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1. Introduction

Many of the bridge infrastructure of world are getting older and a large number these structures are in need of maintenance, rehabilitation or replacement. Hence, condition estimation and proper maintenance of bridges have a widespread significance for continuous research. In recent years, structural reliability analysis has increasingly demonstrated an important role in structural system analysis and design in bridge engineering. In spite of all the merits with reliability concepts and seemingly sound logic for its use, widespread acceptance by the design community has not occurred. This is because precise estimation of individual element reliability and system reliability is almost impossible in many practical situations because of many uncertainties from many sources, which can be classified into three broad categories (Christensen and Murotsu 1986): physical uncertainty, model uncertainty, and statistical uncertainty. In contrary to structural reliability, the concept known as the interval analysis is fundamentally a non probabilistic approach. It deals with mathematical operations numbers which are having a lower and an upper bound (Qiu et al. 2008).

The objective of this paper is to combine structural reliability with interval analysis to overcome some of present problems in condition assessment of bridges. In this study, statistical parameters of variables are considered to lie in a range of suitable values. This layout of interval based reliability is expected to upgrade the accuracy of present reliability analysis in bridge engineering and increase confidence among practicing engineers.

2. Methodology

Depending on the bridge type, a finite number of critical failure modes can be introduced. For these failure modes, safety margins can be introduced (Christensen and Murotsu 1986). Basically, a safety margin consists of a resistance (strength) variable and a load variable. The means (μ_R, μ_S) and the standard deviations (σ_R, σ_S) of resistance and load variables can be found by experimental and structural analysis. In reality, there is a possibility that all these values are positioned in a range or as interval numbers with a lower bound and an upper bound. This can be expressed as for the resistance variable in Eq. (1) and load variable in Eq. (2) as follows

$$\mu_R = [\underline{\mu}_R, \bar{\mu}_R] \quad \sigma_R = [\underline{\sigma}_R, \bar{\sigma}_R] \quad (1)$$

$$\mu_S = [\underline{\mu}_S, \bar{\mu}_S] \quad \sigma_S = [\underline{\sigma}_S, \bar{\sigma}_S] \quad (2)$$

In above, lower and upper strokes denote lower and upper bounds of respective statistical parameters. When variables follow normal distribution, the upper and the lower of the reliability index ($\bar{\beta}, \underline{\beta}$) can be expressed as follows

$$\bar{\beta} = \frac{(\bar{\mu}_R - \bar{\mu}_S)}{\sqrt{(\bar{\sigma}_R^2 + \bar{\sigma}_S^2)}} \quad (3)$$

$$\underline{\beta} = \frac{(\underline{\mu}_R - \underline{\mu}_S)}{\sqrt{(\underline{\sigma}_R^2 + \underline{\sigma}_S^2)}} \quad (4)$$

When both variables follow log normal distribution, upper and lower bounds of reliability index can be expressed as

$$\bar{\beta} = \frac{\ln\left(\frac{\bar{\mu}_S \times \sqrt{\frac{COV_R^2 + 1}{COV_S^2 + 1}}}{\bar{\mu}_R}\right)}{\sqrt{\ln((COV_R^2 + 1) \times (COV_S^2 + 1))}} \quad (5)$$

$$\underline{\beta} = \frac{\ln\left(\frac{\underline{\mu}_S \times \sqrt{\frac{COV_R^2 + 1}{COV_S^2 + 1}}}{\underline{\mu}_R}\right)}{\sqrt{\ln((COV_R^2 + 1) \times (COV_S^2 + 1))}} \quad (6)$$

where COV_R and COV_S represent for coefficient of variations of resistance and load variables. The reliability index can be used to estimate failure probability. Then, the lower and upper bound of failure probabilities ($\underline{P}_f, \bar{P}_f$) can be expressed as

$$\underline{P}_f = \Phi(-\bar{\beta}) \quad (7)$$

$$\bar{P}_f = \Phi(-\underline{\beta}) \quad (8)$$

where Φ is the standard unit normal distribution.

