Static and Seismic Slope Stability Analyses Based on Strength Reduction Method

せん断強度低減法による斜面の静的・動的安定解析

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This study presents the numerical procedure for static and seismic stability analyses of slope based on the shakedown theorem in plasticity[7]. The stability of slope has been defined as a reduction coefficient in shear strength for the limit state of slope mass. Firstly, the applicability of proposed method to the static slope stability analysis was investigated. The obtained factors of safety were compared with those of the conventional methods. It was clearly shown that the proposed method could evaluate the stability of slope properly. Secondly, the seismic stability of slope was investigated against sine wave accelerations excited at the bedrock of subsoil. The proposed method was shown to estimate the factors of safety by taking account of the effects of frequency in enforced acceleration and vibration properties of slope mass as damping and resonance. The obtained factor of safety for low frequency wave acceleration coincided with that of pseudo-static stability analysis and, on the other hand, the factor of safety approached that of static stability analysis for high frequency wave acceleration. Thirdly, the combined stability method[16] which composed of deformation and stability analyses was conducted. The comparison between combined stability and proposed methods in cases of natural slope and embankment was investigated. It was found that the minimum factor of safety for natural slope obtained by the conventional method, however, gave an almost coincident factor of safety by the proposed method. But, the significant difference appeared in case of embankment problem. The possible engineering meaning of time dependent factor of safety obtained by the combined method was clarified.

Key Words : *damping, dynamic, earthquake, numerical analysis, safety factor, slope stability, stability analysis*

1. Introduction

The assessment of slope stability for the static and seismic conditions has been an important task in geotechnical engineering. The static slope stability has been generally evaluated with the limit equilibrium method as the circular arc method, the slice method and others, which introduce the specific failure mode into the analysis. The seismic stability of slope, therefore, has been estimated by the pseudo-static concept which originally comes from concepts of static slope stability. However, the earthquake ground motion is usually variable repeated in nature rather than monotonically and proportionally increasing in one direction.

To assess the seismic stability of structure in more realistic manner, the seismic loading should be treated in the same nature with the earthquake ground motion. Shakedown analysis deals with variable repeated loading like the one generated by earthquake ground excitation. Therefore, in this study a new seismic stability assessment of slope is proposed based on the shakedown analysis technique. It is also exhibited that the proposed method can take the vibration properties of slope into consideration.

Recently, the static slope stability analysis is conducted by the following two methods. One is the stability analysis which directly deals with the limit state at failure such as the limit equilibrium method or the limit analysis. The other is the deformation analysis based on the finite element method. Duncan[5] gave a comprehensive review on both methods for slope stability assessment. Besides two methods, the experimental studies are also conducted. For static slope stability, the shakedown analysis falls into limit analysis that has been used in the conventional methods. Therefore, in this study the comparison between them will be exhibited.

In seismic stability assessment, the application of deformation analysis is more conspicuous. Newmark[11] and Seed[14] proposed the methods of analysis to predict the permanent displacement of earth structure against earthquake. It is because the permanent displacement of structure caused by earthquake is important in design, and the conventional stability analysis based on the pseudo-static concept contains a drawback due to lack of earthquake nature. Toki et al.[16] conducted the stability analysis employing both the limit equilibrium analysis and the deformation analysis. The stability of slope could be evaluated by considering the effects of earthquake record and vibration properties of slope mass. However, the factor of safety was obtained as time dependent and varied in time corresponding to the earthquake excitation. It could not give any direct information on the total stability of slope against the earthquake loading. The relationship between time dependent factor of safety and total stability of the slope is desired to be clarified. Ohtsuka et al.[12] proposed the seismic stability analysis for embankment based on the shakedown theorem. They gave the seismic stability of earth structures dependent on both earthquake record and vibration properties of soil structures. However, the stability of slope was evaluated in terms of load factor while the factor of safety is generally defined as a reduction coefficient in shear strength for the limit state of slope mass at failure.

This study proposes a numerical procedure by means of shakedown analysis technique to assess the static and seismic stability of slopes where the factor of safety can be newly estimated as a reduction coefficient in shear strength. Firstly, the applicability of proposed method to the static slope stability analysis is inquired through the comparison with the conventional methods. Secondly, the seismic stability of slope is investigated against sine wave accelerations excited at the bedrock of subsoil. The effects of frequency in enforced acceleration and vibration properties of slope mass as damping and resonance are discussed in detail. Thirdly, the combined stability method of deformation and stability analyses is discussed in comparison with the proposed seismic stability analysis. The possible engineering meaning of time dependent factor of safety obtained by the combined method is taken into consideration.

2. Shakedown Analysis of Slope Based on Shear Strength Reduction Method

The first general shakedown theorem for the admissible stress field in a three dimensional body was developed by Melan, and the second theorem for the admissible kinematic field was developed by W.T. Koiter[7]. The shakedown theorems directly define the stability of an elasto-plastic body against the repeated loads which vary arbitrary in a prescribed domain. When the effect of repeated loads which may alternate in magnitude and direction is not negligible, then the structures may undergo failure although the static collapse conditions are not attained[1]. Thus, an assumption as monotonically load for earthquake excitation may trigger an instability of structure. With advantages of shakedown analysis, the static and seismic stability of slopes will be investigated through the assessment of their factors of safety. The factor of safety is estimated as a reduction coefficient in shear strength. Since the shakedown analysis falls into conventional method for static loading, the comparison between them can be well investigated. It is expected that both analyses will give the similar results.

