

# New Multiaxial Fatigue Model to Estimate Life of Steel Bridges due to High and Low Amplitude Loadings

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## 1. Introduction

High cycle fatigue (HCF) caused by low amplitude loading has been cited as a major cause of structure failures. However, a number of fatigue failures has been reported in the past that cannot be explained by HCF. Studies on these failures reveal that high amplitude loading such as earthquakes, cyclones is one of the reasons for these failures. During extreme loadings, some members may be subjected to stresses in plastic range causing low cycle fatigue (LCF) damage. This may lead to sudden failures. The commonly used approach of damage prediction is based on von Mises strain, Coffin-Manson strain-life relationship with Miner's rule (Suresh 1998). However, von Mises strain does not correctly represent the fatigue behavior in multiaxial HCF and LCF. Also, HCF and LCF interaction is not correctly given by Coffin Manson curve. Further, Miner's rule does not predict correct results in variable amplitude loadings since it cannot capture the loading sequence effect. Therefore, the objective of the paper is to propose a new multiaxial fatigue model to predict life of structures including bridges caused by high and low amplitude loadings. Initially, the proposed model is introduced. Then, verification of the proposed model is explained with experimental test results of two materials obtained from the literature.

## 2. Proposed fatigue model

The damage variable is based on modified von Mises strain as shown below.

$$\varepsilon_{eq} = (1 + \alpha\phi)(1 + k \sin \varphi)\varepsilon_{VM} \quad (1)$$

where  $\varepsilon_{eq}$  is the equivalent strain,  $\alpha$  and  $k$  are the material parameters. The parameters,  $\phi$  and  $\varphi$ , depend on loading arrangements.  $\varepsilon_{VM}$  is the von Mises strain amplitude.

### 2.1. Strain-life curve

The proposed strain curve consists of two parts as shown in Fig. 1. The first part corresponds fatigue life of plastic strain cycles ( $\varepsilon_{eq} \geq \varepsilon_y$ ) which usually affects LCF. To describe this part, Coffin-Manson strain-life curve is utilized as shown below.

$$\varepsilon_{eq} = \frac{\sigma_f'}{E}(2N)^b + \varepsilon_f'(2N)^c \quad (2)$$

where  $N$  is the number of cycles to failure,  $\sigma_f'$  is the fatigue strength coefficient,  $b$  is the fatigue strength exponent,  $\varepsilon_f'$  is the fatigue ductility coefficient,  $c$  is the fatigue ductility exponent and  $E$  is the elastic modulus of the material.

The second part describes the fatigue life of elastic

strain cycles ( $\varepsilon_{eq} < \varepsilon_y$ ) which usually affects HCF. This part of curve represents hypothetical fully known curve. The shape of the curve is obtained by directly transforming the previous fully known stress-life curve to elastic strain-life curve (Siriwardane et al. 2008) as shown below.

$$\varepsilon_{eq} = \varepsilon_e \left( \frac{N + N_u}{N + N_e} \right)^{b'} \quad (3)$$

where  $\varepsilon_e$  is the strain amplitude of the fatigue limit,  $N_e$  is the fatigue life at  $\varepsilon_e$ . The  $\varepsilon_y$  and  $N_y$  are the yield strain and the corresponding number of cycles to failure. The  $b'$  is the slope of the finite life region.

