Consideration of Non Normality in Reliability Based Condition Prediction of Bridges

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1 Introduction

Road infrastructure has been an integral part of human civilization from ancient times. The quality and efficiency of a transportation system of a given country is an index of its level of economic stand in the world. Everywhere around the world, there is a need for significant investment to repair civil infrastructures, which are deteriorating under heavy use, severe exposure conditions, and age (Atashi et al., 2007). Among these, roads and railway transportation associated structures play an important role in the ground transportation systems and it constitutes a considerable investment in infrastructure and directly affects the productive capacity of a country.

Bridges are the most critical yet vulnerable elements in highway transportation networks. To upkeep with present progress of a society, there is a need to attend the issue of condition estimation and proper maintenance of bridges. The study of bridges has different areas of research. Above all, maintenance of existing bridge infrastructure has significant research interest since construction of new bridges involves with huge resources expenditures. Keeping the bridges as many years as possible will release the burden on bridge authorities.

In reliability based condition evaluation of bridge, one thing that bridge engineers faces is that consideration of non normally variables in the reliability expressions. The estimate of the reliability of a structural system is greatly facilitated if all variables can be assumed as Gaussian or normally distributed. In such a case, the only information needed is mean and standard deviation of the variable. This is because available procedures contain normal variables. Although, the assumption of normality yields results of certain accuracy, consideration of non normality of variables is essential and beneficial in correct condition prediction of bridges. That will be useful for the bridge engineers who are seeking better ways of expending allocated resources in bridge maintenance.

In some situations in reliability expressions, log normal distributions is used for modelling resistance variables as it has the advantage of precluding negative values. In consideration of load variables, it is usually necessary to distinguish between permanent and variable loads. Permanent loads such as self weight can be modelled by normally distributed variables and variable loads are likely to be very closely approximated by an asymptotic extreme value distribution.

In this paper, a methodology is introduced how to convert non-normal variables with equivalent normal variables. Those equivalent normal variables are then inserted in to reliability based expressions and having done that reliability index and failure probability can be found. This procedure can be extended to system level and thereby system reliability index of the concern bridge can be found. As such with this procedure, more accurate predictions of conditions of bridges can be made.

2 Methodology

2.1 General reliability based procedure of condition prediction of bridges

Bridges can failure under different types of failure modes. The critical types of failure modes for a selected bridge can be different from one type of bridge to another. From these failure modes, critical failure modes are selected and for those failure modes, reliability based safety margins can be proposed.

If a bridge has n number of critical failure modes, for them, following reliability expressions can be proposed as shown in the Eq. (1).

$$M_i = Z_{R_i} - Z_{S_i} \qquad i = 1,2...,n \tag{1}$$

Where M_i is the safety margin for i^{th} mode of failure of the bridge. Z_{R_i} is the strength variable and Z_{S_i} is the load variable.

Using the fundamentals of structural reliability theory, the reliability index for i^{th} failure mode can be expressed as indicated by authors (Christensen and Baker, 1982; Christensen and Murotsu 1986),

$$\beta_i = \left(\mu_{ZRi} - \mu_{ZSi}\right) / \sqrt{\left(\sigma^2_{Z_{Ri}} + \sigma^2_{Z_{S_i}}\right)} \qquad i = 1, ,n \tag{2}$$

Elementary failure probability for this mode of failure P_{f_i} can be found as in the Eq. (4). Where ϕ is the standard unit normal distribution function.

$$P_{f_i} = \phi(-\beta_i) \quad i = 1,,n \tag{3}$$

$$P_{f_i} = 1 - \phi\left[\left(\mu_{Z R_i} - \mu_{Z S_i}\right) / \sqrt{\left(\sigma_{Z_{R_i}}^2 + \sigma_{Z_{S_i}}^2\right)}\right] \tag{4}$$

These elementary failure probabilities can be incorporated in to system by using simple bounds as shown in Eq. (5).

$$\underset{i=1}{\overset{n}{Max}}\, P_{f_i} \le P_F \le 1 - \prod_{i=1}^{n} \left(1 - P_{f_i}\right) \tag{5}$$

2.2 Consideration of non-normality of variables

Above procedure can be applied if the variables are normally distributed. If variables are not normally distributed or follow a different probabilistic distribution, then above procedure can not be used. In such case, it is possible to transform original distribution and density functions of resistance and load variables in to equivalent normal distribution function and equivalent normal density functions of resistance and load variables at a considered point.

Assume a fundamental system with two uncorrelated basic variables; resistance variable and load variable. Let mean and standard deviations of variables are μ_R, σ_R, μ_S and σ_S.

Then define the values (s^*, r^*) as follows as shown in Eq. (6) and (7),

$$r^* = \mu_R - \alpha_R \sigma_R \tag{6}$$

$$s^* = \mu_S + \alpha_S \sigma_S \tag{7}$$

The values α_R and α_S are multiplication factors for resistance and load variables. The physical meaning of point (s^*, r^*) is that it the point where original distribution and density function equal to normalized distribution and density functions. This can be mathematically represented in the following Eq. (8) to (11). In notation, $F_R(r^*)$ and $f_R(r^*)$ are original distribution and density functions of resistance variable. Similarly, $F_S(s^*)$ and $f_S(s^*)$ are original distribution and density function of load variable.

$$F_R(r^*) = \phi\left[\left(r^* - \mu_R^1\right) / \sigma_R^1\right] \tag{8}$$

$$f_R(r^*) = 1/\sigma_R^1 \times \varphi\left[\left(r^* - \mu_R^1\right) / \sigma_R^1\right] \tag{9}$$

$$F_S(s^*) = \phi\left[\left(s^* - \mu_S^1\right) / \sigma_S^1\right] \tag{10}$$

$$f_s(s^*) = 1/\sigma_S^1 \times \varphi\left[\left(s^* - \mu_S^1\right) / \sigma_S^1\right] \tag{11}$$

In this case, μ_R^1, σ_R^1, μ_S^1, σ_S^1 are unknowns of the equivalent normal distributions of variables; R^1 and S^1. Above variables should be found in order to describe the non-normal variables as shown in Eq. (12) to (15).

$$\sigma_R^1 = \varphi\left[\phi^{-1}\left(F_R(r^*)\right)\right] / f_R(r^*) \tag{12}$$

$$\mu_R^1 = r^* - \phi^{-1}\left(F_R(r^*)\right)\sigma_R^1 \tag{13}$$

Similarly, for load variables, the parameters of the equivalent normal variables can be found as shown below.

$$\sigma_S^1 = \varphi\left[\phi^{-1}\left(F_S(s^*)\right)\right] / f_S(s^*) \tag{14}$$

$$\mu_S^1 = s^* - \phi^{-1}\left(F_S(s^*)\right)\sigma_S^1 \tag{15}$$

Having found, above variables (μ_R^1, σ_R^1, μ_S^1, σ_S^1) above values can be substituted in to Eq. (2) and reliability index and failure probability can be found as shown Eq. (2) and (4). Furthermore, methodology can be extended in to system model by considering other modes of failures as shown Eq. (5).

3 Conclusion

This paper proposes a methodology to count non normality of variables in structural reliability based condition prediction of bridges. By using the procedure mentioned in methodology section, non normal variables can be converted to equivalent normal variables and thereby the accuracy of the reliability calculations can be improved. With that more precise condition prediction of bridges can be obtained.

4 Reference

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