2.1 Lower Bound Theorem in Shakedown Analysis

The first general shakedown theorem of Melan is also known as the lower bound method. The body is modeled into an elasto-perfectly plastic material. There, the stress $\sigma(t)$ is assumed as the addition of the initial stress $\boldsymbol{\sigma}_o$, the elastic stress $\boldsymbol{\sigma}^e(t)$ and the residual stress $\boldsymbol{\sigma}^r(t)$. The elastic stress $\boldsymbol{\sigma}^e(t)$ is uniquely determined by the external force and the residual stress $\boldsymbol{\sigma}^r(t)$ is defined to cause corresponding to the plastic deformation. The residual stress is self-equilibrate and remains in the body even if the external force is removed. The lower bound theorem of shakedown analysis states that the behavior of the structure will finally settle into an elastic response against the external force through some repeats of arbitrary load in the prescribed load domain (to be shakedown) if any time independent residual stress distribution $\overline{\boldsymbol{\sigma}}^r$ can be found everywhere in the structure which obeys the following yield function[7)]

$$\boldsymbol{\sigma}_s(t) = \boldsymbol{\sigma}_o + \boldsymbol{\sigma}^e(t) + \overline{\boldsymbol{\sigma}}^r, \quad f(\boldsymbol{\sigma}_s(t)) \leq 0 \quad (1)$$

where f denotes a yield function of soil. The stability of structure is defined against the prescribed load domain.

On the shakedown analysis, the failure criterion of structure is referred to whether the structure to be in shakedown condition or not. When the structure falls into shakedown condition, then the structure behaves elastically. It means that the plastic flow can not develop anymore. Since the stability of structure is reflected by the factor of safety, its value refer to shakedown condition. If the factor of safety less than one, the structure is said as failure due to that the shakedown condition can not be attained.

2.2 Finite Element Analysis with Linear Programming

The lower bound theorem in shakedown analysis falls into the maximization problem of the factor of safety. Maier[9)] has been applied the linear programming technique to solve the maximization problem based on the lower bound theorem. By combining the linear programming technique and the finite element analysis, this study proposes a new method to assess the factor of safety in slope stability problem.

The yield function is modeled into the piecewise linear function such as

$$\boldsymbol{f}(\boldsymbol{\sigma}) = N_e^T \boldsymbol{\sigma} - \boldsymbol{K}_e \leq \boldsymbol{0}, \quad (2)$$

where N_e is an assembly of the outward normal vectors to the yield function and $\boldsymbol{K}_e$, an assembly of yield limits corresponding to outward normal vectors. As shown in **Fig. 1**, by handling N_e and $\boldsymbol{K}_e$ in the piecewise linear yield function, the yield surface can be well approximated. The circumscribed piecewise linear yield function was adopted for Drucker-Prager type yield function, in this study.

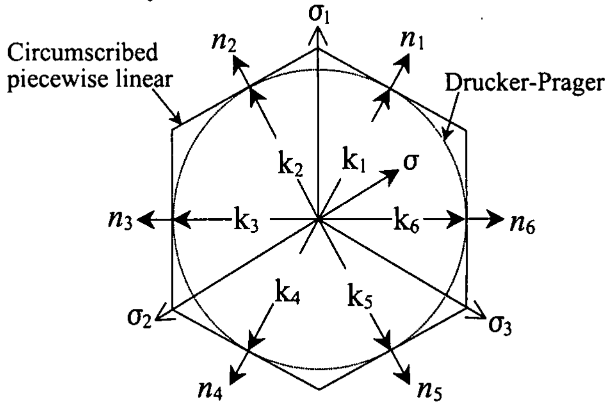

Fig. 1 Piecewise linear yield function.

2.3 Shakedown Analysis Based on Shear Strength Method

In slope stability analysis, the shear strength criterion of Mohr-Coulomb has been widely employed as

$$\tau = c + \sigma_n \tan\phi \quad (3)$$

where c and ϕ indicate the cohesion and the angle of shear resistance, respectively. The factor of safety for slope is commonly defined as a reduction coefficient β in shear strength as shown in **Fig. 2** such that

$$\tau = \frac{c}{\beta} + \sigma_n \frac{\tan\phi}{\beta} = \hat{c} + \sigma_n \tan\hat{\phi} \quad (4)$$

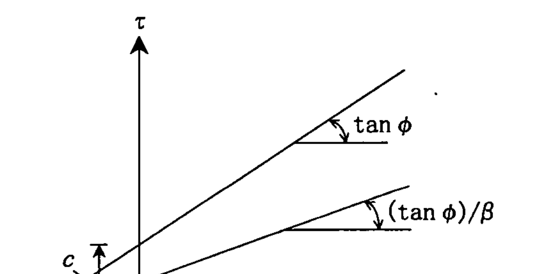

Fig. 2 Definition for factor of safety.

which constitutes the limit state that the slope undergoes failure. The strength parameters $\hat{c}$ and $\hat{\phi}$ for active shear strength are, therefore, functions of factor of safety and an iterative procedure to determine the factor of safety has been employed in the conventional methods of various limit equilibrium methods. It is worth noting that the factor of safety obtained in this paper deals with a global factor of safety.

The Drucker-Prager criterion is employed to assess the factor of safety for slope under the three dimensional condition such that

$$f(\boldsymbol{\sigma}) = \sqrt{J_2} - \alpha I_1 - k \tag{5}$$

where J_2 expresses the second invariant for deviatoric stress and I_1, the first invariant of stress. The parameters of α and k can be expressed by the parameters of $\hat{c}$ and $\hat{\phi}$ in Eq. (4) such as

$$\alpha = \frac{3\tan\hat{\phi}}{\sqrt{9+12\tan^2\hat{\phi}}},\ k = \frac{3\hat{c}}{\sqrt{9+12\tan^2\hat{\phi}}} \tag{6}$$

under the plane strain condition[3]. In the analytical procedure, the Drucker-Prager yield criterion is approached by the circumscribed piecewise linear yield function as shown in **Fig. 1**.

