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A Viscoelastic-Viscoplastic Constitutive Model for Clay

粘土の粘弾ー粘塑性構成モデル

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In the present study, a cyclic viscoelastic-viscoplastic constitutive model for clay based on the kinematic hardening rule was proposed, which incorporates three-element viscoelastic component in the cyclic elastic-viscoplastic constitutive model. From cyclic undrained triaxial test results and numerical simulations, it was found that viscoplastic approach is significant in the large strain level as approaching a failure state while viscoelastic approach is notable in the small strain level. Moreover, it can be seen that the proposed model can well describe both viscoelastic and viscoplastic behaviors of clay in the wide range of strain level and is applicable for earthquake response analysis and/or liquefaction analysis.

Key Words : constitutive model, viscoelasticity, viscoplasticity, clay

1. Introduction

Up to now, many constitutive models for clay have been proposed and studied based on the elasto-plastic or elasto-viscoplastic theory and it has been recognized that the effect of time on the loading process is a salient feature. Since clay behavior is viscoelastic in the infinitesimal and/or small strain levels (see for examlpe: Kondner and Ho, 1965; Murayama and Shibata, 1966; Hori, 1974; di Benedetto and Tatsuoka, 1997), a viscoelastic-viscoplastic model for clay is necessary to explain the actual deformation characteristics of clay in the infinitesimal and small strain levels. After the 1995 Hyogoken-Nambu earthquake, liquefaction phenomena were observed in the wide area, in particular, in reclaimed land such as Port Island, Rokko Island, etc (Shibata, et al.1996). One of the issues of liquefaction analysis of sand-clay layered ground is the effect of clay layers on liquefaction of sandy layers. As for the dynamic analysis of ground during earthquake, it is necessary to account for the strain dependent characteristics of shear modulus and damping in the small strain level. Hence, in order to perform dynamic analysis accurately it is a better way to consider viscoelastic as well as viscoplastic characteristics of clay.

In the present study, we proposed a cyclic viscoelastic-viscoplastic constitutive model to describe the dynamic behavior of clay in the wide range of strain. Then, we simulated cyclic triaxial test results of natural clay using the proposed model. Although the simulations are concerned only with the low frequent dynamic behaviors of natural clay, we think that they are good example to confirm the performance of the proposed model, in particular to simulate the nonlinear dynamic deformation characteristics of clay in the wide range of strain.

2. A Viscoelastic-Viscoplastic Constitutive Model for Clay

It is assumed that the deviatric strain rate tensor $\dot{e}_{ij}$ is composed of elastic, viscoelastic and viscoplastic components.

$$\dot{e}_{ij} = \dot{e}^{e}_{ij} + \dot{e}^{vev}_{ij} + \dot{e}^{vp}_{ij} = \dot{e}^{ve}_{ij} + \dot{e}^{vp}_{ij} \tag{1}$$

where $\dot{e}^{e}_{ij}$: elastic component, $\dot{e}^{vev}_{ij}$: viscoelastic Voigt component, $\dot{e}^{vp}_{ij}$: viscoplastic component, $\dot{e}^{ve}_{ij}$: viscoelastic component. As for viscoelastic component, a three parameter model with Voigt element and elastic spring is adopted. The deviatoric elastic strain rate component tensor is given by

$$\dot{e}^{e}_{ij} = \frac{1}{2G_1}\dot{S}_{ij} \tag{2}$$

where G_1 is the first elastic shear modulus, $\dot{S}_{ij}$ is the deviatoric stress rate tensor. Deviatoric viscoelastic strain rate tensor is given by

$$\dot{e}^{vev}_{ij} = \frac{1}{\mu}(S_{ij} - 2G_2 \cdot e^{vev}_{ij}) \tag{3}$$

where μ is the viscosity coefficient, G_2 is the second elastic shear modulus of Voigt element, S_{ij} is the deviatoric stress tensor. Consequently, the viscoelastic deviatoric strain rate tensor $\dot{e}^{ve}_{ij}$ becomes

$$\dot{e}^{ve}_{ij} = \frac{1}{2G_1}\dot{S}_{ij} + \frac{1}{\mu}(S_{ij} - 2G_2 \cdot e^{vev}_{ij}) \tag{4}$$

For overconsolidated clay, Oka (1982) developed an elasto-viscoplastic constitutive model based on an overstress type viscoplasticity theory and the non-associated flow rule, and the viscoplastic model for overconsolidated clay was extended to the cyclic constitutive model for clay (Oka, 1992). The overconsolidation boundary surface is defined as the boundary in the effective stress space between the normally consolidated($f_b \geq 0$) and the overconsolidated($f_b < 0$) regions shown in Fig.1.

