying the engineering theory of shear stress distribution for a thin segment, the shear stress across the cross section is calculated. Fig.4 shows a segment defined by cutting planes at x and $x+dx$. The normal stress is assumed to be constant through the thickness t . q is the shear flow along the curve s . q_A is the shear flow at point A.

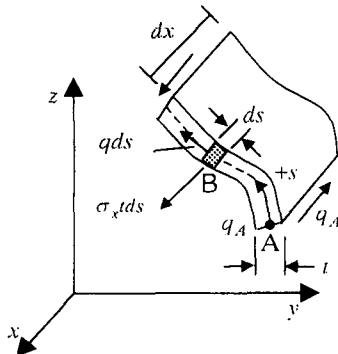


Fig.4 Shear in thin-walled cross section

Integrating the equilibrium equation in x direction, Eq.(5), with respect to s , the following expression for q is obtained as shown in Eq. (6)

$$\frac{\partial q}{\partial s} + \frac{\partial}{\partial x}(\sigma_x t) = 0 \quad (5)$$

$$q = q_A - \frac{\partial}{\partial x} \int_{s_A}^s \sigma_x t ds \quad (6)$$

For a rectangular cross section, the value of shear flow at the edge is zero. Using the normal stress from Eq. (4), Eq. (6) can be written for the shear stress after dividing the shear flow by the thickness of the beam cross section as follows.

$$\sigma_{xy} = q_A + \int_{-h/2}^y E_c y dy \left(\frac{d^3v}{dx^3} \right) \quad (7)$$

where h is the depth of beam and σ_{xy} is the shear stress over the cross section in terms of displacement $v(x)$.

For the constant tangent modulus E_c of concrete, the shear stress distribution across the cross section is of a symmetric parabolic distribution about the centroid of the beam. For variable tangent modulus over the cross section, the shear stress distribution can also be calculated numerically using Eq. (7).

The transverse displacement in the y direction is represented by a cubic interpolation function as in Eq. (2). Therefore, the shear stress calculated by Eq. (7) is constant along the x -axis while it varies over the cross section along the y -axis for the element in the finite element analysis.

After calculating the normal stresses and shear stresses over the cross section of a beam, the principal stresses and the corresponding principal stress directions are calculated. When the principal tensile stress exceeds the tensile strength of concrete, the concrete is assumed to be cracked in a direction perpendicular to the principal tensile stress. Then, the crack pattern is obtained over the cross section. These flexural cracks and diagonal cracks determine the region where the predominant diagonal crack forms in the beam.

2.2 Predominant diagonal crack

In the finite element method, we predict the flexural cracks prior to diagonal cracks. A diagonal crack is formed at an angle. As soon as a diagonal crack reaches the centroidal axis, it tends to become a predominant or continuous diagonal crack that causes shear failure. If the crack width is small at the centroid, the concrete can carry more shear load. The predominant diagonal cracks form connecting the cracks of the bottom of the beam close to the support with the inclined diagonal cracks near the loading point with significant crack opening.

When the diagonal crack crosses the centroid of the cross section, large crack opening and sudden failure occurs in a reinforced concrete beams unless there is a shear carrying mechanism to sustain further load in the cracked beam¹⁾.

To define predominant diagonal crack in the present model, the experimental observations, assumptions and rules are described as follows.

- 1) When predominant diagonal crack crosses the centroidal axis of RC beam, shear deformation is significantly increased¹⁾.
- 2) It is assumed that for monotonic loading, the direction of diagonal crack does not change significantly from experimental evidence⁵⁾.
- 3) It is assumed that the concrete cannot carry additional shear after predominant diagonal crack crosses the centroidal axes of RC beam without shear reinforcement. Then, the shear resistance by the concrete is of a constant value and it is assumed as the ultimate capacity of concrete to resist shear.
- 4) l_s , θ_s in Fig.5 defines the length and the angle of the predominant diagonal crack respectively obtained from analysis results.
- 5) b_s in Fig.5 is the band width or tolerance to define the predominant diagonal crack. This value is set small and constant. In this study, b_s is assumed as $d/10$ where d is the effective depth of reinforced concrete beam.

