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Virtual Work Error Estimator for Statistical Evaluation of Existing Structure from Static Response

仮想仕事型誤差エステメーターによる既存構造物の確率的な損傷同定法の開発

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Abstract: In this paper, a virtual work error estimator is defined to express the discrepancy between a real structure and the analytical model, with which a system identification scheme is developed. Moreover, an adaptive parameter grouping method is applied to deal with the sparsity of data. In order to obtain the relationship between the input error and the output error, Monte Carlo method is used to simulate the measured data with error. Based on the identified results, the normal distribution of estimated parameters can be assumed. As a statistical approach, Hypothesis test is introduced for damage assessment. Using one solution in the sample, the status of an existing structure is statistically evaluated by locating and assessing the damage of elements. The proposed scheme is proved effective through numerical example.

Key Words: virtual work, constitutive parameter, parameter estimation, Hypothesis test

1. Introduction

The damage in all load-carrying structures, such as building, bridges, air-crafts, spacecraft and offshore platforms, may be continuously accumulated during their service. Structural damage often occurs in one or several individual locations of a structure with the degradation of stiffness. Some damage assessment methods based on the system identification (SI) techniques have been developed to detect the damage in structural systems during the last decade (e.g., Sanayei and Onipede (1991), Hajela and Soeiro (1990), Hjelmstad and Shin (1997), Yeo. and Shin (2000)). A SI-based damage assessment algorithm consists of system identification and damage assessment. First, the stiffness properties of a given structure are estimated by a SI algorithm and then the damage status of the structure is identified by comparing the changes in stiffness of the structure. Therefore, a stable SI algorithm is essential for a reliable damage assessment. It will be reasonable to say that SI plays an important role for the establishment of maintenance theory.

Either static or dynamic response can be used in SI-based damage assessment algorithms, which are divided into two major categories: dynamic and static. The intent of parameter estimation is to adjust the parameters of the analytical finite element model (FEM) to match the real structure with measured data.

Although there have been many successful examples by applying dynamic parameter identification methods in civil engineering, they also has some disadvantages to this kind of methods. Firstly, a large amount of dynamic data is needed to derive an accurate response of the structure. Generally, an estimated damping matrix must be used, which induces error in the system identification. Moreover, the identification process usually is not carried out usually at the element level, so that the damage locations can not be exactly known.

For static parameter estimation, on the other hand, to express the discrepancy between the real structure and the analytical model, both force error estimator and displacement error estimator are defined respectively, with which several models of structural identification have been proposed. Sheena et al. (1982) developed an identification method on the assumption that the displacements of all degree of freedom must be measured completely. But due to its complexity, they choose limited number of measured displacements to calculate the remaining displacement measurements based on spline theories. This introduces a major source of error for the stiffness matrix of structures. The drawbacks of Sanayer and Scampoli's method are that the displacements should be measured at the same locations where the external loads were applied. In the paper of Banan and Hjelmstad (1993), the unknowns comprise both constitutive parameters and unmeasured displacements. Therefore the number of unknown variables increases and the stability of calculating process decreases. Sanayei and Onipede (1990) proposed an algorithm in which the unmeasured displacements were condensed, but its limitation is that the degrees of freedom of measured displacements are fixed still in all load cases. The main difference among those methods is how to deal with incomplete measurements or measurement sparsity problem and to chose what schemes are used to solve the minimization problems. Although those methods are capable of identifying the structural parameters of structures, they could not yet deal with noisy and sparse measured data successfully.

This paper focuses on detecting and assessing the damages in structures from measured static response. The use of static response has practical value, since the static displacements can be measured with sufficient accuracy, due to the development of recent measurement technology. The static identification has fewer theoretical complications, and provides clear view of damage detection. Due to those considerations, we propose a virtual work error estimator and adopt an adaptive parameter grouping scheme to develop the system identification scheme. Monte Carlo method is used to simulate the measured data with error. We investigate the relationship between the input error and output error in detail and adopt Hypothsis test to statistically evaluate the status of the existing structures.

2. Structural Modeling and System Identification

In this section, we define a virtual work error estimator to express the discrepancy between the real structure and its analytical model, and then develop the solving algorithm. In order to deal with the sparsity problem of measured data, an adaptive parameter group subdivision method at the same time is adopted also.