$$f_b = \bar{\eta}^*_{(0)} + M^*_m \ln\frac{\sigma'_m}{\sigma'_{mb}} = 0 \tag{5}$$

where $\bar{\eta}^*_{(0)}$ is the relative stress ratio defined by

$$\bar{\eta}^*_{(0)} = \left\{(\eta^*_{ij} - \eta^*_{ij(0)})(\eta^*_{ij} - \eta^*_{ij(0)})\right\}^{\frac{1}{2}} \tag{6}$$

$$\sigma'_{mb} = \sigma'_{mbi}\exp(\frac{1+e}{\lambda-\kappa}v^p) \tag{7}$$

The parameter,σ'_{mb} varies with changes in the viscoplastic volumetricstrain with the initial condition of $\sigma'_{mb} = \sigma'_{mbi}$. In Eqs.(5)-(7), v^{vp} is the viscoplastic volumetric strain, σ'_{mbi} is the initial value of consolidation yield stress σ'_{mb}, e is the void ratio, κ is the swelling index, λ is the compression index, σ'_m is the mean effective stress($\sigma'_m = \frac{1}{3}\sigma'_{kk}$), η^*_{ij} is the stress ratio tensor($\eta^*_{ij} = \frac{S_{ij}}{\sigma'_m}$), $\eta^*_{(0)}$ shows the value of η^*_{ij} at the end of consolidation and M^*_m is the value of $\sqrt{\eta^*_{ij}\eta^*_{ij}}$ at the maximum compression.

Considering the nonlinear kinematic hardening rule, static yield function is given as follows. For changes in the stress ratio, the following yield function is used as

$$f_y = \{(\eta^*_{ij} - \chi^*_{ij})(\eta^*_{ij} - \chi^*_{ij})\}^{1/2} - R_D = 0 \tag{8}$$

where, χ^*_{ij} is the kinematic hardening tensor and R_D is the scalar variable. The evolutional equation for kinematic hardening tensor χ^*_{ij} is given by

$$d\chi^*_{ij} = B^*(A^* de^{vp}_{ij} - \chi^*_{ij} d\gamma^{vp}) \tag{9}$$

where, de^{vp}_{ij} is the deviatoric viscoplastic strain increment tensor, A^* and B^* are material constants ($A^* = M^*_f$, $B^* = \frac{G^p}{M^*_f}$) and $d\gamma^{vp}$ is the second invariant of the viscoplastic deviatoric strain increment tensor. Namely,

$$d\gamma^{vp} = \sqrt{de^{vp}_{ij} de^{vp}_{ij}} \tag{10}$$

B^* follows Eq.(11) in order to express a non-linear characteristics of B^* with a strain level.

$$B^* = B_s + (B_0 - B_s)exp(-B_t\gamma^{vp*}) \tag{11}$$

The plastic potential function shown in Fig.1 is given by

$$f_p = \{(\eta^*_{ij} - \chi^*_{ij})(\eta^*_{ij} - \chi^*_{ij})\}^{\frac{1}{2}} + \tilde{M}^* \ln(\frac{\sigma'_m}{\sigma'_{ma}}) = 0 \tag{12}$$

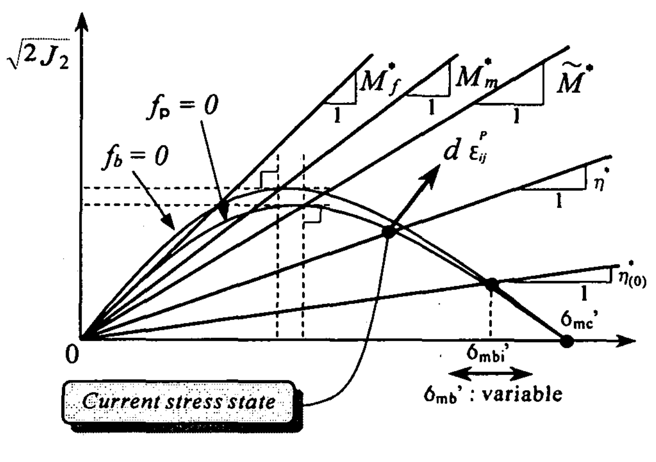


Fig.1 The concept of plastic potential function and overconsolidation boundary surface