From the definition on factor of safety, N_e and $\boldsymbol{K}_e$ in Eq. (2) are defined as the functions of reduction coefficient as $N_e(\beta)$ and $\boldsymbol{K}_e(\beta)$ in the same way with Eqs. (5) and (6). In the finite element technique, the stresses vector $\boldsymbol{\sigma}$ are transformed into discretized stresses vector $\boldsymbol{\Sigma}$ defined in the whole structure. The yield function for the whole structure is, therefore, expressed in terms of discretized stress vector such that

$$N(\beta)^T\boldsymbol{\Sigma} - \boldsymbol{K}(\beta) \le \boldsymbol{0} \tag{7}$$

where $N(\beta)$ and $\boldsymbol{K}(\beta)$ are the assemblage matrix and vector of $N_e(\beta)$ and $\boldsymbol{K}_e(\beta)$, respectively. The stability analysis will be conducted by using a linear programming method to solve the maximization problem. The reduction coefficient β is a subject of maximization. In the process of maximization of β, the initial value of β_o is assumed temporarily. The stability analysis in the first iteration can be expressed as follows:

$$f_1 = max\left\{ s \left| \begin{array}{c} s\ N(\beta_o)^T\boldsymbol{\Sigma}_o \\ +N(\beta_o)^T\overline{\boldsymbol{\Sigma}^r} \le \boldsymbol{K}(\beta_o) \\ B^T\overline{\boldsymbol{\Sigma}^r} = \boldsymbol{0} \end{array} \right. \right\} \tag{8}$$

In the above equations, $\boldsymbol{\Sigma}_o$ and $\overline{\boldsymbol{\Sigma}^r}$ express the initial and constant residual discretized stress vectors of the whole structure. The first equation denotes the yield function for the load factor s against the initial stress equilibrating with the gravity force and the second equation, the self-equilibrium condition on the residual discretized stress vector. A value of f_1 indicates the margin of structure for the limit state at failure under the assumed shear strength parameters of $\hat{c}(\beta_o)$ and $\hat{\phi}(\beta_o)$. The second iteration will be conducted by replacing β_o by $\beta_1 = f_1 \cdot \beta_o$ in Eq. (8), f_2 can be obtained in the same way. In the nth iteration, taking $\beta_{n-1} = f_{n-1} \cdot f_{n-2} \cdots f_1 \cdot \beta_o$, the stability analysis constitutes a form as,

$$f_n = max\left\{ s \left| \begin{array}{c} s\ N(\beta_{n-1})^T\boldsymbol{\Sigma}_o \\ +N(\beta_{n-1})^T\overline{\boldsymbol{\Sigma}^r} \\ \le \boldsymbol{K}(\beta_{n-1}) \\ B^T\overline{\boldsymbol{\Sigma}^r} = \boldsymbol{0} \end{array} \right. \right\} \tag{9}$$

When the value of f_n converges to 1.0, the reduction coefficient of β_{n-1} coincides with β_n and the structure attains to the limit state under the employed shear strength parameters of $\hat{c}(\beta_n)$ and $\hat{\phi}(\beta_n)$. From the definition of slope stability, the factor of safety is given by the reduction coefficient as $Fs = \beta_n$ finally.

2.4 Static Slope Stability Analyses

Shakedown analysis comes down to limit analysis in stability evaluation against a monotonous load application. Whereas, the limit analysis has been widely employed in the conventional slope stability analyses. It is important to verify the applicability of proposed method based on shakedown analysis through the comparison with the conventional method. The limit equilibrium method which is based on limit analysis has been most widely employed in static slope stability assessment. In the early development, Taylor[15] investigated the stability number for simple slope under various soil constants with the use of a circular arc method (after Nash[10]). Chen[3] conducted the upper bound solutions based on the limit analysis and compared his results to Taylor's. The stability number is a non-dimensional number and describes the stability of general slope for a specific slope angle. Therefore, the factor of safety can be derived from the stability number through an iterative procedure. In the present paper, the factors of safety obtained by shakedown analysis for simple slope are compared among Taylor[15] and Chen[3] methods.

Investigation on the factor of safety for simple slope was done by using soil parameters shown in **Table 1**. The slope angle, β and the angle of shear resistance, ϕ are treated as variable. The height of slope was kept constant as 10.0 m. The results on the factor of safety are shown in **Table 2**. In the same geometry of slope, by using the stability numbers proposed by Taylor[15] and Chen[3], the factors of safety can be found. The results are also shown in **Table 2** for comparison. It is readily seen that the proposed method gives the almost same factor of safety with the conventional methods. The proposed method affords a little larger factor of safety in comparison with the conventional methods. It is because the proposed method employs the circumscribed piecewise linear yield function of the Drucker-Prager criterion with comparatively small number of piecewise linearization as shown in **Fig. 1**. If the number of piecewise linearization increases, then the circumscribed piecewise linear yield function approaches the Drucker-Prager yield function. It means that the obtained factor of safety will be decreased, such that the factors of safety by the proposed and conventional methods to be more closer.

Table 1 Soil parameters.

E	40 MPa	γ_t	16.0 kN/m^3
γ_w	9.8 kN/m^3	ν	0.4
c	17.5 kPa	ϕ	7.5°

3. Seismic Slope Stability Analysis Taking Account of Earthquake Motion

It is well understood that the earthquake motion has some important characteristics. In the slope stability analysis, many efforts have been spent to take all of these characteristics into consideration. The present paper is an effort in this track. One of the important characteristics of earthquake motion is that the earthquake motion behaves as variable repeated loading. Unfortunately, in the seismic stability analysis this characteristic is usually ignored. Therefore, it is benefit to evaluate the seismic stability of slope by taking account of (1) variable repeated loading of enforced acceleration time history, (2) frequency property of earthquake, (3) vibration property of slope mass and (4) resonant effect. The items from (2) to (4) have been well known in the dynamic analysis. Many researchers have conducted the assessment of seismic slope stability by combining deformation and limit equilibrium analyses where the items from (2) to (4) included in their analyses. However, the factor of safety is given as a function of time and it does not certainly give a direct information on the stability of slope against the whole dynamic load itself. Ohtsuka et al.[12] proposed the seismic stability analysis on the embankment problem by introducing the failure definition against the dynamic load based on the shakedown concept where the load factor was treated as safety factor. The factor of safety for slope is, however, commonly defined as the reduction coefficient in shear strength against the limit state at failure. Thus, this paper is presented with this common definition of factor of safety.