2.1 The Virtual Work of Error Estimators

Consider a linear structure subjected to static load case $\{Af\}$. The dimension of measured displacement vector $\{\Delta\}$ is *nmd*, standing for the number of measured displacements. If we assume that there is virtual force vector $\{f\}$ applied along the directions of measured displacements, whose dimension is also *nmd*. Then the corresponding virtual work of the structure could be expressed as:

$$VW1 = \{f\}\cdot\{\Delta\}^T \tag{1}$$

Through a finite element method, the real structure is parameterized to be an analytical model in which the relationship between the applied forces $\{Af\}$ and corresponding displacements under the static load case can be described as follows:

$$\{Aff\} = [K(p)]\cdot\{u\} \tag{2}$$

where $\{Aff\}$ $(N\times 1)$ is the nodal vector of the applied forces $\{Af\}$, $\{u\}$ $(N\times 1)$ is corresponding response vector in the finite element model of system, and $[K(p)]$ is the parameterized stiffness matrix which can be formulated by decomposing the stiffness matrix into constitutive parameters and constant matrices for each element,

$$[K(p)] = \sum_{m=1}^{N_m}\sum_{\mu=1}^{M_m} p_{\mu m}[\lambda_m][B_m]^T[D_{\mu m}][B_m] \tag{3}$$

in which N_m is the number of the elements, others are messages about element m: M_m the total number of parameters, $P_{\mu m}$ the constitutive parameter, $[D_{\mu m}]$ the parameter-independent constitutive kernel matrices, $[\lambda_m]$ the location matrix, and $[B_m]$ the translative matrix. For a given structure, $[B_m]^T\cdot[D_{\mu m}]\cdot[B_m]$ is a constant matrix depending on element geometry only.

Therefore, the virtual work of the analytical model is

$$VW2 = \{ff\}\cdot[K(p)]^{-1}\cdot\{Aff\} \tag{4}$$

where $\{ff\}$ is the nodal vector of virtual forces $\{f\}$.

For an the identification problem, there is a parameterized finite element model, which is called analytical model above and measured displacement data of a real structure from static tests. We need an index to express the discrepancy between the measured data of real structure and the calculated data from the analytical model. We define the discrepancy of virtual work between the real structure and the analytical model as an index to examine the fitness of estimated results.

$$E(p) = \{ff\}^T \cdot [K(p)]^{-1} \cdot \{Aff\} - \{f\} \cdot \{\Delta\}^T \quad (5)$$

The essence of parameter estimation is to find a set of parameters, which can minimize the absolute value of $E(p)$. If the structural stiffness matrix exactly captures the properties of the system and if the measured data were free from errors, then Equ.(5) would be zero. Although the structure is linear, because of the inversion of matrices, they change into a nonlinear problem.

2.2 Parameter Estimation Algorithm

We adopt the square of error value as a criterion of judgment.

$$J(p) = E(p)^2 \quad (6)$$

Now the smaller the $J(p)$, the better accuracy of fitting we get. The mathematical model of the structure identification is

to find $\{p_i,\ i=1,2,\ldots,nup\}$
so as to minimize

$$J(p) = (\{ff\}^T [K]^{-1} \{Aff\} - \{f\} \cdot \{\Delta\}^T)^2 \quad (7)$$

To solve this nonlinear optimal problem, one can use any of a number of available optimization methods. Here we use the improved Newton method[2)] to develop recursive quadratic programming algorithm, which requires the gradient (Jacobi vector) and the Hessian matrix of the error function with respect to unknown parameters. They are given respectively as follows:

$$\text{Jacobi vector } \{G\} = \frac{\partial J(p)}{\partial \{p\}} \quad (8)$$

In which the ith component can be expressed as:

$$\frac{\partial J(p)}{\partial p_i} = 2(\{ff\}^T \cdot [K]^{-1} \cdot \{Aff\} - \{f\} \cdot \{\Delta\}^T) \cdot \{ff\}^T \cdot [K]^{-1} \cdot \frac{\partial [K]}{\partial p_i} \cdot [K]^{-1} \cdot \{Aff\} \quad (9)$$
$$i = 1,2,\cdots\cdots nup$$

$$\text{Hessian matrix } [H] = \frac{\partial^2 J(p)}{\partial \{p\} \partial \{p\}} \quad (10)$$

where the jth component in ith line is:

$$\begin{aligned}\frac{\partial^2 J(p)}{\partial p_i \partial p_j} = {} & 2\{ff\}^T \cdot [K]^{-1} \cdot \frac{\partial [K]}{\partial p_j} \cdot [K]^{-1} \cdot \{Aff\} \cdot \\ & \{ff\}^T \cdot [K]^{-1} \cdot \frac{\partial [K]}{\partial p_i} \cdot [K]^{-1} \cdot \{Aff\} + \\ & 2(\{ff\}^T \cdot [K]^{-1} \cdot \{Aff\} - \{f\} \cdot \{\Delta\}^T) \cdot \\ & [\{ff\}^T \cdot [K]^{-1} \cdot \frac{\partial [K]}{\partial p_j} \cdot [K]^{-1} \cdot \frac{\partial [K]}{\partial p_i} \cdot [K]^{-1} \cdot \{Aff\} + \\ & \{ff\}^T \cdot [K]^{-1} \cdot \frac{\partial [K]}{\partial p_i} \cdot [K]^{-1} \cdot \frac{\partial [K]}{\partial p_j} \cdot [K]^{-1} \cdot \{Aff\}]\end{aligned}$$
$$i = 1,2,\cdots\cdots nup$$
$$j = 1,2,\cdots\cdots nup$$
$$(11)$$

in which nup is the number of unknown parameters. Now the recursive procedure can be set up for static identification.

$$\{\Delta p\} = -[H]^{-1}\{G\} \quad (12)$$

$$\{p\}^{i+1} = \{p\}^i + [\alpha_i]\{\Delta p\} \quad (13)$$

in which i idicates the iteration number and $[\alpha_i]$ is a damping coefficient matrix to assure that $J(p_{k+1})$ is smaller than $J(p_k)$. For the general Newton method, it is a unite matrix. Besides two criteria are chosen to check the algorithm for convergence. The first one is the change in the scalar error function, $J(p)$ and the second one is changes in the parameters, p_j^{i+1}/p_j^i, where i is the iteration number and j is the order number of parameters. As to measure the goodness of fit between the real structure and analytical model, the first one is more suitable. Tolerance limits are set for two criteria. When any of the limits are reached the algorithm is considered to have converged. These limits can be also used to control the desired accuracy in the identified parameters.

2.3 Adaptive Parameter Grouping Algorithm

To localize damage in a systematic manner, we introduce an adaptive parameter group subdivision algorithm[10] to the proposed parameter estimation model. The main idea of the scheme is to separate damaged parts in finite-element model by subdividing parameter groups sequentially starting from a known baseline grouping. (Note: when we refer to "baseline" values we mean values determined by a prior application of the algorithm, i.e., values obtained through parameter estimation with measured data). After each subdivision, a new set of parameter groups and their group parameter are established and estimated. By subdividing a suspicious parameter group, parameter become more sensitive and more representative of the real values. Because several damaged regions with different levels of severity may coexist in a structural system, the subdivision should be continuously carried out until all the damaged members are completely extracted. In this process, the parameterized stiffness matrix, shown in Eq. 3, is rewritten as follows:

$$[K(p)] = \sum_{n=1}^{N} \sum_{m \in \Omega_n} \sum_{\mu=1}^{M_m} p_{\mu m} [\lambda_m][B_m]^T [D_{\mu m}][B_m] \tag{14}$$

where N indicates the number of parameter groups; the index set Ω_n contains all of the element numbers associated with parameter group n. The subdivision of parameter groups implies that the number of groups, and hence the number of parameters, is a variable in the parameterization of the model.

In order to develope a parameter group updating scheme one must decide which parameter group should be subdivided. In selecting the candidate group, Natke and Cempel (1991) suggest to use the error between the estimated value and the baseline value of a parameter to measure the need for subdivision. With this measurement, the group whose estimated parameter is the most distant from its baseline value would be selected as the candidate subset. The schematic flow chart of this method is illustrated in Fig. 1.

1. a. Set up the initial grouping $\Omega^{(0)}$ and its parameters $\{p^{(0)}\}$.
   b. Set k=1.
2. Estimation parameters $\{p^{(1)}\}$ by minimizing $J(p^{(1)})$.
3. a. Set $J(p)$min=$J(p^{(k)})$
   b. If $|p_i^{(k)}-p_i^{(k-1)}| \le$ tolerance,
      remove the ith parameter from further estimation.
   c. If all the parameters are fixed, STOP.
   d. Otherwise, update $\Omega^{(k)}$ and its parameters $\{p^{(k)}\}$.
4. a. Set $J(p)$old = $J(p)$min.
   b. Determine the deepest level of grouping.
   c. If all the groups have been investigated, STOP.
   d. Otherwise, determine the candidate subset.
      do i=1, N
         . subdivide the ith possible candidate subset
         . estimate $\{p_j^{(k)}\}$ by minimizing $J(p^{(k)})$
         . if ($J(p_j^{(k)}) \le J(p)$min) then
            $J(p)$min = $J(p^{(k)})$
            $j$candidate = j
            endif
         continue
         if ($J(p)$min $\le J(p)$old) then
            update $\Omega^{(k)}$ and $\{p^{(k)}\}$by subdividing $j$candidate
            update group level
         else
            return to the parent grouping
         endif
   e. Set k=k+1.
   f. Go to 3.