where,

$$\tilde{M}^* = -\frac{\eta^*}{\ln(\sigma'_m/\sigma'_{mc})} \tag{13}$$

$$\sigma'_{mc} = \sigma'_{mb}\exp\left(\frac{\eta^*_{(0)}}{M^*_m}\right) \tag{14}$$

in which $\eta^* = \sqrt{\eta^*_{ij}\eta^*_{ij}}$, and $\eta^*_{(0)} = \sqrt{\eta^*_{ij(0)}\eta^*_{ij(0)}}$. Also, $\eta^*_{ij(0)}$ and σ'_{mb} are the values of η^*_{ij} and σ'_m at the end of anisotropic consolidation, respectively. $\tilde{M}^*$ can be determined by the current stress and σ'_{mc}. For the change of σ'_m, another type of yield and potential functions are used (Oka, 1992). In the present study, however, we neglect second yield function for simplicity.

The viscoplastic strain rate tensor $\dot{\varepsilon}^{vp}_{ij}$ corresponding to $f_y = 0$ is assumed to be given by

$$\dot{\varepsilon}^{vp}_{ij} = <\Phi_{1ijkl}(F)> \frac{\partial f_p}{\partial \sigma'_{kl}} \tag{15}$$

where, $<\Phi_{1ijkl}(F)> = \Phi_{1ijkl}(F) : F > 0$, and $<\Phi_{1ijkl}(F)> = 0 : F \leq 0$.

$$F = f_y = \left\{(\eta^*_{ij} - \chi^*_{ij})(\eta^*_{ij} - \chi^*_{ij})\right\}^{\frac{1}{2}} - R_D \tag{16}$$

in which $F = 0$ denotes the static yield function. In Eq.(15), $\Phi_{1ijkl}(F)$ is the function of F, $F = 0$ denotes the static yield function. In Perzyna's theory(1963), $\Phi_{1ijkl}(F)$ was dealt with as the scalar function. However, it is herein assumed that the first material function is the fourth order isotropic tensor function.

$$\Phi_{1ijkl}(F) = C_{ijkl}\Phi'_1(F) \tag{17}$$

The concrete shape of the material function is determined referring the previous work(Adachi and Oka,1982; Oka,1982)

$$\frac{\Phi'_1(F)}{\sigma'_m} = \exp\left\{m'_o\left\{(\eta^*_{ij} - \chi^*_{ij})(\eta^*_{ij} - \chi^*_{ij})\right\}^{1/2}\right\} \tag{18}$$

where, m'_o is the viscoplastic parameter and C_{ijkl} is the fourth order isotropic tensor.

$$C_{ijkl} = a\delta_{ij}\delta_{kl} + b(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}) \tag{19}$$

where, a and b are material constants ($C_{01} = 2b$, $C_{02} = 3a + 2b$).

Finally, total strain rate is obtained as:

$$\begin{aligned}\dot{\varepsilon}_{ij} &= \frac{1}{2G_1}\dot{S}_{ij} + \frac{1}{\mu}\left(S_{ij} - 2G_2 \cdot e^{vev}_{ij}\right) + \frac{\kappa}{3(1+e)}\frac{\dot{\sigma}'_m}{\sigma'_m}\delta_{ij} \\ &+ C_{01}\frac{<\Phi'_1(F)>}{\sigma'_m}\frac{(\eta^*_{ij} - \chi^*_{ij})}{\bar{\eta}^*_x} \\ &+ C_{02}\frac{<\Phi'_1(F)>}{\sigma'_m}\left\{\tilde{M}^* - \frac{\eta^*_{mn}(\eta^*_{mn} - \chi^*_{mn})}{\bar{\eta}^*_x}\right\}\frac{1}{3}\delta_{ij}\end{aligned} \tag{20}$$

3. Viscoelastic Characteristics of Clay in the Small Strain Range

The linear viscoelastic approach is valid for the behavior in the range of small strain, while viscoplastic modeling of soils is useful in the range of large strain including failure. In this study, a cyclic viscoelastic-viscoplastic model for clay based on the nonlinear kinematic hardening rule and three parameter viscoelastic theory is proposed and the structure of the model is shown in Fig.2.