With the above experimental observation, assumptions and rules, the predominant diagonal crack in Fig.5 (dotted line AB) is determined in the present model as follows.

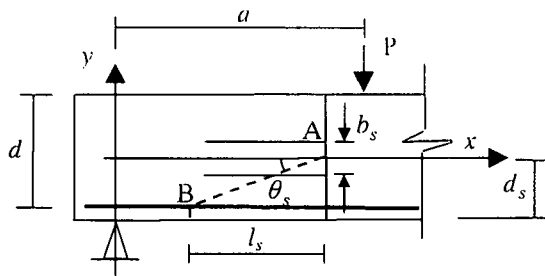


Fig.5 Formation of predominant diagonal crack

In Fig.5, d is the effective depth, a is the shear span, d_s is the height of the diagonal crack. When the first diagonal crack crosses the centroidal axis of the cross section for one element in the beam, the crack pattern is checked for other adjacent elements whether there is additional cracks in adjacent elements. Flexural crack of the first element near to the support and the first diagonal crack which crosses the centroidal axis of the RC beam is connected in which this dotted line AB is as

shown in Fig.5 with the length l_s . This region has a number of elements along the x -axis. Therefore, the angle θ_s is the slope of the dotted line. If all elements have a crack along the dotted line AB in Fig.5 with the tolerance b_s , a predominant diagonal crack is assumed with length l_s . Line AB contains several elements with cracks. When one of the elements does not have a crack on the line AB with the tolerance b_s , the analysis is carried out without assuming a predominant diagonal crack. A diagonal crack can form even after the first diagonal crack crosses the centroidal axis of RC beam within the tolerance b_s with the cracks predicted for each element in the updated region l_s for the next load step. Once a predominant diagonal crack is formed, shear type elements replace flexural type elements over this region. The analysis is continued with the predominant diagonal crack. If the crack exceeds the band width b_s at the centroid and reaches near to the loading point, then the diagonal crack is moved towards the loading point increasing l_s with additional shear type elements. The angle θ_s of a diagonal crack is obtained with the angle in between two values as follows.

$$\alpha < \theta_s < \beta \quad (8)$$

β is less than 90 degrees and α is greater than zero degree. It can be obtained depending on the condition of cracks defined by the region of predominant diagonal crack with the length l_s .

The present procedure is efficient once one can determine the region of the predominant diagonal crack. Changing the element in the region of l_s using shear type element, the deformation analysis of the beam is continued until failure.

2.3 Shear type beam element

In order to model shear deformation in the region explained in 2.2, the linear displacement field⁹⁾ is assumed as follows.

$$v(x) = (1 - \xi)v_1 + \xi v_2 \quad (9)$$

$$\theta(x) = (1 - \xi)\theta_1 + \xi\theta_2 \quad (10)$$

where $\xi = x/L$. Using the displacement field along the centroidal axes of the element given in Eq. (1), Eq. (9) and Eq. (10), the displacement at any point across the cross section is given by Eq. (11). Then, the strain-displacement relation obtained from Eq. (11) is given in Eq. (12). Calculation of stiffness matrix⁹⁾ is based on the principle of virtual work for the beam element.

$$U(x, y) = u(x) - y\theta(x); V(x) = v(x); \theta = \frac{dv}{dx} + \gamma(x) \quad (11)$$

$$\varepsilon = \frac{\partial U}{\partial x} = \frac{du(x)}{dx} - y \frac{d\theta(x)}{dx};$$

$$\gamma = \frac{\partial U}{\partial y} + \frac{\partial V}{\partial x} = -\theta + \frac{dv(x)}{dx} \quad (12)$$

where ε is the normal strain, γ is the shear strain, $U(x,y)$ is the displacement at any point on the beam across the cross section along the x -axis and θ is the angle of rotation at the centroid along the x -axis.