Fig. 1 Flowchart of Parameter Grouping Algorithm

Numerical simulation studies have shown that the estimation error generally exhibits a significant decrease when damaged elements are clearly separated from undamaged elements. However, when noise exists in the measured data, one cannot be certain that a group contains the greatest degree of damage (or any damage at all) simply because the deviation in the estimated parameter is the greatest. Noise in the measured data may cause a group to behave as if it contains damaged elements even when it does not. Based on this observation, the current algorithm seeks a candidate subset by carrying out parameter estimation M times, where M is the current number of parameter groups. The subdivision that gives the smallest value of the error function $J(p)$ is then permanently subdivided. The M possible candidate groups are those at the deepest level in the subdivision hierarchy. The groups at lower levels in the hierarchy are not considered any longer as candidates until depth is probed in the current group.

3. Data Perturbation Method

If the measured data on the real structure were free of error, a single cycle of calculation using the algorithm described above would be enough to track damage out. Any parameter estimation that different from the intact value would be associated with damage. The amount of reduction in the value of the parameter would be indicative of the severity of damage.

Measurements are, unfortunately, never free from error. Practically, measured data always contain certain levels of noise. Those noises include not only true measurement errors, but also those caused by the difference of load and displacement boundary between the real structure and assumed model. The modeling error, as it known, may also include other effects, such as manufacturing inconsistencies, residual or thermal stresses, or material flaws. Because the modeling error is not the topic of this discussion, it is not considered in this paper.

Now we only deal with the errors of measurements. Those errors will cause the SI algorithm to estimate parameter values different from the actual properties of the structure. If the experiment were repeated the estimated values would be different. Even though the experiment is repeated under the identical conditions, the measured data show random distribution. Therefore, the parameters calculated from the measured data should be also considered as random variables. Here, we use Monte Carlo to simulate the input data and investigate the relationship between the input error and output error.

3.1 Modeling of Input Error

The input data consist of force vectors and displacement vectors. If the force is applied on only one dimension of the freedom space at one time, except for that forced dimension, all other dimensions apparently will have zero in force vector components. On the other hand, the displacement will be formed along all dimensions. In this way, the force can be assumed to have no errors and only displacement vector contains noise.

Although we consider that there are noises only in the measured displacements, it is difficult, if not impossible, to mathematically model measurement noise. However, for numerical experimentation, we can simulate them by varying the calculated displacement measurement values slightly, as shown in Fig. 2, with a known probability distribution.

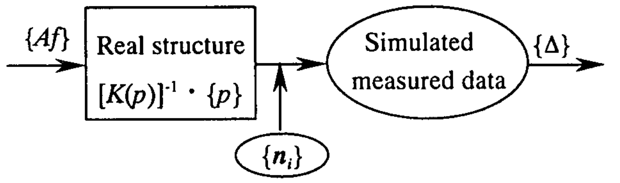

Fig. 2 Simulated Measured Displacements

The most commonly used distribution is the normal distribution, which represents a higher probability of noise level closer to the mean, and a lower probability of larger noise. To range the distribution, it is convenient to use a 95% confidence interval. Therefore, $k_{\alpha/2}$, or I_e equals to 1.96σ with 95% confidence, in which I_e is input error. Namely, the variance of measurement noise is

$$\sigma^2 = \frac{I_e^2}{1.96^2} \approx \frac{I_e^2}{4} \tag{15}$$

Having determined the distribution parameterized on I_e, Equ. (15) is used to generate a set of normal random vector $\boldsymbol{n}_i$ with a mean of zero and standard deviation σ. Then,

$$\{\Delta\} = [Q][K(p)]^{-1}.\{Aff\} + \{n\} \tag{16}$$

where $\{p\}$ is the vector of true values of real structure and $[Q]$ is the Boolean matrix[4] that extracts the measured response from the complete displacement vector. The response $\{\Delta\}$ is taken as the measured response of the real structure.

3.2 Statistical Indices

For noisy response $\{\Delta_k,\ k=1,2,\ \ldots,\ NOBS\}$, the estimation simulation produces a sample $\{p_{j,k},\ k=1,2,\ \ldots,\ NOBS,\ j=1,2,\ \ldots,\ nup\}$, where $p_{j,k}$ is jth parameter of the results from Kth observation and $NOBS$ stands for the number of observations. So the sample size of every variable is $NOBS$. By increasing the sample size $NOBS$ and using the method of Maximum Likelihood, the mean and standard deviation of the sample converge to the mean and standard deviation of the population.