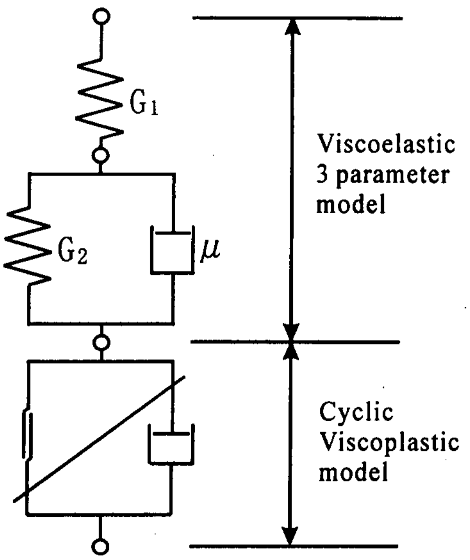


Fig.2 The structure of the proposed model

Until now, viscoelastic constitutive models have been used to modeling of engineering materials such as polymer, concrete, metal and soil. Mainly, linear viscoelastic model such as Maxwell model, Voigt model and three parameter model have been used for the analysis of viscoelastic behavior. Kondner and Ho(1965), Hori(1974), and Di Benedetto and Tatsuoka(1997) reported that three parameter model which consists of the free spring as the instantaneous elasticity and the Voigt element as the retardation elasticity in parallel can explain approximately the dynamic behavior of clay. Moreover, Murayama and Shibata(1964) proved about time dependent of clay in high frequency region considering the distribution of relaxation time. When strain level is low the time dependent behavior of clay can be expressed by viscoelastic model, but if strain level is high viscoplastic model will be needed for the effect of plasticity. Therefore, when one consider to construct a constitutive model for clay, in particular in the wide strain level, viscoelastic-viscoplastic approach is noticeable method from the above fact. In this study, therefore, a cyclic viscoelastic-viscoplastic model based on the nonlinear kinematic hardening rule and three parameter theory is proposed but the viscoelastic component is deemed just deviator component. Akai and Hori(1974) reported that the behavior of mixed material as elastic solid and elastic fluid with Darcy's law is resemblent the behavior of three parameter viscoelastic model.

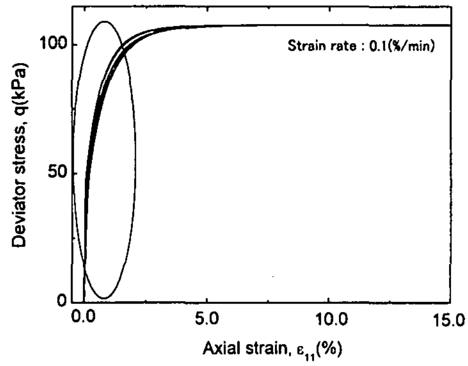


Fig.3 Stress-strain relations by the proposed model

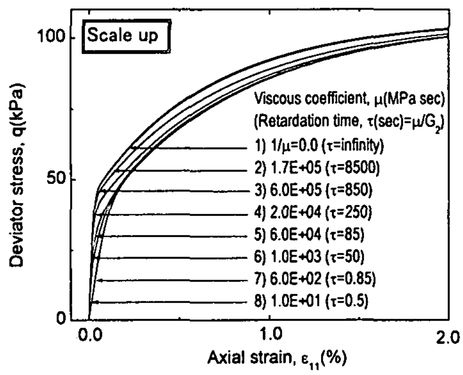


Fig.4 Stress-strain relations in small strain level

Consequently, from experimental works up to date, regarding viscoelastic behavior of real volumetric component, there is some difficulties to distinguish between the viscoelastic behavior of soil skeleton and resemblent viscoelastic behavior due to the interaction of water and soil skeleton. Therefore, the viscoelastic component is treated for, just, deviator component and the volumetric viscoelastic strain rate component should be studied later.

Fig.3 appears monotonic stress-strain relations by the viscoelastic-viscoplastic model and Fig.4 is a figure by scaling up Fig.3. If viscous coefficient is infinity, for example, in the case of 1) in Fig.4, the model become an elasto-viscoplastic model since there is only first shear modulus as shown in Fig.2. Table 1 shows parameters used in the simulation.

Table 1. Parameters used in the simulation

Elastic modulus, $E(MPa)$	180
First shear elastic modulus, $G_1(MPa)$	60
Second shear elastic modulus, $G_2(MPa)$	20
Stress ratio at failure state, M_f^*	1.3
Stress ratio at critical state, M_m^*	1.2
Viscoplastic parameter, $C_{01}(1/sec)$	9.0E-08
Viscoplastic parameter, $C_{02}(1/sec)$	1.0E-08
Viscoplastic parameter, m_0'	15.0
Viscoplastic modulus parameter, B_0	70
Viscoplastic modulus parameter, B_s	69
Viscoplastic modulus parameter, B_t	0.001
Compression index, λ	0.4
Swelling index, κ	0.04
Initial mean effective stress, $\sigma_m'(kPa)$	122.2
Consolidation pressure, $\sigma_{mb}'(kPa)$	183.3
Initial void ratio, e_0	1.2
Strain rate, $\dot{\varepsilon}(\%/min)$	0.1