2.4 Stress on shear reinforcement and modeling

To model the region of a predominant diagonal crack, shear type elements are used. This region with either shear reinforcement or no shear reinforcement would be modeled using shear type elements as follows.

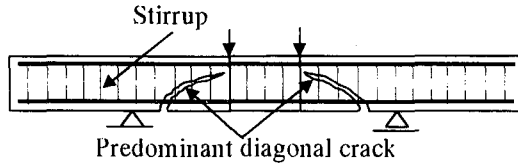


Fig.6 Beam after diagonal crack

Case-1 : Beam without shear reinforcement

If there is no shear reinforcement in the region of predominant diagonal crack, the region is replaced with shear type elements. If the shear stiffness of the elements in the region is very small, then the shear failure suddenly occurs. The shear failure of concrete is sudden and without warning, because the failure is caused by diagonal tension.

Case-2: Beam with shear reinforcement

Upon formation of a predominant diagonal crack, redistribution of shear stress to shear reinforcement occurs while some part of the shear force is carried by concrete. In order to incorporate the shear reinforcement in the present model, it is decided to include the shear stiffness of the reinforcement in addition to the concrete shear stiffness as follows.

$$E_{sr}^* = E_c m; m = E_{sr} / E_c \quad (13)$$

where E_{sr} is the Young's modulus of shear reinforcement, E_c is the tangent modulus of concrete and E_{sr}^* is the tangent modulus of shear reinforcement to be included in the shear stiffness of shear type elements. The contribution of shear stiffness by the shear reinforcement obtained from Eq. (13) is reflected in Eq. (14)

To calculate the stress on the shear reinforcement after the formation of a predominant diagonal crack, the following method is used. The shear force F_v carried by the concrete is calculated using the shear stiffness¹⁰⁾ as follows.

$$k_{sr} = 2A_s E_{sr}^* / S \quad (14)$$

$$F_v = (GA/L)(v_1 - v_2) + (GA/2)(\theta_1 + \theta_2);$$

$$GA = \int_A G dA \quad (15)$$

$$G = G_o (E_c / E_{co}) \quad (16)$$

where GA/L is the shear stiffness in the vertical direction, G is the tangent shear modulus of concrete calculated from Eq.(16), E_c is the tangent modulus of concrete, G_o , E_{co} are the initial values, S is the stirrup spacing, A_s is the area of single shear reinforcement, and k_{sr} is the vertical shear stiffness added in the stiffness matrix of the shear type beam element.

After calculating the shear resistance by Eq. (15) for the element, the shear resistance by the shear reinforcement is obtained as the difference between the shear resistance in Eq. (15) and the external applied shear force.

It is assumed that the shear resistance by the concrete remains constant after the diagonal crack crosses the centroid of the cross section unless there is a sudden failure in the concrete. Incremental external shear is assumed to be taken by the shear reinforcement.

2.5 Shear capacity by design code

The ACI 318¹⁰⁾ equation for the ultimate strength of concrete in shear V_{uc} is given as follows.

$$V_{uc} = \left(0.16 \sqrt{f_c} + 17.2 \rho_w \frac{V_u d}{M_u} \right) b_w d_o \leq 0.29 \sqrt{f_c} b_w d \quad (17)$$

where ρ_w is the longitudinal tensile steel ratio (A_{st}/bd), V_u , M_u are shear force and moment at the critical section and f_c is the compressive strength of concrete.

3. Analysis procedure

The present computational procedure is to be applied for simply supported reinforced concrete beams using the finite element method. The analysis procedure can be described as follows.

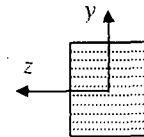


Fig.7 Discretization of beam cross section

- 1) The beam is divided into a number of flexural type elements. The cross section is also divided into a number of layers as shown in Fig.7 to take into

account the material nonlinearity and shear stress distribution over the cross section.