To compare our proposed estimation algorithms and to find trends in the behavior of these estimators, we will use statistical indices to characterize our results. Taking pe_j as the intact value of the jth parameter, the intact rate of the kth observation of the jth parameter is.

$$E_{j,k} = \frac{p_{j,k}}{pe_j} \tag{17}$$

The mean of intact rate and standard deviation of the percentage error of the jth parameter is

$$ME_j = \frac{1}{NOBS} \sum_{k=1}^{NOBS} E_{j,k} \tag{18}$$

$$SD_j = 100 \cdot \sqrt{\frac{1}{NOBS} \sum_{k=1}^{NOBS} (E_{j,k} - ME)^2} \tag{19}$$

For each unknown parameter p_j, there will be $NOBS$ values. In all, there will be $NUP \times NOBS$ estimated parameters. It is desirable to reduce this large number to a single grand mean (GM), and a single grand standard deviation percentage error (GSD) for ease of comparison. GM and GSD will be used to investigate the relationship between the input error and the output error.

$$GM = \frac{1}{NOBS \cdot NUP} \sum_{j=1}^{NUP} \sum_{k=1}^{NOBS} E_{j,k} \tag{20}$$

$$GSD = \sqrt{\left[\frac{1}{NUP \cdot NOBS - 1}\right] \sum_{j=1}^{NUP} \sum_{k=1}^{NOBS} (E_{j,k} - GM)^2} \tag{21}$$

In the same sense that a sample size of 1 is not valid statistically, reducing all these experiments to two scalar values is not an accurate representation; in particular, the GM does not show maximums or minimums, but it is merely a mean. Although it is possible to use different levels of measurement error for each applied force and each measured displacement, the input error is selected as a single percentage value representing all possible sources of error. In this sense, it is possible to establish an input-output error relationship with a given I_e, from which a single value of GM and GSD are obtained.

3.3 Damage Assessment Using Hypothesis Test

After the mean and standard deviation values of each element in the current structure have been obtained from the data perturbation trials, normally distributed parameters can be assumed[3]. Suppose that the measurements are obtained under the same conditions for both the current structure and the associated undamaged structure. Therefore the statistical distributions of system parameters in the undamaged structure can be reasonably assumed the same as those of current structure. The assumed normal distribution $N_b(1, \sigma^2)$ will be called the baseline distribution for the system parameter, wherein 1 represents the intact status of member in undamaged structure. The random variable is $E_{j,k}$. Hypothesis test can be applied to determine damaged members by useful properties of the normal distribution. The hypothesis test is defined as follows:

H_0: $m=1$
H_1: $m<1$
Statistic: $E_{j,k}$

Rule form: accept H_0 if $E_{j,k} \geq C$

Otherwise, accept H_1
Significance level: α

Acceptance region: $P(-k_\alpha \leq \frac{E_{j,k} - 1}{\sigma}) = 1 - \alpha$

Result: $C = 1 - k_\alpha \cdot \sigma$

Using Hypothesis test, the damage status of a member in the current structure is evaluated as Fig. 3 illustrates. A member that accepts H_0 is taken as undamaged with 100×(1−α)% confidence; in the same way, a member that accepts H_1 is taken as damaged with 100×(1−α)% confidence. The damage index I_D, which represents the damage status of a member with the significance level of α, is defined as follows:

$$I_D = \begin{cases} 0 & if\ H_0\ acceptd\ (x \geq c) \\ 1 & if\ H_1\ acceptd\ (x \leq c) \end{cases} \tag{22}$$

The severity of damage S_D, which indicates how seriously a member is damaged with the significance level of α, is defined as a relative distance of the estimated one from the intact value

$$S_D = (1 - x) \times I_D \times 100\,\% \tag{23}$$

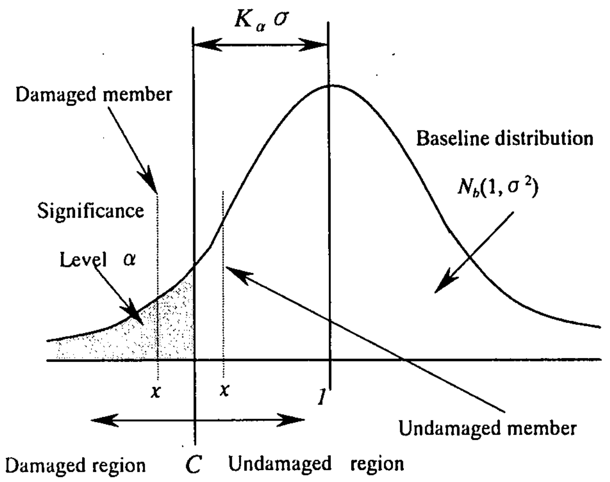

Fig. 3 Interval Estimation for Damage Assessment

4. Simulation Study on a Frame Structure

Consider a 5-story, two-bay steel frame shown in Fig. 5 is used as an example. The frame was divided into 25 frame elements. Nodes were assigned at every joint, and each node has three degrees of freedom. Elements 1-15 make up the columns, and elements 16-25 make up the beams of the frame. The cross-sectional areas, moment of inertia of elements are listed in table 1 and the elastic modulus of every element of the structure is assumed as follows.