Table 2 Factor of safety in comparison with conventional methods.

β	ϕ	Factor of safety, Fs		
		Taylor Method[15]	Chen Method[3]	Proposed Method
30°	0	0.701	0.605	0.714
	5°	0.996	0.999	1.023
	10°	1.252	1.274	1.301
	15°	1.476	1.500	1.567
	20°	1.752	1.768	1.816
	25°	2.008	2.017	2.109
40°	0	0.658	0.605	0.654
	5°	0.877	0.890	0.909
	10°	1.064	1.085	1.136
	15°	1.253	1.276	1.347
	20°	1.426	1.453	1.536
	25°	1.619	1.616	1.740
50°	0	0.618	0.604	0.620
	5°	0.792	0.800	0.837
	10°	0.940	0.953	1.013
	15°	1.087	1.102	1.181
	20°	1.232	1.247	1.368
	25°	1.376	1.393	1.520

3.1 Slope Stability by Seismic Coefficient Method

Seismic stability of slope against earthquake has been estimated based on the pseudo-static concept (e.g., Sarma[13], Chang et al.[2]). It is also known as seismic coefficient method. This method is generally simple, but the effect of alternating direction of horizontal excitation in earthquake nature is not introduced into stability analysis. The inertia force caused by horizontal seismic coefficient is taken into account by replacing the initial stress vector $\mathbf{\Sigma}_o$ in Eqs. (8) and (9) by $(\mathbf{\Sigma}_o + \mathbf{\Sigma}^e)$, where $\mathbf{\Sigma}^e$ denotes the elastic discretized stress vector against the enforced inertia force. In the seismic coefficient method, the stress vector $\mathbf{\Sigma}^e$ is the response against the enforced inertia force which acts in one direction only, i.e., toward slope surface. However, the horizontal excitation of earthquake is naturally acting in both sides as variable repeated force. Thus, it is interesting to investigate how the effect of repeated force affects the stability of slope.

With the use of superior property of shakedown analysis, the effect of alternating horizontal excitation on the slope stability is investigated. The equivalent force vector $\boldsymbol{\Omega}(\beta)$ corresponding to the variable load $\boldsymbol{F}$ in the prescribed load domain Δ, i.e., the possible alternating horizontal excitation, is introduced after Maier[9] such that

$$\boldsymbol{\Omega}(\beta) = max \left\{ \begin{array}{c} N(\beta)^T \left(\boldsymbol{\Sigma}_o + \boldsymbol{\Sigma}^e\right) \middle| B^T \boldsymbol{\Sigma}^e = \boldsymbol{F}, \\ \boldsymbol{F} \text{ is variable in } \Delta \end{array} \right\} \tag{10}$$

$\boldsymbol{\Sigma}^e$ is uniquely determined for the variable load $\boldsymbol{F}$. The vector $\boldsymbol{\Omega}(\beta)$ is obtained by maximization of each component for possible load $\boldsymbol{F}$ in the load domain Δ.

The basic equation of Eq. (9) in the iterative procedure is replaced by the following equation.

$$f_n = max \left\{ s \left| \begin{array}{c} s\,\boldsymbol{\Omega}(\beta_{n-1}) + \\ N(\beta_{n-1})^T \overline{\boldsymbol{\Sigma}}^r \leq \boldsymbol{K}(\beta_{n-1}) \\ B^T \overline{\boldsymbol{\Sigma}}^r = \boldsymbol{0} \end{array} \right. \right\} \tag{11}$$

where f_n indicates the margin in shear strength for the limit state at failure under the assumed shear strength parameters of $\hat{c}(\beta_{n-1})$ and $\hat{\phi}(\beta_{n-1})$. The factor of safety is determined by $Fs = \beta_n$ when β_n converges as $\beta_n = \beta_{n-1}$ in the iterative procedure.

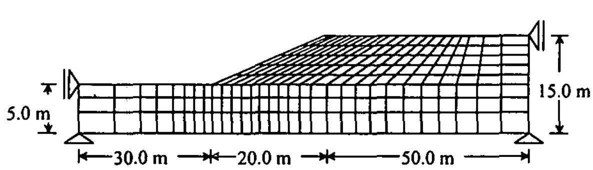

Fig. 3 Boundary condition for case study.

Fig. 3 shows the boundary condition of slope employed for the case study. The soil parameters are the same with **Table 1**. The factor of safety under the static condition is compared among the conventional and proposed methods in **Table 3**. The computed results are found to be a little larger than those of the conventional methods. It might be partly due to the effect of the boundary condition at bedrock. The proposed method employs the continuity assumption and the finite element discretization so that the computation accuracy might be much affected in the case of base failure mode.

Table 3 Factor of safety under static condition.

Method	Factor of safety, Fs
Taylor[15]	1.214
Chen[3]	1.217
Kuroda[8]	1.130
Yamagami et al.[17]	1.207
Proposed Method	1.258

In the seismic coefficient method, the magnitude of inertia force is denoted by the seismic coefficient as $k_h = g_h/g$. The horizontal excitation working for leftward in **Fig. 3** is defined as positive and vice versa. The conventional method usually deals with the positive seismic coefficient only. However, the earthquake motion may alternate from positive to negative directions. Therefore, it is important to examine the effect of alternating earthquake load.