- 2) After calculating the normal strain of the section, the normal stress for concrete is obtained using the stress-strain curve of concrete¹¹⁾ with the modified Park model¹²⁾ (see Fig.9 (a)) which is applicable to high strength concrete as well up to 100 MPa and the normal stress for steel is obtained with the GMP model¹³⁾ (see Fig.9 (b)). Using the tangent stiffness calculated from the material law in Fig.9 (a), the concrete shear stress is obtained from Eq. (7).
- 3) The principal tensile stresses and the angle of the principal direction are calculated. When the principal tensile stress of concrete exceeds the tensile strength, the crack is assumed to occur.
- 4) When the first diagonal crack crosses the centroid, it is checked for the possibility of shear deformation. Once it is determined that there is a predominant diagonal crack, the region l_s shown in Fig.5 is changed to shear type elements.
- 5) If the cracks further develop beyond the band b_s shown in Fig.5, and if there are more cracks, the region is updated and shear type elements are added.
- 6) Until failure, the analysis is continued. The mode of failure can be identified from the analysis whether it is a flexural or a shear failure.

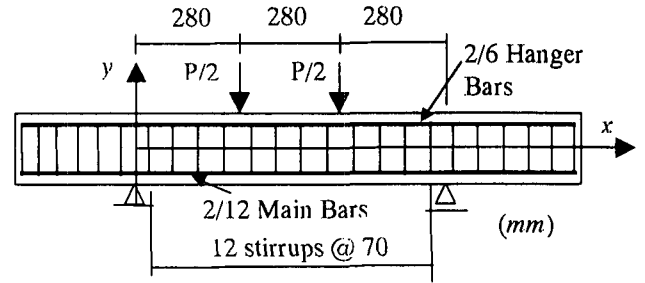
Using this analysis procedure, RC beam is analyzed for the deformation. A part of the flexural type elements is changed to shear type elements in the present model for RC beam.

4. Numerical results

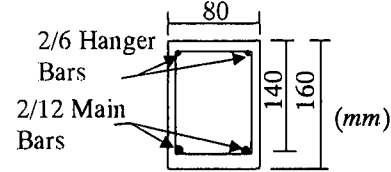
A simply supported reinforced concrete beam under two-point loading with the shear span to effective depth ratio of 2 is analyzed using the present model (see Fig.8). Table 1 shows the material properties used in the analysis. The tensile strength of concrete is assumed as $0.1f_c'$, and $E_w=24\text{GPa}$, $G_o=11\text{GPa}$.

Table 1 Material properties

Concrete	Young's modulus (MPa) - 24000
	Compressive strength(MPa) - 30
Steel	Young's modulus(MPa) - 228000
	Yield strength(MPa) - 410
Shear reinforcement	Young's modulus(MPa) - 202000
	Yield strength (MPa) - 370



a) Longitudinal section of beam



b) Cross section of beam

Fig.8 Geometry of simply supported beam

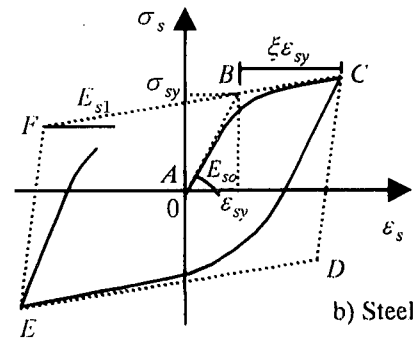
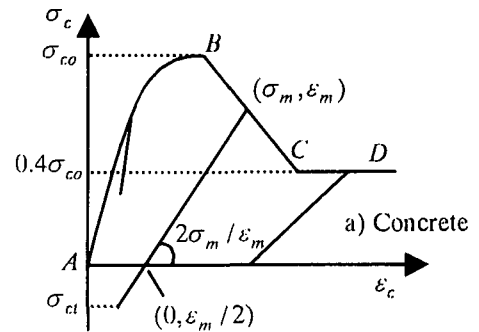


Fig.9 Material models used for the present analysis model

As seen in Fig.10 for the diagonal crack crossing the centroid of the cross section, l_s is the length of the region of the predominant diagonal crack. The angle θ_s of diagonal crack is obtained as 20 degrees and the b_s is set in the analysis as $d/10$ where d is the effective depth (140mm) of RC beam. When the load is increased beyond this level, the crack path moves toward the loading point. However, the tensile longitudinal reinforcement in the middle yields causing a flexural failure of the beam.