a. Undamaged structure

Young's modulus for all elements=206.8 Gpa

b. Current structure (or real structure)

Damage in the structure is assumed as a reduction in the Young's modulus of element, details of which will be stated late for different cases of the study. All of other elements are considered to be intact.

Table 1 Cross Sectional Properties

Member	Area (cm^2)	Moment of inertia (cm^4)
1~15	1065.	442246
16~21	1606.	442246
22~25	1406.	422246

In the frame structure, M_m=2, and $P_{\mu m}$, $[D_{\mu m}]$, $[B_m]$ in Eq. 2 are as follows.

$$P_{1m} = \frac{(EA)_m}{l_m}, \quad P_{2m} = \frac{(EI)_m}{l_m}$$

$$D_{1m} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad D_{2m} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 4 & 2 \\ 0 & 2 & 4 \end{bmatrix}$$

(axial) *(flexural)*

$$[B_m] = \begin{bmatrix} -\cos\theta_m & -\sin\theta_m & 0 & \cos\theta_m & \sin\theta_m & 0 \\ -\frac{\sin\theta_m}{l_m} & \frac{\cos\theta_m}{l_m} & 1 & \frac{\sin\theta_m}{l_m} & -\frac{\cos\theta_m}{l_m} & 0 \\ -\frac{\sin\theta_m}{l_m} & \frac{\cos\theta_m}{l_m} & 0 & \frac{\sin\theta_m}{l_m} & -\frac{\cos\theta_m}{l_m} & 1 \end{bmatrix}$$

The purpose of this simulation study is twofold. First, the specific application will show clearly the meaning of some of the quantities that have been defined by illustrating how they are used. Second, by putting an additional layer of Monte Carlo simulation on the example we can examine the performance of the algorithm with the consideration of measurement errors. To meet these objectives, the two different cases will be studied. We set *NOBS*=30. The number of parameter groups is set not bigger than 5. The locations of the applied force and the corresponding measured displacement are shown in Fig. 4. Fig. 5 shows the flowchart of numerical simulations.