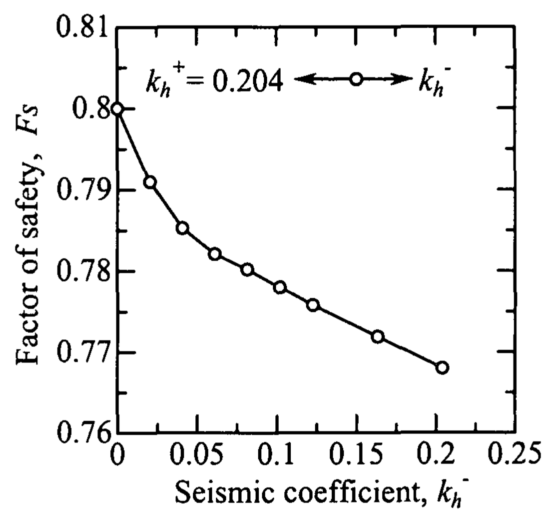

Fig. 4 Effect of alternating load for slope stability.

The effect of alternating horizontal excitation on slope stability is evaluated with the proposed method as shown in **Fig. 4**. For numerical example, the seismic coefficient k_h=0.204 is considered. By conventional method, the factor of safety is obtained as Fs = 0.801. This is indicated in **Fig. 4** for $k_h^- = 0$, i.e., no alternating load. The domain for seismic coefficient in alternating load is arranged as $k_h^- \leq k_h \leq 0.204$ and k_h^- is varied to change the domain for possible seismic coefficient. The figure shows that the factor of safety deteriorates with increasing k_h^-. It directly indicates that the repeated alternating seismic coefficient affects the stability of slope.

However, the degree in effect seems not so large in comparison with that for embankment problem reported by Ohtsuka et al.[12]. It is because the possible failure mode is strictly restricted to the prescribed direction (leftward in **Fig. 3**) in the case of natural slope so that the effect of alternating seismic coefficient appears lower. Nevertheless, the reduction in factor of safety attains to 5% at most and it is still necessary to take account of the effect of possible change in horizontal excitation. It is noted that the effect of alternating seismic coefficient will get greater with both the expansion in possible seismic coefficient domain and the increasing in ϕ.

3.2 Seismic Stability Analysis with Shakedown Theorem

Although the shakedown concept was firstly developed for static or quasi-static loading, further developments have brought out the dynamic shakedown concept into view. It assures that the structures may attain to the shakedown condition against dynamic loading. Ho[6] and Ceradini[1] have shown the proofs of dynamic shakedown. Ohtsuka et al.[12] proposed the failure definition of structure based on the shakedown concept by referring whether the structure reaches an elastic response to the prescribed dynamic load through some repeats of the dynamic load as shown in **Fig. 5**. The repetitive prescribed dynamic load is developed in the computation in order to find out the limit state of structure. It means that the obtained factor of safety is only due to the prescribed dynamic load without any repetition. Introduced definition of stability was proved based on the dynamic shakedown by Ho[6]. The stability analysis for the dynamic load could be formulated by means of a linear programming method.

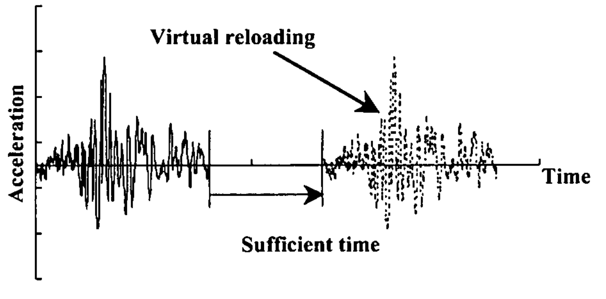

Fig. 5 Imaginary repeats of dynamic load.

Taking the dynamic load as a successive load application, the effect of dynamic load is transformed into the following equivalent force vector in the same way with Eq. (10) such that

$$\boldsymbol{\Omega}(\beta) = max\left\{N(\beta)^T\left(\boldsymbol{\Sigma}_o + \boldsymbol{\Sigma}^e(t)\right)\,\Big|\,B^T\boldsymbol{\Sigma}^e(t) = \boldsymbol{F}(t) - C\dot{\boldsymbol{U}}(t) - M\ddot{\boldsymbol{U}}(t),\ 0 \le t \le T\right\} \quad (12)$$

in which the reduction coefficient β for shear strength parameters is employed. The conditional equation indicates the equation of motion for the prescribed dynamic load of $\boldsymbol{F}(t)$ $(0 \le t \le T)$. The notation of C and M denote the damping and mass matrices, respectively. $\boldsymbol{\Sigma}^e(t)$ is a discretized stress vector dependent of time. $\boldsymbol{\Omega}(\beta)$ is obtained by a maximization of each component in Eq. (12). $\boldsymbol{\Omega}(\beta)$ denotes the equivalent force vector to the dynamic load with regard to the yield limit. It reflects the loading history $\boldsymbol{F}(t)$ and the vibration property of ground. Finally, the factor of safety can be assessed for the dynamic load with an iterative method such as shown in Eq. (11).

3.3 Seismic Stability of Natural Slope for Sinusoidal Acceleration Wave

The earthquake ground motion is naturally considered as a dynamic loading, where the frequency of motion plays an important role. So that, the stability of natural slope against earthquake should be affected by the frequency of earthquake itself. In order to clarify the applicability of the proposed method to assess the seismic stability of slope where the effect of frequency of motion can be taken into account, a sinusoidal acceleration wave is used instead of earthquake motion.