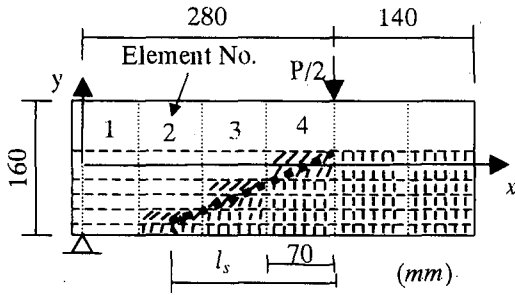


Fig.10 Crack pattern at the formation of diagonal crack crossing the centroid

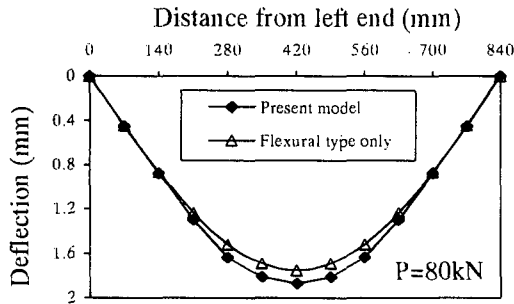


Fig.11 Load-deflection curve at yielding of longitudinal reinforcement for two-point loading

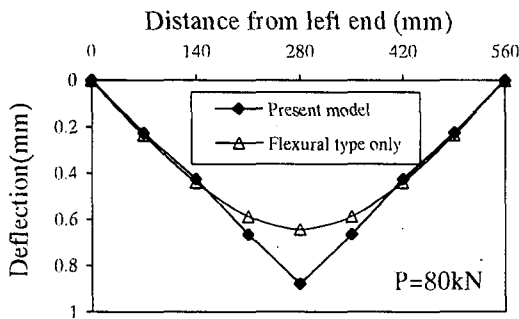


Fig.12 Load-deflection curve at yielding of longitudinal reinforcement for one-point loading

Fig. 11 shows the load-deflection profile at yielding of longitudinal reinforcement for two-point loading. After changing to the shear type elements in the region of the diagonal crack, the displacement increases in the middle region compared to the deflection profile with flexural type elements only. From this, it is confirmed that by introducing the shear type elements in the analysis model, the shear deformation is more properly taken into account.

Fig.12 shows the load-deflection curve for one-point loading by eliminating uniform flexure part of the test beam with the same shear span to effective depth ratio. To compare the value of shear capacity and deflection profiles for two-point loading and one-point loading, this analysis is carried out. Concrete shear capacity of 1.5MPa and the total shear load of about 40 kN are

obtained with the similar crack pattern in the shear span as shown in Fig.10. These values are about the same (within 2% error) as those of two-point loading case of Fig.11. Using the shear type elements the deflected shape of one-point loading is obtained as linear in the mid-span region.

In order to see the effect of the shear type elements used in the region of the predominant diagonal crack on the shear stiffness, the shear stiffness-load curve for the 4th element in Fig. 10 is plotted in Fig.13.

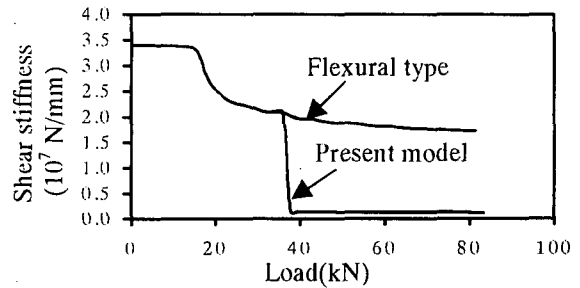


Fig.13 Load-stiffness curve of element

The difference in the reduction of the stiffness of the element is observed. If there is a severe reduction in shear stiffness due to the failure of the concrete, more shear stress would be taken by the shear reinforcement.