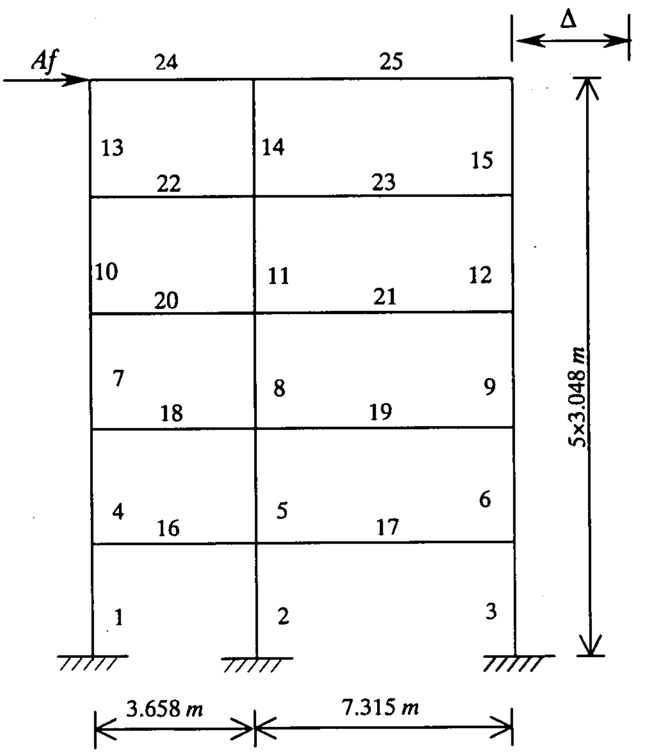

Fig. 4 A 5-story, two-bay steel frame

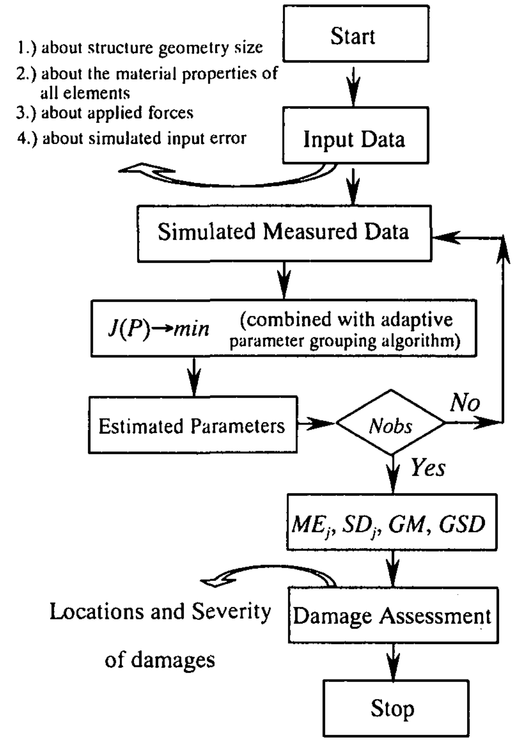

Fig. 5 Flow Chart of Numerical Simulation

Case 1. One Member Damaged

It is assumed that element 13 is damaged with 40% reduction in Young's modulus. To generate the measured data of the structure, we perform a finite element analysis. The computed results are added with uniformly distributed noise, whose amplitude is 5%.

The estimated intact rates of all elements, averaged over 30 Monte Carlo trials in the face of 5% measurement error, are shown in Fig. 6. It can be clearly seen that element 13 is seriously damaged. Although slight damage also happens in element 11 and 16, it is not dominant compared to element 13.

In a real application one has a single set of measured data, a single sample of estimated results can be obtained. Based on the results as shown in Fig. 6, we assume σ =8% in the baseline distribution $N_b(1, \sigma^2)$ in the face of 5% measurement error, then we set α=5%, so K_α=1.65 and C=0.84. We take one from 30 estimated samples, the status of current structure of all members is evaluated and the results are plotted in Fig. 7. By the Hypothesis test, the damaged member is identified as damaged and two undamaged members are assessed as damaged members. Since the damage severities of other undamaged members are small compared to the damaged member, it is concluded that It is unlikely to damage in those members. Those results have 95% confidence.

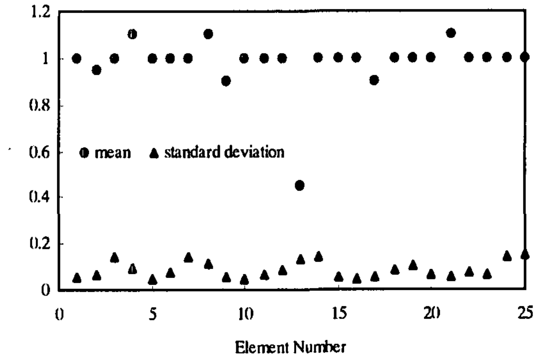

Fig. 6 Mean and Standard Deviation of Intact Rate in all Elements with 5% Input Error

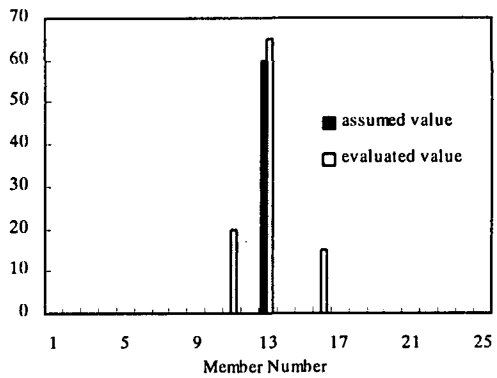

Fig. 7 Severity Damage Charts

Case 2. Two Members Damaged

Let us consider the case in which two members are damaged to different degrees. The stiffness deterioration is 50%, 25% in member 5 and 21 respectively. The estimated intact rates of all elements, averaged over 30 Monte Carlo trials under 2.5% measurement error, are shown in Fig. 8. One can clearly see that member 5, 21 are damaged, although there appear to be two candidates for characterization lightly damaged.

To examine the input-output error relationship, it is desired to plot the GM and GSD values against I_e values, which are shown in Fig. 9 and Fig. 10 respectively. They are used to estimate the output error for a given input error, and also can be used to determine the allowable I_e by limiting output error for the experiment design. The measurement noise tolerance is expected to vary from structure to structure based on the locations of measurements and the topology of the structure. Even if the results depend on cases, we can determine the stability of algorithm approximately based on Fig. 9 and Fig.10.