A simple slope as shown in **Fig. 3** instead of natural slope is subjected to sine wave acceleration which acts horizontally at the bedrock of subsoil. The parameters of soil are given in **Table 1**. The amplitude of sine wave acceleration is taken as $a_x = 200$ gal. The Rayleigh damping model is employed in the elastic dynamic deformation analysis, such that matrix C in Eq. (12) can be expressed as

$$C = aM + bK \quad (13)$$

The Rayleigh damping model assumes that a damping is proportional to mass M and stiffness K of the structure. The parameters of a and b are coefficients for the Rayleigh damping model.

Christian et al.[4] proposed the relationship between the coefficients of a and b and the damping ratio as

$$h_i = \frac{a}{2\omega_i} + \frac{b\omega_i}{2} \tag{14}$$

where h_i and ω_i are the damping ratio and frequency at mode i, respectively. The parameters a and b for specific damping ratio could be determined by assuming the fundamental natural frequency of slope ω_o governs the major behavior. If h is the desired damping ratio, it is reasonable to make $h_i = h$ at ω_o. This can be achieved by setting

$$a = h\,\omega_o \quad \mathit{and} \quad b = \frac{h}{\omega_o} \tag{15}$$

For example, let the desired damping ratio $h =$ 10%, the fundamental natural frequency of slope $\omega_o = 10.275$ rad (1.64 Hz), then the parameters $a = 1.0275$ and $b = 0.0097$. It is worth noting that the parameters a and b can be given to determine the damping ratio h.

Fig. 6 shows the relationship between the obtained factor of safety and frequency in sine wave acceleration. The obtained factor of safety was computed by using the Eqs. (11) and (12), where the dynamic load was given by sine wave acceleration. The damping ratio is taken from $h = 10\%$ to 50% in order to examine the effect of damping on the seismic stability. In the figure, the factor of safety for static condition and by seismic coefficient method are illustrated with dotted lines. It is noted that the factor of safety obtained by seismic coefficient method is computed for inertia force corresponding to the maximum amplitude of enforced acceleration which acts alternately in range $-0.204 \leq k_h \leq 0.204$ ($k_h = 0.204$ corresponds to $a_x = 200$ gal).

It is readily seen from the figure that the obtained factor of safety approaches that of static condition at high frequency in enforced acceleration and, on the other hand, it coincides with that seismic coefficient method at low frequency. It seems quite natural that the effect of dynamic load on the stability is lower at high frequency. On the contrary, the seismic coefficient method gives a good approximation to the motion of soil mass in slope at low frequency. Taking notice of the change in factor of safety with frequency, the factor of safety firstly decreases with the increasing in frequency, but it turns to increase after the frequency of about 2 Hz is achieved. Moreover, the fundamental natural frequency of slope was found as $\omega_o = 1.64$ Hz prior to computation of safety factor. The minimum factor of safety is obtained at frequency which is very close to the fundamental natural frequency. Therefore, it is reasonable to conclude that the minimum factor of safety is triggered by resonant phenomenon. It is important to note here that the frequencies which are near to resonant frequency afford the factors of safety which are lower than that obtained by the seismic coefficient method. The similar trend was found for the seismic stability of embankment in term of load factor that was reported by Ohtsuka et al.[12].

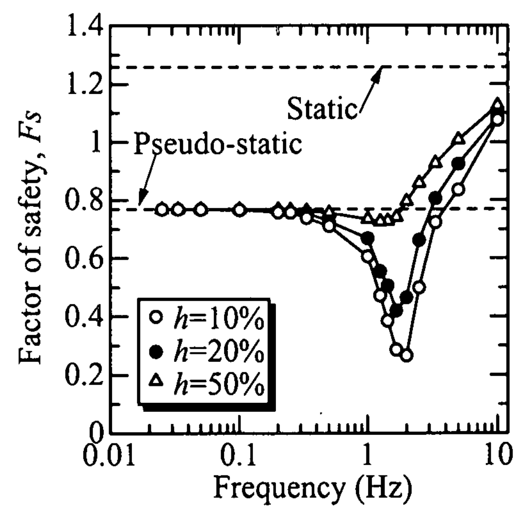

Fig. 6 Effect of frequency in sine wave acceleration.

In the same figure, the effect of damping ratio is clearly exhibited. At the damping ratio of $h = 10\%$, the factor of safety seems drastically falling into a minimum value at resonance frequency, but, at $h = 50\%$ such performance is not discernible. It indicates that the effect of damping works properly.

The effect of vibration property of slope on the seismic stability is examined in the same way as before. Taking the damping ratio as $h = 10\%$, the elastic modulus of slope that reflects its vibration property is changed to see its effect to distribution of safety factor. **Fig. 7** illustrates the relationship between the obtained factor of safety and frequency by enforcing the sine wave acceleration. It can be seen that the frequency on the minimum factor of safety is shifted according to elastic modulus. If the elastic modulus of system is changed, then its fundamental natural frequency is also changed. For elastic moduli of 1, 5 and 40 MPa, the fundamental natural frequencies are 0.26, 0.47 and 1.64 Hz, respectively. The figure clearly shows that the resonance frequencies follow the fundamental natural frequencies of slope according to their elastic moduli. It confirms that the developed method could evaluate the seismic stability rationally.

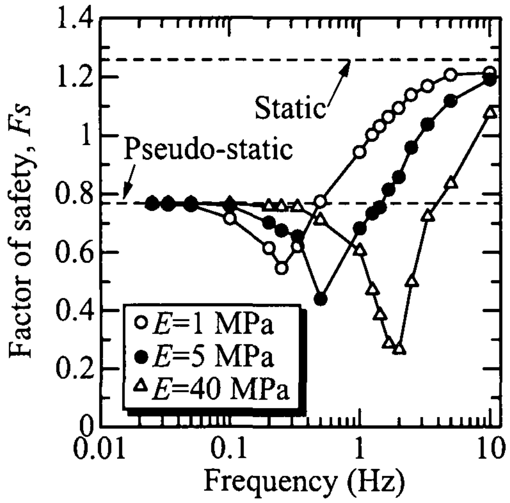

Fig. 7 Effect of vibration property of slope.