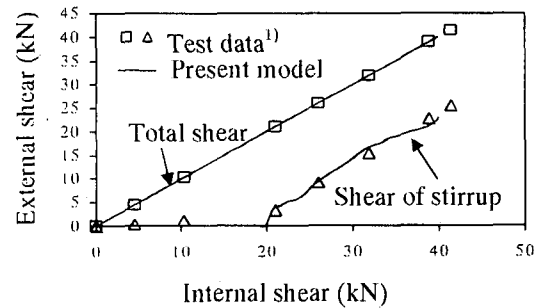


Fig.14 External shear and internal shear

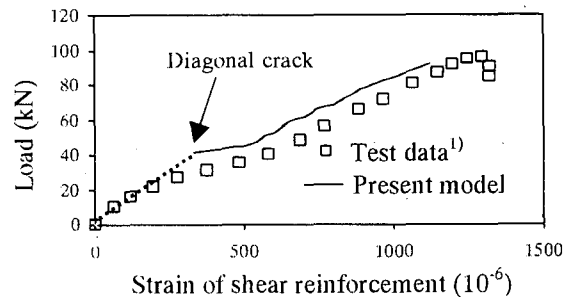


Fig.15 Load-strain curve of shear reinforcement

After predominant diagonal crack forms, the shear force carried by stirrup is shown in Fig.14 in which this shear force by stirrup is divided by the cross sectional area of shear reinforcement in the region of l_s in Fig.10. Until the diagonal crack crosses the centroid, the shear taken

by the reinforcement is neglected. External applied shear and internal shear resisted by the shear reinforcement are shown in Fig.14. It shows reasonably good agreement with the test results. About 17 kN in average is carried by the concrete from the analytical model and it is 17.75 kN in the test data.

Fig.15 shows the load-average strain of shear reinforcement, which is in the region of l_s defined in Fig.10.

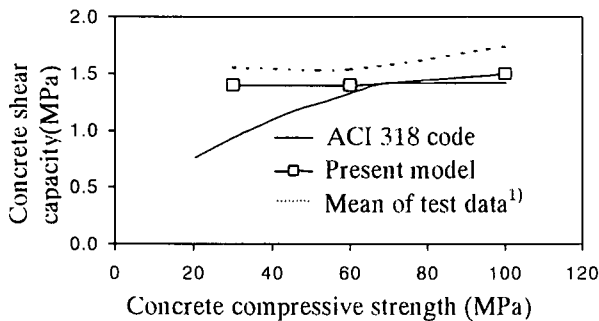


Fig.16 Concrete shear capacity and concrete compressive strength

The ultimate capacity of the shear strength of concrete is calculated with ACI code¹⁰⁾ as 0.93 MPa while it is from the analytical and test data is close to 1.5 MPa. The value obtained from ACI code in Eq. (17) is in the safe side.

Finally for different values of concrete strengths, the ultimate shear capacity is obtained. The comparison with the test data and the ACI code is shown in Fig.16. Numerical results obtained from the present model are in agreement with the test data and the ACI code values.

5. Conclusions

The numerical simulation model presented in this paper is capable of predicting the crack pattern, shear deformation, failure mode and stress on shear reinforcement.

From the numerical simulation, the following conclusions are obtained.

- 1) The present model is capable of predicting the predominant diagonal crack in a reinforced concrete beam.
- 2) The use of shear type elements with flexural type elements is effective to model the beam to take into account the shear deformation.
- 3) The ultimate shear capacity carried by the concrete and the stress carried by the stirrup can be simulated and compared with design code.

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