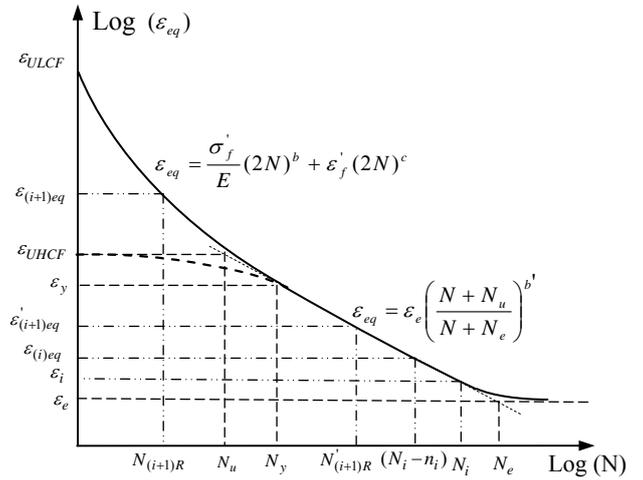


Fig. 1. Proposed strain-life curve

### 2.2. Damage indicator

Suppose a component is subjected to a certain equivalent strain amplitude ( $\varepsilon_i$ ) of  $n_i$  number of cycles at load level  $i$ ,  $N_i$  is the fatigue life (number of cycles to failure) corresponding to ( $\varepsilon_i$ ) (Fig. 1). Therefore, the reduced life at the load level  $i$  is obtained as  $(N_i - n_i)$ . The equivalent strain ( $\varepsilon_{(i)eq}$ ) (Fig. 1), which corresponds to the failure life  $(N_i - n_i)$  is defined as  $i^{\text{th}}$  level damage equivalent strain. Hence, the new damage indicator,  $D_i$  is stated as below.

$$D_i = \frac{(\varepsilon_{(i)eq} - (\varepsilon)_i)}{(\varepsilon)_u - (\varepsilon)_i} \quad (4)$$

and  $(\varepsilon)_u$  is expressed

$$(\varepsilon)_u = \begin{cases} \varepsilon_{ULCF} & (\varepsilon)_i \geq \varepsilon_y \\ \varepsilon_{UHCF} & (\varepsilon)_i < \varepsilon_y \end{cases} \quad (5)$$

Assuming the end of  $i^{\text{th}}$  loading level, damage  $D_i$  has been accumulated (occurred) due to the effect of

$(\varepsilon)_{i+1}$  loading cycles, the damage is transformed to load level  $i+1$  as below.

$$D_i = \frac{(\varepsilon)'_{(i+1)eq} - (\varepsilon)_{i+1}}{(\varepsilon)_u - (\varepsilon)_{i+1}} \quad (6)$$

Then,  $(\varepsilon)'_{(i+1)eq}$  is the damage equivalent strain at loading level  $i+1$  is calculated. Then, the corresponding equivalent number of cycles to failure  $N'_{(i+1)R}$  is obtained from the strain-life curve as shown in Fig. 1. From that, the corresponding residual life at load level  $i+1$  (number of cycles is  $n_i+1$ ),  $N_{(i+1)R}$  is calculated as,

$$N_{(i+1)R} = N'_{(i+1)R} - n_{(i+1)} \quad (7)$$

Therefore, strain  $(\varepsilon)_{(i+1)eq}$ , which corresponds to  $N_{(i+1)R}$  at load level  $i+1$ , is obtained from the strain-life curve as shown in Fig. 1. Then the cumulative damage at the end of load level  $i+1$  is defined as,

$$D_{(i+1)} = \frac{(\varepsilon)_{(i+1)eq} - (\varepsilon)_{i+1}}{(\varepsilon)_u - (\varepsilon)_{i+1}} \quad (8)$$

At the first cycle the equivalent strain  $(\varepsilon)_{(i)eq}$  is equal to  $(\varepsilon)_i$  and the corresponding damage indicator becomes  $D_i=0$ . Similarly at the last cycle, the damage indicator becomes  $D_i=1$  when  $(\varepsilon)_{(i)eq}$  is equal to  $(\varepsilon)_u$ . Therefore, the damage indicator is normalized to one ( $D_i=1$ ) at the fatigue failure of the material. Hence, the above procedure is followed until  $D_i=1$ .

### 3. Verification of the proposed fatigue model

Seventeen fatigue tests of Haynes 188 conducted by Bonacuse and Kalluri (2002) were used to verify the proposed model as shown in Fig. 2.