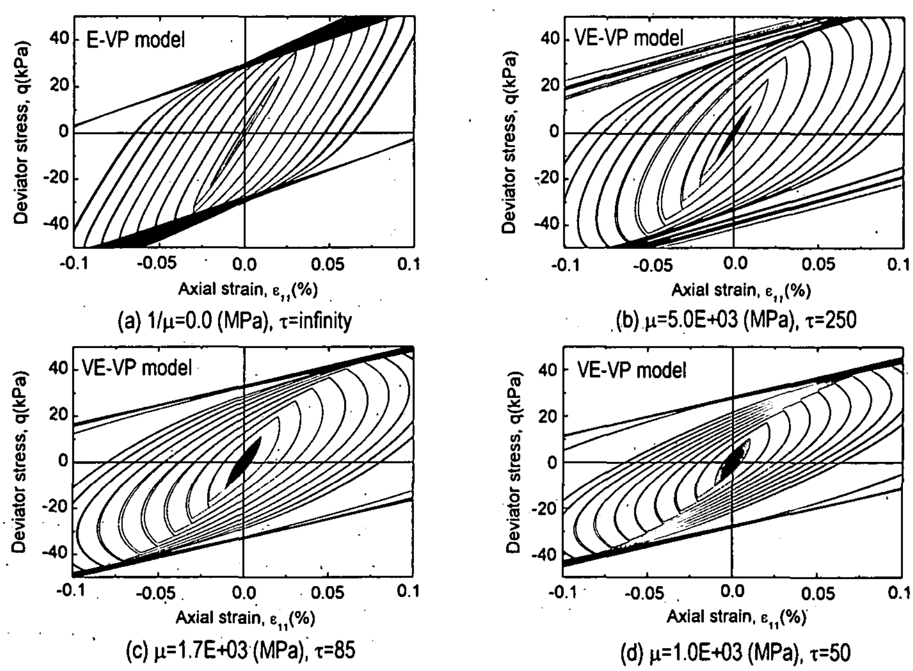


Fig.5 Cyclic stress-strain relations in small strain level with different viscous coefficient

If viscous coefficient reach to almost zero, for example the similar case of 7) and 8) in Fig.4, the model indicates elasto-viscoplastic behavior since the behavior of viscous compont indicate elastic behavior because of extremely strong viscous effect so that all component in three parameter model appear elastic behavior. Fig.5 appears cyclic stress strain relations in small strain level with different viscous coefficient by an elasto-viscoplastic and a viscoelastic-viscoplastic model. Fig.5(a) is given by elasto-viscoplastic model approach and Fig.5(b),(c) and (d) are given by viscoelastic-viscoplastic model approach. It should be noted that the clear difference can be seen in the stress-strain relations in small strain level obtained by these two models. For example, in the case of elasto-viscoplastic approach shown in Fig.5(a) elastic behavior becomes predominent within 0.02 % strain level and then viscoplastic behavior comes out with increasing of strain level. Moreover, hysteresis curve is flatter in small strain level and there is no strain level dependencies of shear modulus within 0.03 % strain level as we can see in Fig.5(a). One the other hand, the case of viscoelastic-viscoplastic approach, as is seen in the case of Fig.5(d), is well discribed viscoelastic behavior and the skeleton curve is similar to "S" shape with circular hysteresis curve and, namely, the model can explain strain level dependencies of shear modulus.

4. Cyclic Loading Tests and Simulation Using a Viscoelastic-Viscoplastic Model

The phenomenon which clay layer get to the rupture due to dynamic load is, generally, not clearly studied but some cases such as the 1985 Mexico earthquake and 1989 Loma Prieta earthquake show us dynamic deformation characteristics of soft clay ground is one of the important reason to reach failure. Therefore, it is needed to consider ground behavior during earthquake that not only sand layer but also clay layer is affected to dynamic motion. In this study, in order to determine the strength and deformation characteristics of natural clay with various cyclic loading conditions, two kinds of cyclic triaxial tests were conducted. One is the conventional cyclic undrained triaxial test to determine the relationships between the single amplitude of cyclic stress ratio and the numbers of cycles required to reach failure state. The other is the cyclic triaxial test to determine the deformation properties, i.e. equivalent Young's modulus and hysteretic damping. The specimen tested in this study was sampled at Komatsujima Port, Tokushima Pref., in Japan.