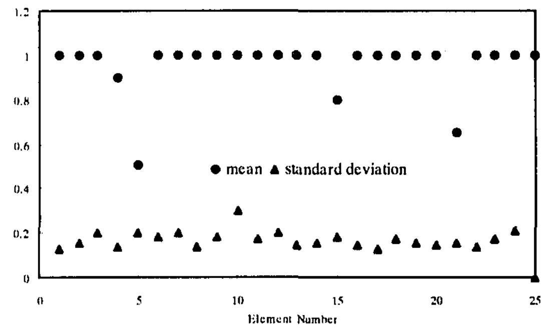

Fig. 8 Mean and Standard Deviation of Intact Rates in all Elements with 2.5% Input Error

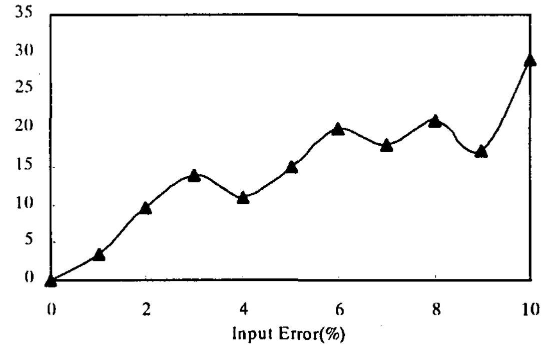

Fig. 9 Variations of single grand standard deviation percentage with noise amplitude(%)

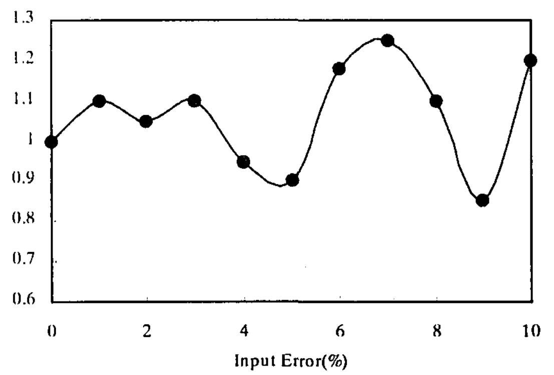

Fig. 10 Variations of single grand mean with Noise Amplitude(%)

Based on the results as shown in Fig. 8, we can assume σ =15% in the baseline distribution $N_b(1, \sigma^2)$ in the face of 2% measurement error, then if we set α=5%, so K_α=1.65 and C=0.85.

By using one sample from estimated result, the status of current structure is presented in Fig. 11. By Hypothesis test, two damaged members are identified as damaged. In Fig. 11, when x<1, we also calculate their severity of damage. In this way, only one undamaged members are taken as damaged. Since the damage severity of undamaged member is small comparing to that of the damaged members, it is concluded that there is little possibility to damage in those members. Those results have 95% confidence.

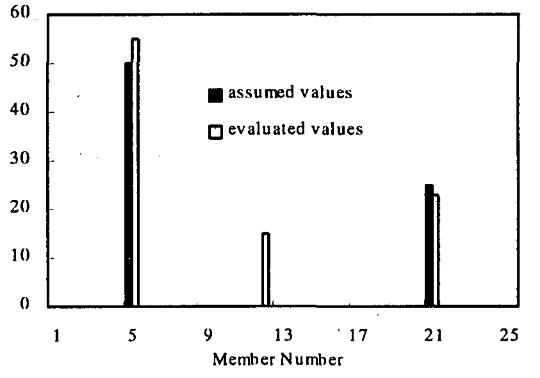

Fig. 11 Severity Damage Charts(%)

5. Summary

This paper focuses on a new detection and assessment algorithm, which is based on the virtual work error estimator with an adaptive parameter grouping scheme and Hypothesis test. The procedure is illustrated and tested using Monte Carlo simulation theories in numerical simulations. The conclusions can be drawn as follows.

1.) It has been recognized in all literatures related to static identification that the number of parameters to be estimated should not exceed that of the number of independent measurements. However, this restriction is no longer a problem with the algorithm derived based on the virtual work error estimator proposed in this paper.
2.) With the decrease of measuring numbers, the output error can be reduced on a large scale. The level of acceptable measurement errors will also be greatly improved comparing with the results in Ref.[1].
3.) Hypothesis test is an effective tool to evaluate the damage status of existing structures based on the estimated results.

System identification is a highly nonlinear unconstrained optimal problem, even though the structure is linear one. The loading of computation increases with the increase of element number. Its solvability is greatly depends on the structural characteristics, applied load and measured displacements selected for numerical simulation. More effort should be made to discuss the uniqueness of solution.

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