3. Case Study

Four span brick masonry arch bridge, constructed in 1833, is selected from Sri Lanka as a case study. The selected bridge (No. 90/1) is located in the route A1. Arch barrels and spandrel walls of all four spans were built of brick masonry. Piers and abutments were built of dressed Granite stones. A side view of the bridge is shown in Fig. 1 and the geometric details of the bridge are given in Table 2. In this study, resistance variable is considered as provisional axle load (PAL) and load variable is considered as actual axle load (AAL).



Fig. 1. A side view of the A1 90/1 bridge

Table 1: Geometric details of the bridge

Geometric parameter	Value
Bridge length (L_b)	70 m
Clear span of an arch (L)	15 m
Thickness of the barrel (d)	1.4 m
Height of the compacted fill from the crest of the barrel (h)	1.05 m
Rise of the arch at mid span (r_c)	4.20 m
Number of arches (n)	4

Six modifying factors are estimated by referring tables and figures in UIC code (UIC 1995). Finally, the mean of PAL of outer and inner arches are estimated as given in Table 2.

Table 2: Modifying factors and mean of PAL for route A1 90/1 bridge

Modifying factor	Outer arch	Inner arch
Arch shape factor	1.0	1.0
Material factor	1.0	1.0
Joint factor	1.0	1.0
Condition factor	1.0	1.0
Number of spans factor	0.9	0.8
Dynamic factor	1.25	1.25
Mean of PAL (kN)	720	640

According to a previous study (Frangopol 1999), coefficient of variation (COV) of PAL is considered as 0.10. Mean of AAL is obtained from Road Development Authority of Sri Lanka as 85.5 kN. Referring the same literature, coefficient of variation (COV) of AAL is considered as 0.3 since live loading has more variation.

From these statistical parameters, failure probabilities of the arches were estimated. Then, using series system, reliability index of the bridge is estimated for nine possible cases when variables follow normal distribution as shown in Table 3.

Table 3: Change of the reliability index of the bridge for normally distributed variables

Case	Description	Reliability index	% of change in reliability index
I	No change	7.99	-
II	10% increase of mean of PAL	8.92	11.7
III	10% decrease of mean of PAL	7.05	-10.5
IV	10% increase of mean of AAL	7.87	-1.5
V	10% decrease of mean of AAL	8.12	1.6
VI	10% increase of S.D. of PAL	7.34	-8.1
VII	10% decrease of S.D. of PAL	8.75	9.5
VIII	10% increase of S.D. of AAL	7.88	-1.4
IX	10% decrease of S.D. of AAL	8.10	1.3

(S.D. = Standard deviation)

Table 4: Change of the reliability index of the bridge for log normally distributed variables

Case	Description	Reliability index	% of change of reliability index
I	No change	6.56	-
II	10% increase of mean of PAL	6.93	5.7
III	10% decrease of mean of PAL	6.13	-6.1
IV	10% increase of mean of AAL	6.80	3.6
V	10% decrease of mean of AAL	6.35	-3.2
VI	10% increase of S.D. of PAL	6.48	-1.1
VII	10% decrease of S.D. of PAL	6.62	1.1
VIII	10% increase of S.D. of AAL	6.05	-7.7
IX	10% decrease of S.D. of AAL	7.16	9.2

(S.D. = Standard deviation)

From these results, it is clear that some of statistical parameters affect reliability index more than others. This observation is valid when PAL and AAL behave as normally distributed and log normally distributed variables. When variables follow normal probability distribution (Table 3), variation of mean of PAL affects the reliability index mostly. Further, variation of standard deviation of AAL has the least effect on reliability index of the bridge.

When variables follow log normal distribution (Table 4), change of standard deviation of AAL load affects mostly while change in standard deviation of PAL has the least effect on reliability index of the bridge.

4. Conclusions

Interval reliability analysis is applied to condition assessment of bridges when their strength and load variables follow normal distribution and log normal distribution respectively. Practical applicability of the introduced condition assessment is checked with a case study. In both situations, critical statistical parameter which affects mostly in terms of reliability indices was identified. Hence, in estimating statistical parameters, possible scenarios of reliability index variation can be visualized.

References

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