In cyclic undrained triaxial tests, a symmetrical cyclic loading was applied to isotropical consolidated clay specimen by a sinusoidal load with constant frequency of 0.1Hz. The definition of failure is a point when a double axial strain amplitude reaches 10% during cyclic loadings. Table 2 shows physical properties of the tested clay.

Table 2. Physical properties of the tested specimen

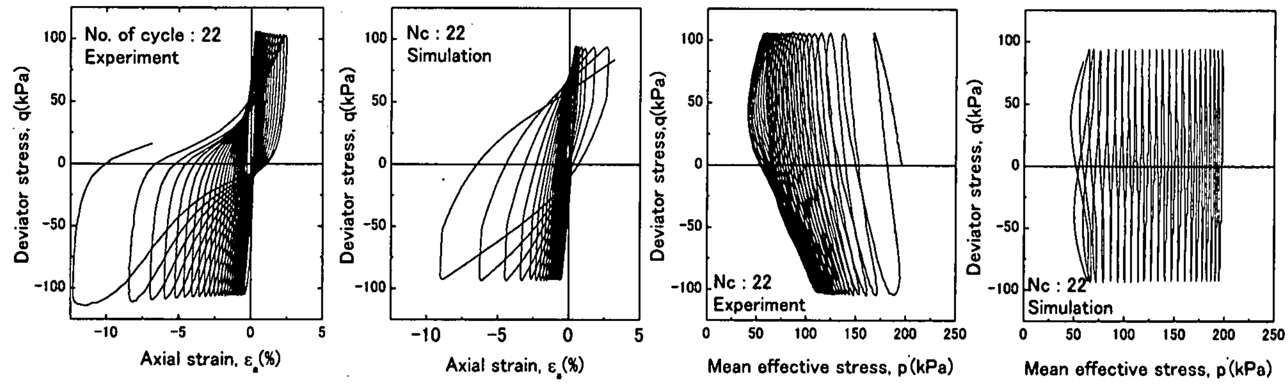
Sample number / Parameter	T-1	T-2	T-3	T-4
Depth (m)	22.0 -22.8	32.0 -32.8	35.0 -35.8	32.0 -32.8
Soil type	Clayey silt	Clayey silt	Silty clay	Clayey silt
Density, $G_s(g/cm^3)$	2.774	2.713	2.725	2.713
Water content, $w(\%)$	42.6	46.9	40.2	46.9
Liquid limit, $W_L(\%)$	44.9	48.5	39.4	48.5
Plastic index, I_p	18.7	22.0	16.7	22.0
Compression index, C_c	0.422	0.520	0.350	0.520
Consoildation yield stress, $P_c(kPa)$	510	314	392	314
Effective confining pressure, (kPa)	196	196	196	196
B value	0.934	0.993	0.962	0.919
Frequency, (Hz)	0.01	0.01	0.01	0.05
Single ampl. cyclic deviator stress, σ_d	105.1	126	127	-
$\sigma_d/2\sigma_c'$	0.268	0.332	0.324	-
Number of cyclic	22	22	4	-
Water content, $w(\%)(initial)$	46.1	47.0	43.5	48.11
Water content, $w(\%)(after)$	45.4	47.7	43.3	46.76
Specific gravity	2.774	2.713	2.725	2.713
OCR	2.6	1.6	2.0	1.6

In a cyclic triaxial test to determine deformation properties, 0.05Hz sinusoidal load is used in each strain level stages. Thirty stages cyclic loading was performed covering in the strain range of less than 0.0005% to more than 0.3%. A total of eleven cyclic loading by axial load controlled manner was applied in each stages. The deformation properties are determined using 10th hysteretic loop of stress-strain relationship. At the beginning of each stage, a drainage valve in test apparatus was opened to dissipate the excess pore water pressure in specimen.

Fig.6 - Fig.8 show stress-strain relationships and effective stress paths obtained by the cyclic undrained triaxial tests using samples T-1, T-2 and T-3. In these figures, the results of element simulations using the proposed viscoelastic-visocoplastic model are also shown for comparison. A lot of conventional cyclic undrained triaxial tests were conducted and most of

results were similar to the result of T-1 in Fig.6, which shows the remarkable increase of extensive axial strain during cyclic loading.

