Evolution of water surface elevation induced by seabed topography

Keywords: wave, sea bed, erosion

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Introduction

Rarely can one find a body of water opens to the atmosphere that does not have waves on its surface. These waves are a manifestation of forces acting on the fluid tending to deform it against the action of gravity and surface tension, which together act to maintain a level fluid surface (G. Dean & A. Dalrymple, 2002). It is well known that water waves propagating on the ocean can create significant dynamic pressure in non-cohesive marine sediments (D.S. Jeng, 1996). The coastline responds to various forcing mechanisms that provide the energy to drive the littoral processes. The prediction of areas eventually affected by coastal erosion is needed to fight against disasters. This study discusses the evolution of water surface elevation induced by a seabed topography. The initial wave and the shape of the bed are represented by mathematical relations so that it is easy to change parameters such as the wave amplitude and the bed's shape. The aim of this study is to develop an integrated model, based on COMSOL Multiphysics, for a better understanding of wave motion induced by seabed topography.

Methods

Two cases will be considered: a variable bed and a flat bottom. The Saint-Venant's shallow water equations is used in this example (Comsol Application Gallery). The equations are given by:

$$\frac{\partial z}{dt} + \nabla \cdot (zv) = 0 \qquad \frac{\partial (zv)}{dt} + \nabla \cdot (zv \cdot v) + gz\nabla z - \nu\Delta(zv) = 0$$

where z is the thickness of the water layer, v is the horizontal velocity, g the gravitational constant and ν the kinematic viscosity. The initial condition is a wave profile, which defines the following expression:

$$Z_0 = 0.02 - Z_f + 0.005e^{-(x-3)^2}$$

where Z_f is the analytical expression for the sea bed profile (**Figure 1**). The elevation of the water surface is $z+zf$.
The assumption of a rigid and impermeable seabed is assumed in both cases. The expression of the sea bed profile is:

$$Z_f = ae^{-(x-x_0)^2} + k_1 x$$

where X_0 is the sea bed ridge position, k_1 is the sea bed slope parameter and a is the sea bed ridge height.

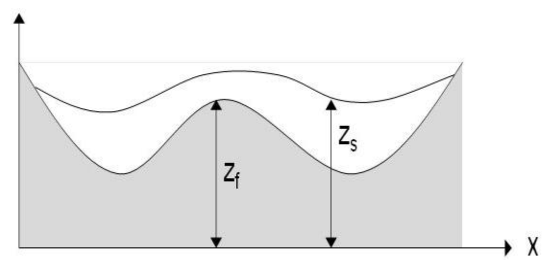
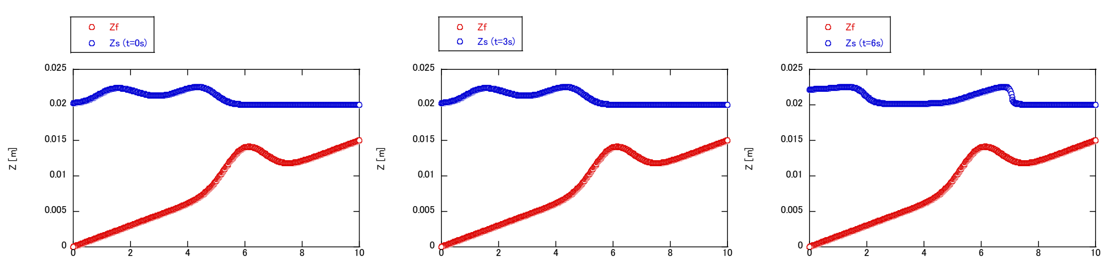


Figure 1 Vertical section through the fluid domain showing the bed of a lake and the water surface (Comsol Application Gallery).

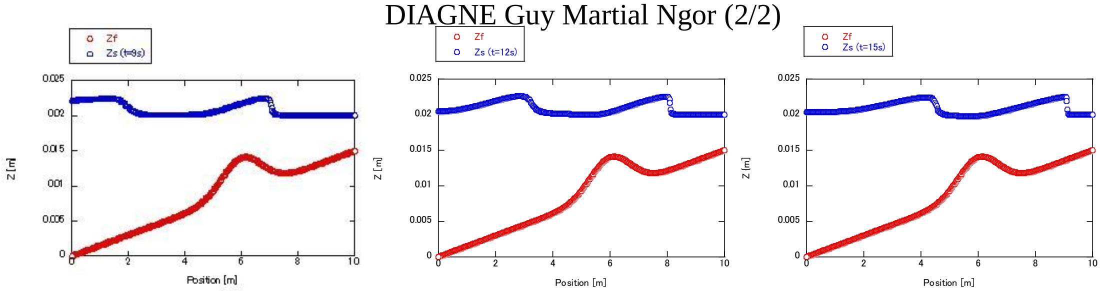
The modelisation is conducted by COMSOL Multiphysics (5.3a version). The main features of COMSOL Multiphysics adopted to set up the model are: (1) 1D space dimension, (2) General Form PDE(g) to set up the differents equations.