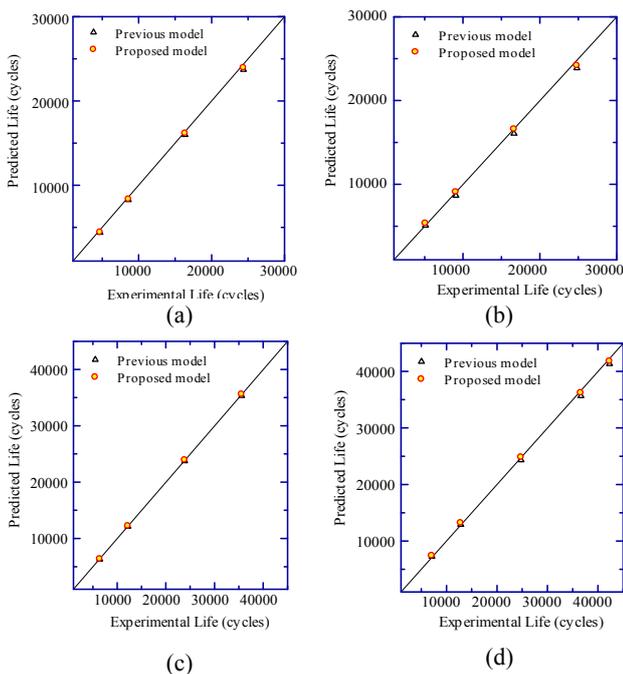


Fig. 2. Comparison of predicted and experimental lives for Haynes 188 (a) AA (b) AT (C) TA (d) TT tests

There, axial (A) and torsional (T) loadings were used in four different sequences (AA, AT, TA and TT). Then experimental lives were compared with the proposed model predictions. In addition, Miner's rule based previous model was also used to predict the results. The results are graphically shown in Fig. 2. Percentage variations of the previous and proposed models were compared with the experimental results as 0.73% and 0.64%. Therefore, proposed model predicts more accurate results than the previous model.

Secondly, eleven fatigue tests of S45C steel conducted by Chen et al. (2006) were used to verify the proposed model. There, axial (A) and torsional (T) loadings were used in two different sequences (AT and TA). The obtained results are shown in Fig. 3. Failure number of cycles of these tests was predicted by the proposed model. In addition, Miner's rule employed previous model was used to predict the number of cycles to failure. The obtained results are plotted in Fig. 3.

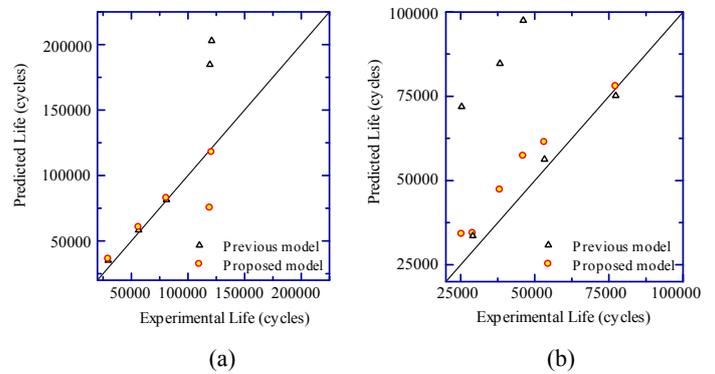


Fig. 3. Comparison of predicted and experimental lives for S45C steel (a) AT (b) TA tests

Percentage variations of the previous and proposed models were compared with the experimental results as 23.9% and 6.3%. Therefore, proposed model predicts more accurate results than the previous model.

### 4. Conclusions

New multiaxial fatigue model was proposed to estimate fatigue life due to high and low amplitude loadings. Verification of the model was conducted by comparing the predicted lives with experimental lives of two materials. Proposed model can successfully be utilized in life predictions of structures such bridges in high and low amplitude loading situations.

### References

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