4. Consideration on Conventional Seismic Stability Analysis with Dynamic Response Analysis

In the conventional seismic stability analyses, two methods are usually conducted. One is the seismic coefficient method, which is based on pseudo-static concept. This method takes the loading as time independent. The other is dynamic response analysis, where the loading is time dependent. However, both methods have no different in the principle of loading. In other words, both methods enforce load to structure for specific instant only. Therefore, both methods could not give a satisfied answer with regard to question about how the stability of structure against the load in entire time domain. The presented paper proposes a new perspective to overcome the problem by means of shakedown analysis technique.

Apart from conventional seismic stability analysis, the shakedown analysis could consider the load in entire time domain. In the conventional dynamic analysis, the obtained factors of safety change from moment to moment, but, in shakedown analysis, the factor of safety is a single value which represents the stability against load in total time. In the conventional method, the minimum factor of safety during loading is usually adjusted as the total stability. Based on the circumstances, it is interesting to investigate the comparison between conventional and proposed methods.

4.1 Time Dependent Slope Stability by Combined Analysis

The combined analysis is intended to predict the seismic stability of slope by means of dynamic deformation and stability analyses simultaneously. By conducting the dynamic deformation analysis, the stress distribution in slope can be estimated for every moment according to the dynamic load by taking account of loading history and vibration property of slope. The factor of safety is, therefore, defined for every moment by performing the stability analysis against the specific stress distribution in slope.

As before, the boundary condition of **Fig. 3** and soil parameters in **Table 1** are used for case study to perform the combined analysis. The obtained factor of safety in time is illustrated in **Figs. 8 and 9** for periods of sinusoidal wave acceleration $T = 0.25$ s and 0.75 s, respectively. Both figures illustrate the periodic change in factor of safety, however, the change process in time is apparently different. The factor of safety for $T = 0.25$ s almost forms a sine wave, but that for $T = 0.75$ s does not. In the case of $T = 0.75$ s the behavior of factor of safety seems much complicated, however, it settles to the specific pattern with time. The minimum factor of safety during the excitation is obtained as $Fs = 0.801$ and 0.473 for $T = 0.25$ s and 0.75 s, respectively. The fundamental natural period of slope is 0.61 s so that the factor of safety obtained for $T = 0.75$ s changes more largely than that for $T = 0.25$ s, since the excitation period is more closer to fundamental natural period.

The feature of this method is to take account of loading history of enforced acceleration and vibration properties of slope mass, such that the history of safety factor is also clearly shown. However, the factor of safety is obtained as a function of time, which does not directly afford any information on the stability of slope against the entire acceleration load. Moreover, the combined method further requires numerous times of computation to obtain a time dependent factor of safety.

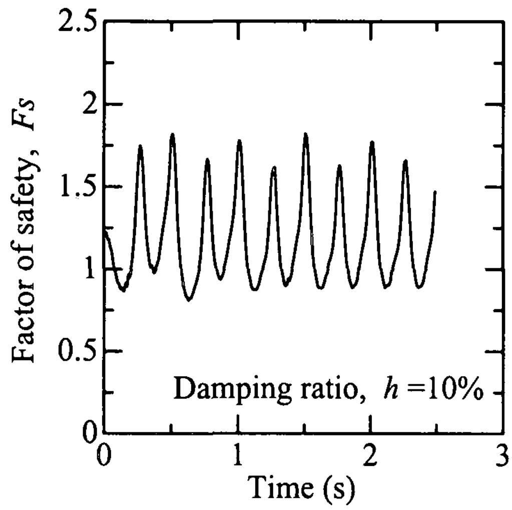

Fig. 8 Time dependent factor of safety for time period, $T = 0.25$ s.

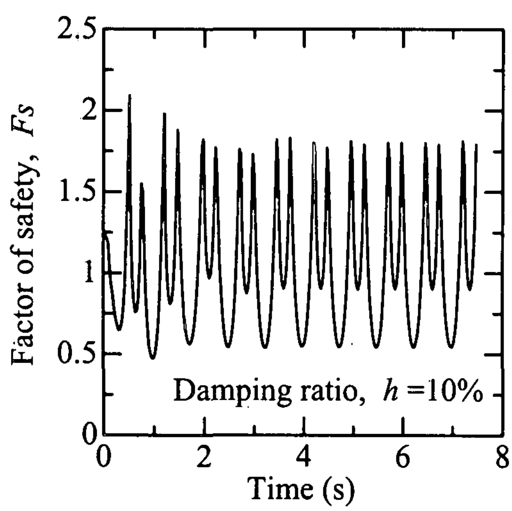

Fig. 9 Time dependent factor of safety for time period, $T = 0.75$ s.

4.2 Comparative Discussion with Shakedown Analysis

It has been shown that the proposed method can evaluate the seismic stability of slope by taking account of dynamic properties of earthquake and slope mass. However, the combined method of deformation and stability analyses could also consider the similar aspects of dynamic properties in safety factor assessment. In order to investigate the relationship in factor of safety between the proposed and combined methods, a comparison between them is conducted.

Table 4 shows the comparison in factor of safety between two methods. The damping ratio is taken as $h = 10\%$ and 20%. In the combined method, the minimum factor of safety in overall time duration is assumed as the total factor of safety as shown in the table. It is readily seen that the proposed method gives the lower factors of safety in comparison with the combined method for both time periods of $T = 0.25$ s and 0.75 s. It is because the combined method can not consider the overall time duration factor of safety directly. Besides the effect of alternating magnitude and direction in excitation is negligible. However, the difference in factor of safety between these methods is found not so large. Considering the effect of alternating acceleration on factor of safety is at most 5%, it can be said that the combined method gives a good estimation on factor of safety in case of natural slope.