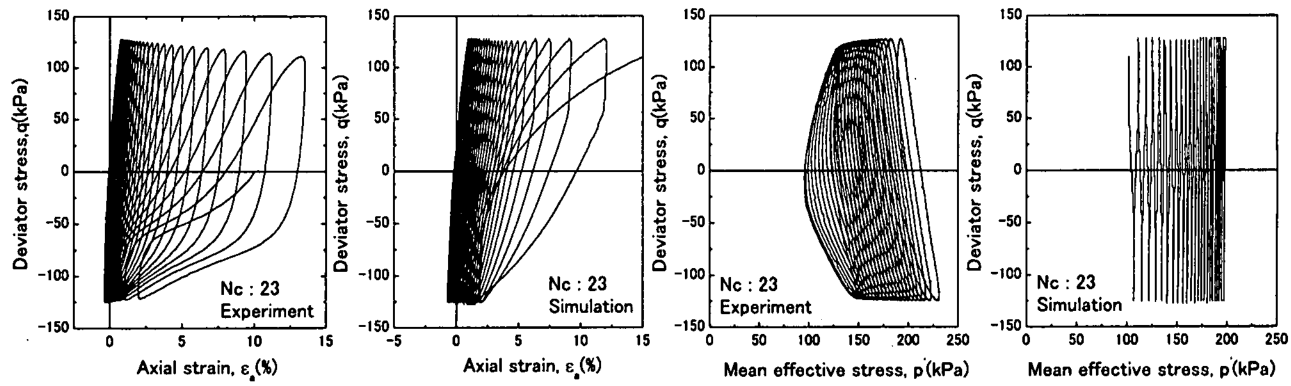
In the case of T-2, the compressive axial strain is larger than extensive axial strain as shown in Fig.7. Some cases were reached failure state very quickly with small number of cyclic loading as T-3 in Fig.8.



(a) Stress-strain relation

(b) Stess path

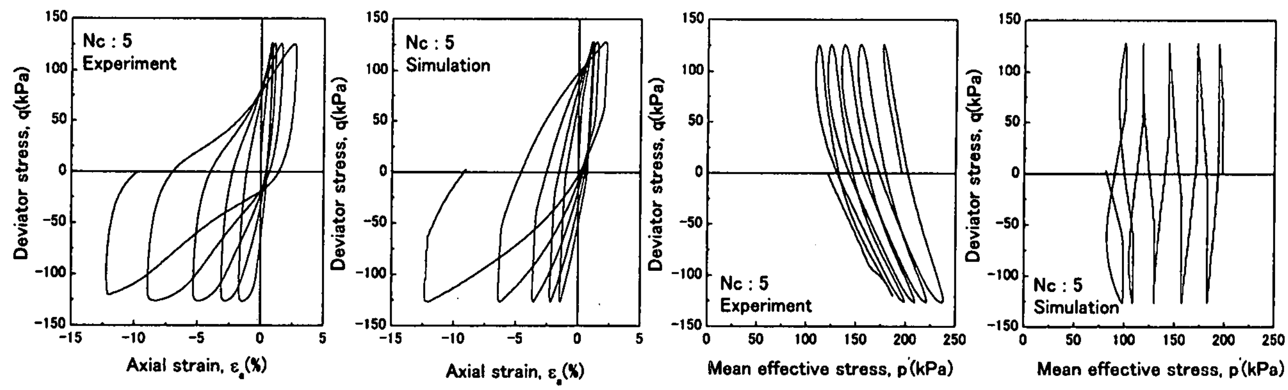
Fig.6 Experiment and element simulation results of case T-1 by the proposed model



(a) Stress-strain relation

(b) Stess path

Fig.7 Experiment and element simulation results of case T-2 by the proposed model



(a) Stress-strain relation

(b) Stess path

Fig.8 Experiment and element simulation results of case T-3 by the proposed model

From Figs.6-8, it can be seen that stress-strain relationships during cyclic loading are well described by the proposed model. In particular, strain level dependent characteristics in both compression and extension conditions are well reproduced with same cyclic number. The parameters used in the simulation are listed in Table 3.