Results

First the simulation is runned for the variable sea bed and it runs for 60 seconds. The following figures show the water surface elevation and slope of the sea bed at six output times toward the biginning of the simulation (**Figure 2**).



(a) Profile at t=0s (b) Profile at t=3s (c) Profile at t=6s

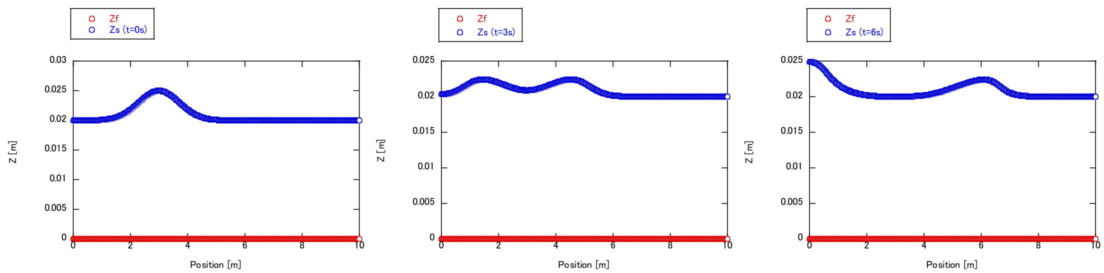


(d) Profile at t=9s (e) Profile at t=12s (f) Profile at t=15s

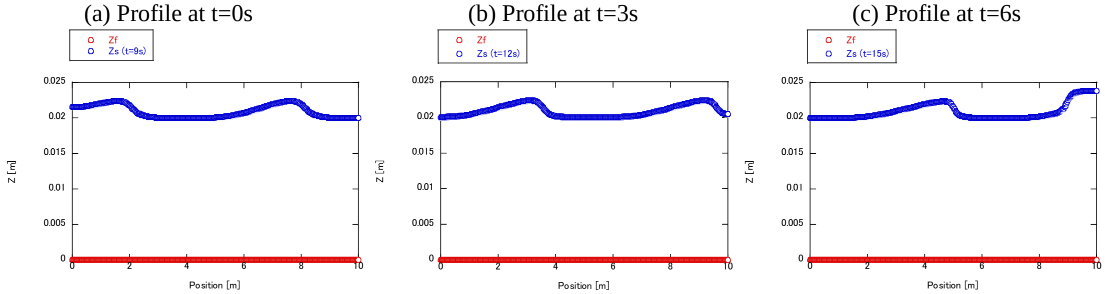
Figure 2. The water level and the slope of the sea bed at six output times

The simulation clearly shows the influence of the topography of the sea bed on the elevation of the water surface. For most of the cases, the elevation of water decreases beyond the crest of the seabed. The maximum values are located at t = 12s and they reach an altitude of 0.0226m.

Second, the simulation is runned for a flat bottom with the same time (**Figure 3**). In that case, the sea bed ridge height (*a*)and the seabed slope parameter (k_1) are equals to zero.



(a) Profile at t=0s (b) Profile at t=3s (c) Profile at t=6s



(d) Profile at t=0s (e) Profile at t=3s (f) Profile at t=6s

Figure 3. The water level and the flat bottom of the sea bed at six output times

In this situation, the bed is setted as invariable and the results show a variation of the water surface elevation. The absence of a crest at the seabed, increases the water heights at certain intervals of time. For example at t=6s, the water level is reaching 0.025m at the zero position.

Conclusion

An integrated model, based on COMSOL Multiphysics, has been developed to study the evolution of water surface elevation induced by seabed topography. The results obtained between both case are different and we can notice that the seabed topography can have a positive impact on the altitude of the water. For a better appreciation, sea bed topography must be included in the simulations. In this paper, only the results for water profil are presented. More detailed parametric studies will be conducted in the future

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