Table 4 Comparison between combined and shakedown analyses for slope stability.

	Factor of safety, Fs			
	Shakedown Analysis		Combined Analysis	
T (s)	$h = 10\%$	$h = 20\%$	$h = 10\%$	$h = 20\%$
0.25	0.754	0.857	0.810	0.902
0.75	0.434	0.536	0.473	0.560

It is important to note, even though the effect of alternating acceleration is not significant in case of natural slope, such effect appeared significantly in case of embankment problem, as reported by Ohtsuka at al.[12]. It is easy to understand why such effect does not appear significantly in case of natural slope, since the deformation due to excitation is restricted to one side only (toward slope surface). But, on the embankment case, the deformation is freely oscillated to the both sides of slope.

In order to make clear the effect of alternating acceleration on the embankment problem, the combined analysis was applied to assess the seismic stability of embankment in the same case study with Ohtsuka et al.[12]. By taking damping ratio $h = 10\%$, the obtained factor of safety in time domain is shown in **Fig. 10** for period $T = 8$ s. The minimum factor of safety was obtained as $Fs = 1.621$. Since the fundamental natural period of embankment is 8.1 s, this factor of safety is expected to be near with the resonant condition. For the same case study of embankment, the comparison between shakedown and combined analyses is shown in **Table 5**. This table indicates that both analyses assess the different factors of safety. It confirms the significant effect of alternating acceleration on the stability of embankment, since the factor of safety obtained by shakedown analysis are far lower than combined analysis.

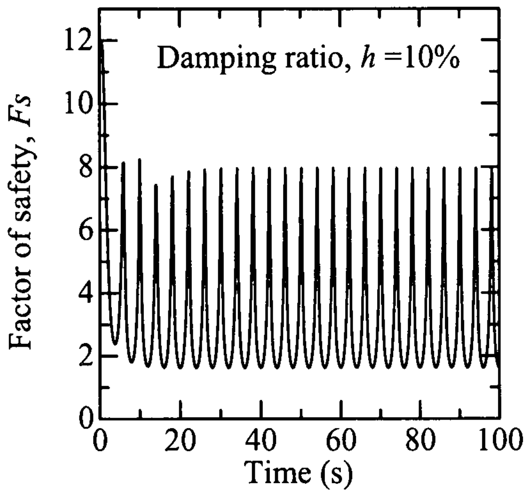

Fig. 10 Time dependent factor of safety for time period, $T = 8$ s.

Table 5 Comparison between combined and shakedown analyses for embankment.

	Factor of safety, Fs			
	Shakedown Analysis		Combined Analysis	
T (s)	$h = 10\%$	$h = 20\%$	$h = 10\%$	$h = 20\%$
4.0	3.600	4.860	4.150	5.204
8.0	1.104	2.166	1.621	3.127

5. Conclusions

This study proposed the new procedure for static and seismic stability analysis of slope based on the shakedown theorem in plasticity. In slope stability assessment, the conventional factor of safety has been defined as a reduction coefficient in shear strength against the limit state at failure. The proposed procedure could evaluate the factor of safety in the same way with the conventional way.

The following results were obtained in this study:

1. The applicability of the proposed stability analysis was firstly examined through the comparison with the conventional methods of Taylor[15] and Chen[3] under the static condition. The factor of safety was investigated for various conditions of geometry and soil parameters for slope. The obtained results were fairly coincident with those of the conventional methods.
2. The seismic stability of slope has been evaluated based on the pseudo-static concept. With the superior property of shakedown theorem, the effect of repeated alternating magnitude and direction in possible seismic coefficient on the slope stability was taken into consideration. Ohtsuka et al.[12] reported that this effect was significant and attains to at most 30% of factor of safety for embankment problem. The computed results for natural slope, however, remained almost the same factors of safety (at most 5%) with the conventional methods. It was because the natural slope usually failed downward, which made the effect of alternating seismic coefficient as lower. It is still noted that this effect will get greater as the expansion in possible seismic coefficient domain.
3. The seismic stability analysis of slope was developed with strength reduction method. It could estimate the factor of safety by taking account of earthquake record and vibration property of slope mass. The applicability of the proposed method to seismic stability assessment was examined through the case study for various frequencies in sine wave acceleration excited at bedrock of subsoil. The factor of safety was obtained dependent of frequency and varied from that of pseudo-static stability analysis to that of static stability analysis.
4. The applicability of combined method was investigated through the case study. The combined method gave the time dependent factor of safety, the meaning of which had been left to be clarified. However, in comparison with the proposed method, the minimum factor of safety by combined method in overall time duration was found to give a good estimation on seismic factor of safety in case of natural slope. But, in case of embankment problem, the factor of safety obtained by the proposed method was far lower than the combined method. It confirms that the combined method ignored the effect of alternating loads, which may generate on the embankment during an earthquake.
5. In case of natural slope, even though the combined method gave the almost same factor of safety with the proposed method, it

is still a distinguishing of computation time between them. The combined method conducts the computation of safety factor for every moment in the time duration of excitation to find a minimum safety factor for one period of excitation. It means that the computation of safety factor should be done in hundreds times for only one period of excitation. Meanwhile, the proposed method conducts the computation of safety factor once only for one period of excitation. Thus, the proposed method saves the computation time much more than the combined method.

6. The change in stiffness and damping ratio of soils during the excitation should be introduced into the proposed method to assess more realistic stability of soil structures. It is left to the further subject of this research.

ACKNOWLEDGMENT

This research was financially supported by grant-in-aid for Scientific Research on Priority Areas (B) from The Ministry of Education, Science, Sports and Culture of Japanese Government. It is gratefully acknowledged.

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(Received April 21, 2000)