Table 3. Parameters used in the element simulation

Sample number / Parameter	T-1	T-2	T-3	T-4
Elastic modulus, E(MPa)	157	157	157	157
First shear modulus, G_1(MPa)	54.1	54.1	54.1	52.7
Second shear modulus, G_2(MPa)	10.8	10.8	10.8	17.6
Viscous coefficient, μ(kPa sec)	6.7E+04	6.7E+04	6.7E+04	2.9E+04
Viscoplastic parameter, C_{01}(1/sec)	1.0E-07	4.0E-07	1.0E-07	4.0E-07
Viscoplastic parameter, C_{02}(1/sec)	1.0E-10	1.0E-10	1.2E-10	2.5E-10
Viscoplastic parameter, m'_0	45	45	45	45
Stress ratio at failure state, M^*_f	1.3	1.11	1.3	1.3
Stress ratio at critical state, M^*_m	1.2	1.05	1.2	1.2
Compression index, λ	0.1815	0.1948	0.2236	0.196
Swelling index, κ	0.0215	0.029	0.0301	0.029
Poisson's ratio, ν	0.45	0.45	0.45	0.49
Initial void ratio, e_0	1.369	1.331	1.225	1.299
Frequency(Hz)	0.01	0.01	0.01	0.05
OCR	2.6	1.6	2.0	1.6
Viscoplastic modulus parameter, B_0	60	52	27	40
Viscoplastic modulus parameter, B_s	0.5	0	3	0
Viscoplastic modulus parameter, B_t	1.5	1	2.2	1

The above behavior observed from cyclic undrained triaxial tests is mainly related to the non-linearity of soil properties in large strain level. Therefore, these simulations can be carried out with changing only the viscoplastic parameters, since the viscoelastic parameters only affect the clay behavior in small strain level.

Fig.9 shows the relationship between axial strain single amplitude and equivalent Young's modulus and hysteretic damping ratio, which was obtained by the cyclic triaxial test to determine a deformation properties and thier simulations. The simulated results using the proposed model are also shown in this Figure. As it is well-known, relationships shown in Fig. 9 are widely used as input information for the various earthquake response analyses. From Fig.9, it is seen the gradually decrease of equivalent Young's modulus with increase of strain in small strain range well matches the simulated results using the viscoelastic-viscoplastic constitutive model with proper viscoelastic parameter. The proposed model seems to be suitable for reproducing non-linear deformation characteristics of clay in the wide range of strain.

In view of the results so far achieved, it is seen that viscoplastic behavior is significant in the large strain level as approaching a failure state while viscoelastic behavior is notable in the small strain level. From the present study, it is revealed that the proposed viscoelastic-viscoplastic constitutive model is well applicable for earthquake response analysis and/or liquefaction analysis.

5. Conclusions

1) A viscoelastic-viscoplastic constitutive model for clay based on the kinematic hardening rule was proposed, which incorporates three-element viscoelastic parts in the cyclic elastic-viscoplastic constitutive model.

2) By comparison of cyclic loading tests and simulation results, viscoplastic behavior of clay is significant in the large strain level as approaching a failure state and the proposed model can discride it very well whenever.

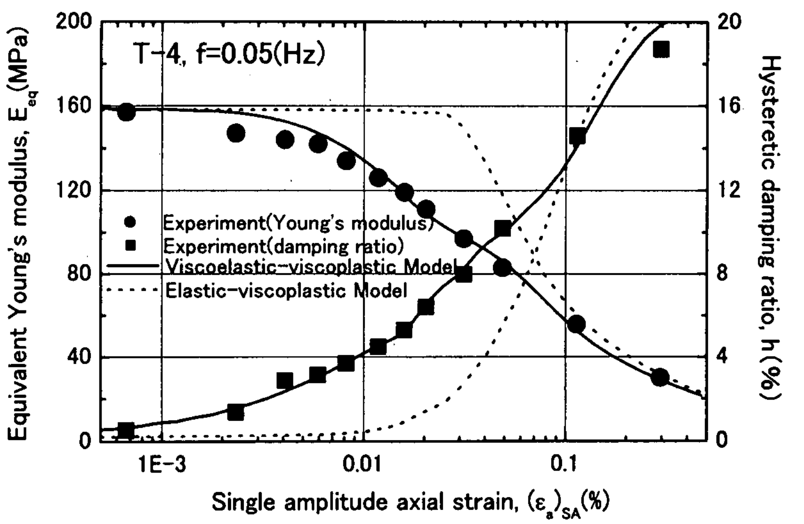


Fig.9 Strain level dependencies of equivalent Young's modulus and hysteretic damping ratio in case T-4

3) In the simulation of the cyclic deformation test in infinitesimal strain level the viscoelastic approach is required to accurately simulate dynamic response. Strain level dependencies of shear modulus and hysteretic damping ratio for natural clay can well be expained by a viscoelastic-viscoplastic constitutive model.

The application of the proposed model to high frequent dynamic problem of clay, i.e. earthquake response, will be shown in the future paper.

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